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A new multiscale prediction-error filter for sparse data interpolation

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ABSTRACT

Prediction-error filters (PEFs) have been used to successfully interpolate seismic data. Using conventional methods, PEFs often cannot be estimated on sparse, irregularly sampled data. We implement an algorithm in which we resample the data to various scales to estimate a single PEF. We show that compared to PEFs estimated from single data scales, our PEF provides a robust first guess for a nonlinear interpolation scheme.

INTRODUCTION

Data interpolation can be formulated as an inverse problem. If the data and model are in the same space, the data fitting goal is simply a mask operator, which guarantees that the model fits the data at the known points. The model is unconstrained where there are unknown data, leaving a family of possible models with a large nullspace. Unfortunately, there is an ambiguity in how to constrain the solution so that a reasonable result is produced. Claerbout (1999) suggests that the covariance of the known data can be used as the covariance of the model. The data covariance can be characterized by a prediction-error filter, which is estimated from the known data. In the second stage of this method, the PEF is then used to regularize the inversion and constrain the nullspace.

In some cases, the sparse data may be so sparsely sampled as to make conventional PEF estimation impossible. For the special case of interpolating between regularly sampled traces, Crawley (2000) spaces the coefficients of the PEF at multiple scales, and successfully interpolates aliased events. Fomel (2001) uses a nonlinear method to estimate dips within data. The method works well, but is computationally expensive. A test case for sparse data interpolation has been developed by Brown, et al. (2000), which consists of a single plane wave that is sparsely and irregularly sampled.

We develop a method that correctly interpolates a more difficult test case, and provides an overall strategy to interpolate sparse, irregular data when existing methods fail. To do this, we develop a PEF estimation scheme where a single PEF is estimated with multiple scales of regridded data, by simultaneously autoregressing for a common filter. We show that this method is more robust than estimating a PEF on a single scale of data, and provides more

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equations than estimating the PEF with multiple scales of filters. Once we estimate the PEF, we use the interpolated data as a starting guess for a nonlinear iterative method, which gives promising results. This method is then compared favorably to starting guesses based upon Laplacian interpolation and PEFs estimated from a single scale of data.

BACKGROUND

Sparse data interpolation is a very important problem in geophysics, due to the high cost associated with data collection. Irregular sampling introduces another level of complexity to the problem. Data interpolation can be implemented using a two stage linear method, the second of which is the minimization of the model when convolved with a filter. The first step is determining an appropriate filter, such as a prediction-error filter (PEF) (Claerbout, 1999).

A PEF can often be estimated by minimizing the output of convolution of an unknown filter with known data. The one dimensional case,

$$\mathbf{0} \quad \approx \quad \mathbf{r} = \begin{bmatrix} y_2 & y_1 & y_0 \\ y_3 & y_2 & y_1 \\ y_4 & y_3 & y_2 \\ y_5 & y_4 & y_3 \\ y_6 & y_5 & y_4 \end{bmatrix} \begin{bmatrix} 0 & \cdot & \cdot \\ \cdot & 1 & \cdot \\ \cdot & \cdot & 1 \end{bmatrix} \begin{bmatrix} 1 \\ a_1 \\ a_2 \end{bmatrix} + \begin{bmatrix} y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{bmatrix} \tag{1}$$

can easily be extended to multiple dimensions with the helical coordinate (Claerbout, 1998). Here a represents filter coefficients, y represents data, and $\mathbf{r}$ is the residual, which we minimize. When there are unknown data, the equations containing missing data are not used. Very sparse data complicates the issue, since there are no fitting equations which determine a PEF. Looking at the above equation, if y_2, y_3 and y_4 were missing, a PEF could not be determined.

A method to overcome the need for contiguous data is to stretch the filter (Crawley, 1998) so that the filter coefficients would fall upon data points. The stretching can be done at multiple scales, due the the scale-invariance of the PEF (Claerbout, 1999). This method works well as long as the data are regularly sampled, and the stretching is isotropic. If the data are irregularly sampled, the method fails, since the filter coefficients no longer fall upon data points, and again we are left without fitting equations.

$$\begin{array}{cccccccc} \cdot & a & \cdot & b & \cdot & c & \cdot & d \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \end{array}$$

A test case has been developed (Brown et al., 2000), which consists of a single plane wave oriented at 22.5^o, irregularly sampled. We extend this test case, and add complexity to it by adding another plane wave with a different frequency and orientation. In addition we add a substantial amount of Gaussian noise. Crawley's (1998) method of scaling the filter does not work for this case, as the irregular sampling leads to a lack of fitting equations. The sampling is much like the middle case in Figure 1 where the filter does not lie on known data, regardless of how much it is stretched.

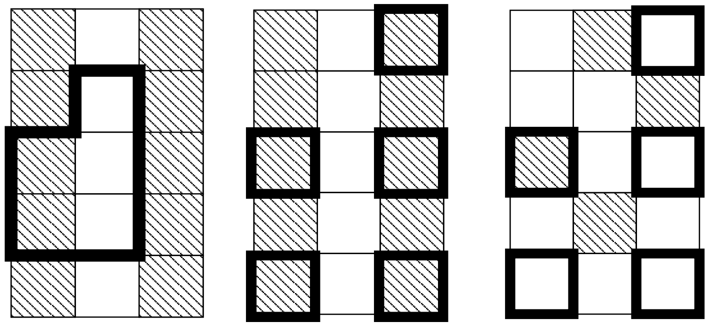

Figure 1: Figure of three cases for a regression equation. The diagonal lines represent gridded data, the white squares are empty bins, and the bold lines represent the PEF. Left: Original PEF on interlaced data, where an equation is not possible. Center: scale-expanded PEF on interlaced data, with a possible equation. Right: scale-expanded PEF on irregularly spaced data, no equation is possible. bill1-peffail [NR]

MULTISCALE PEF ESTIMATION

Instead of scaling the filter to fit the data, the data can be scaled so that it fits the filter. This is accomplished by a regridding algorithm, which we base upon normalized linear interpolation. The data can be regridded at multiple scales, so that the number of fitting equations increases. The success of this method is dependent upon the scale-invariance of the data instead of that of the PEF. If we assume stationarity, the regridding is an acceptable solution, until we get to very large bin sizes where adequate sampling becomes an issue.

The calculation of a multiscale PEF can be described as multiple simultaneous PEF estimations, one for each scale, with its own mask,

$$\begin{bmatrix} \mathbf{K_0 D} \\ \mathbf{K_1 S_1 D} \\ \mathbf{K_2 S_2 D} \\ \cdots \\ \mathbf{K_n S_n D} \end{bmatrix} \mathbf{f} = \begin{bmatrix} \mathbf{d} \\ \mathbf{S_1 d} \\ \mathbf{S_2 d} \\ \cdots \\ \mathbf{S_n d} \end{bmatrix}. \tag{2}$$

In equation (2), $\mathbf{D}$ signifies convolution with the data, $\mathbf{S_n}$ is a regridding matrix, which regrids to the nth scale. $\mathbf{K_n}$ is a weighting vector which is 1 where all data are present in the equation at the nth scale and 0 where there are missing data, $\mathbf{f}$ is the filter, and $\mathbf{d}$ is the data.

There are multiple benefits to this approach of estimating the PEF. First, the data can be sampled very irregularly. Secondly, the PEF scaling approach leads to a smaller number of scales that can be used than when the data are scaled. The PEF must be scaled by an integer value, so that the number of scales is constrained by the size of the data divided by the size of the PEF to be estimated. Conversely, if the data are scaled, this restriction is not present.

A simpler alternative would be to use a single scale of data, where there are sufficient

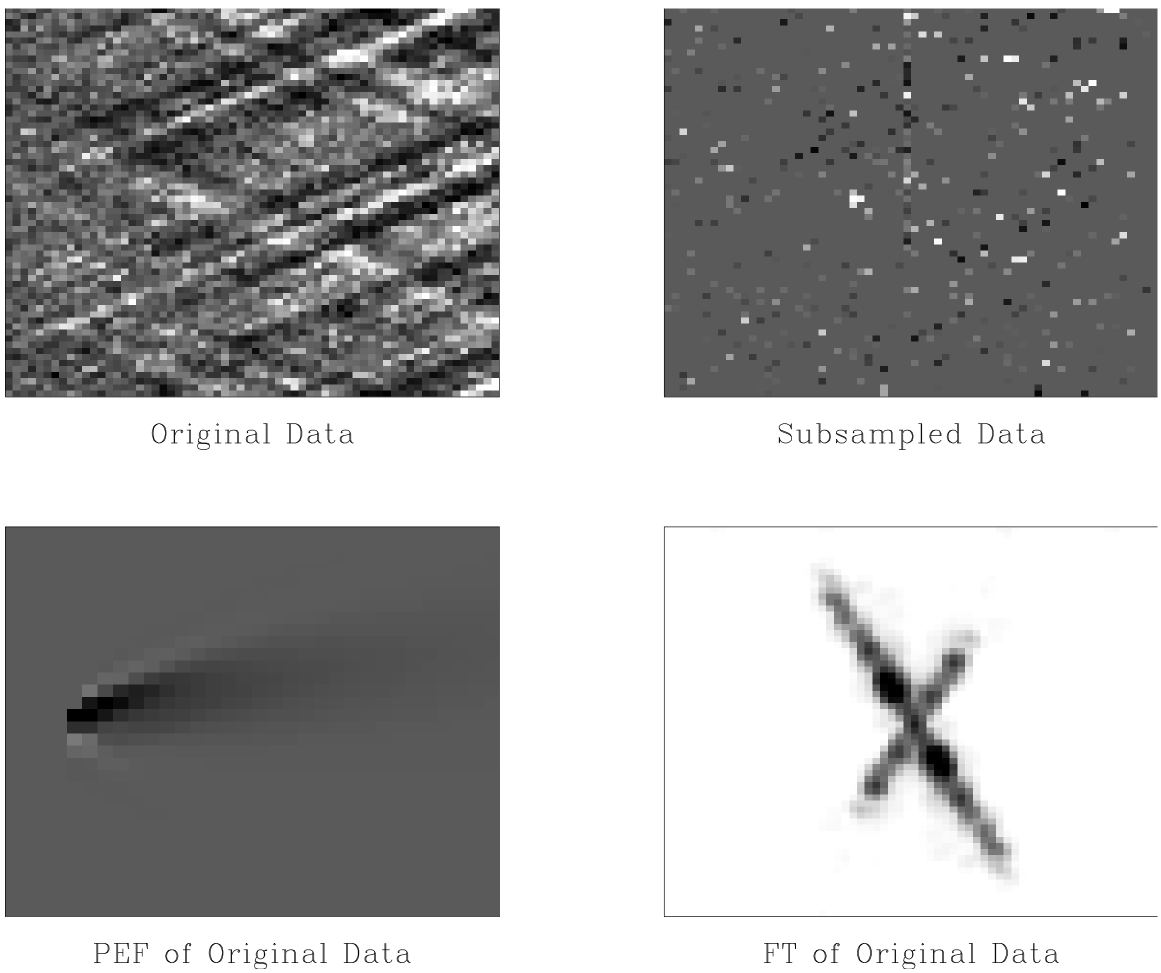

Figure 2: Clockwise from top left: Fully sampled test data; Sparsely sampled test data; Envelope of Fourier Transform of fully sampled test data, Inverse impulse response of PEF from all data. bill1-data [ER]

fitting equations to adequately constrain the estimation of the PEF. Unfortunately, while the PEF is theoretically scale-invariant, there is some variation in the estimated PEF from scale to scale, which makes the choice of scale a challenge, as shown in Figure 3. Also, using multiple scales allows for more fitting equations, which will better constrain the estimation and minimize the effect of erratic data.

There are several degrees of freedom with the multiscale approach. First of all, the size of the PEF can be changed to accommodate a range of situations. A two-column PEF can annihilate a single plane wave, a three-column PEF can annihilate two plane waves, and so on. The height of the PEF determines the dip of plane waves that can be annihilated. Also, the size of the PEF also determines the number of fitting equations for each PEF, with a larger PEF meaning not only that each coefficient is less constrained, but that overall there are less fitting equations for the entire system, due to edge effects and more equations containing missing data. In this case a 5x3 PEF was chosen, so that it can eliminate two plane waves with dips ranging from $+63^o$ to -90^o.

Another degree of freedom is the choice of scales. Certain large scales do not have enough equations to adequately constrain the PEF, and the PEF becomes unstable. Furthermore, at large scales the normalized linear interpolation returns distorted data, where the data are copied into nearby bins. Conversely, there becomes a point where the size of the PEF is approaching the size of the data, where the estimation of the PEF suffers from sampling issues and a lack of equations. Both of these cases are illustrated in Figure 3. In this example, the ranges were chosen to be from one half of the original scale to one quarter of the original scale, so that the the bins would be adequately filled without the detrimental effects of normalized linear interpolation at the larger scales, and sampling issues at the lower scales. The scales which are used are ultimately a function of the sparseness of the data as well as the size of the PEF.

The result of the multiscale PEF estimation is not perfect, as shown in Figure 4. However, the estimate does contain the relevant dips, so it can reliably be used as a starting point for the nonlinear scheme, described next.

NONLINEAR PEF ESTIMATION

Once an initial estimate of the PEF is made, this estimate can be used to interpolate the data, from which a new PEF can be estimated. This nonlinear approach can be repeated until it converges to a final solution, shown in Figure 5. Unfortunately, like most nonlinear methods, the choice of a starting guess is crucial to the success and efficiency of the method. When the starting interpolation is far from the ideal solution, convergence to the best solution is not likely. Figure 6 shows the nonlinear method used on the data, with various different original guesses.

The nonlinear method appears to create wildly different solutions depending on the starting guess. Both the Laplacian interpolation as well as one of the single scale PEF interpolations produced poor results. Both the multiscale PEF and the other single scale PEF produced pleasing results, which are both very close to the ideal solution. The ideal solution is where

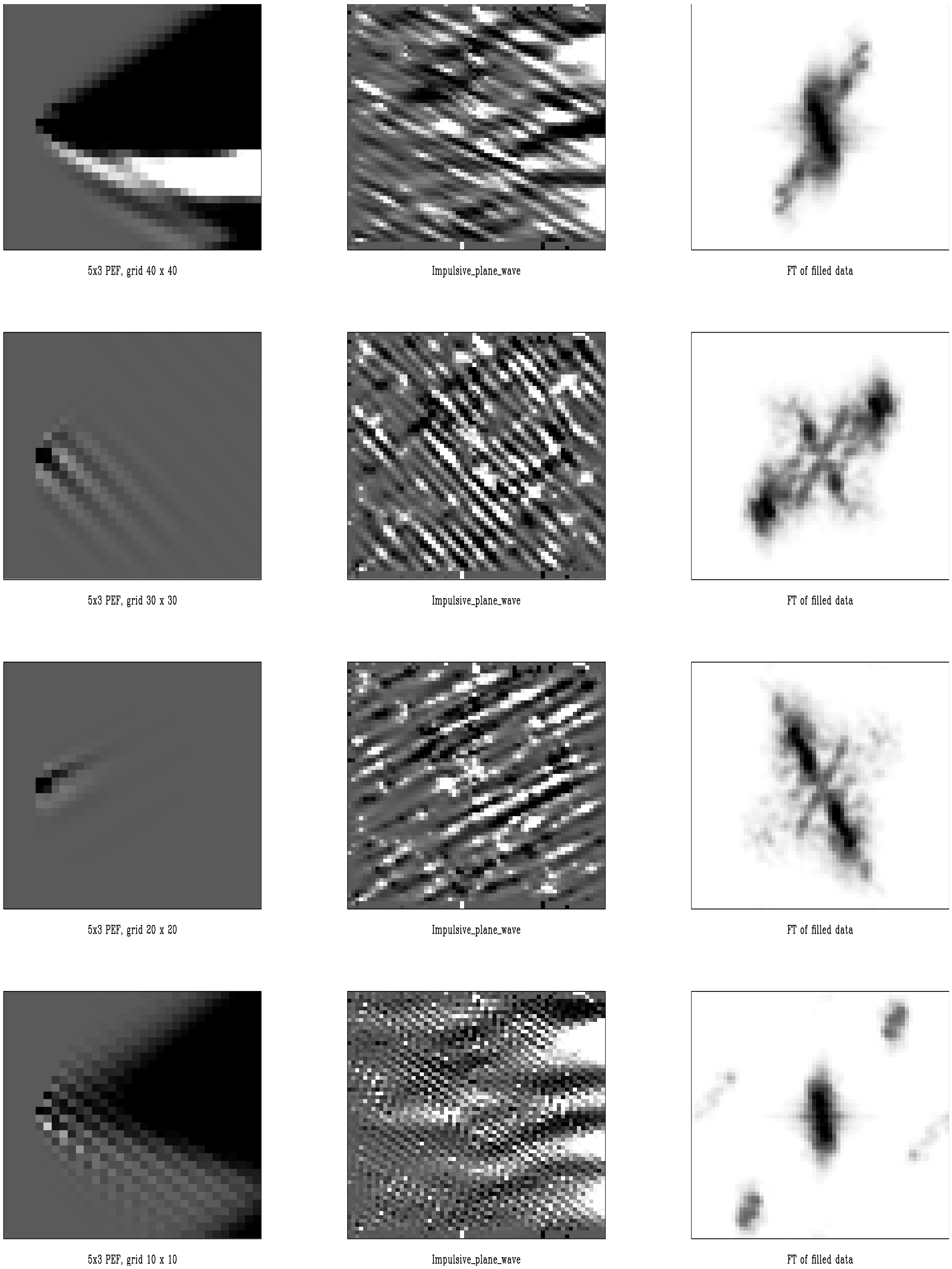

Figure 3: From left to right: Regridded data; inverse impulse response of estimated PEF; envelope of Fourier Transform of data filled with estimated PEF. From top to bottom: 40x40 grid; 30x30 grid; 20x20 grid; 10x10 grid bill1-refpefs [ER,M]

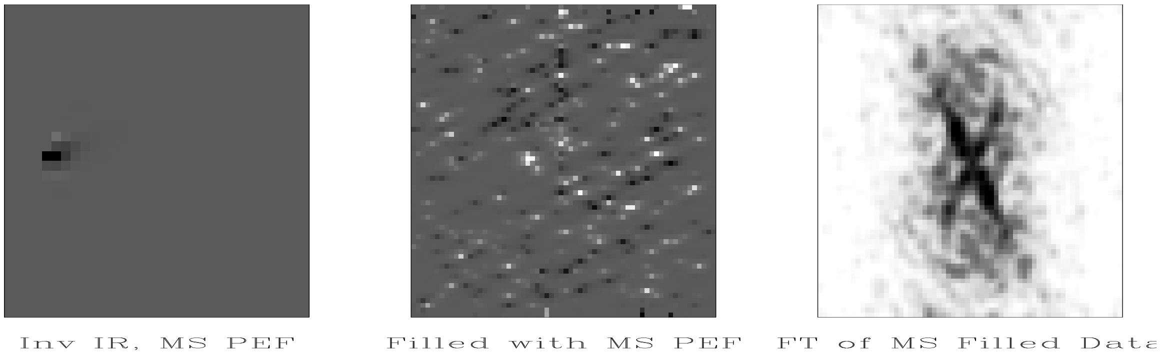

Figure 4: Multiscale estimation, from left to right: Inverse impulse response of PEF; data filled with PEF; envelope of Fourier Transform of filled data. The PEF was calculated with 9 scales of data ranging from 1/2 of the original scale to 1/4 of the original scale. bill1-multipef [ER]

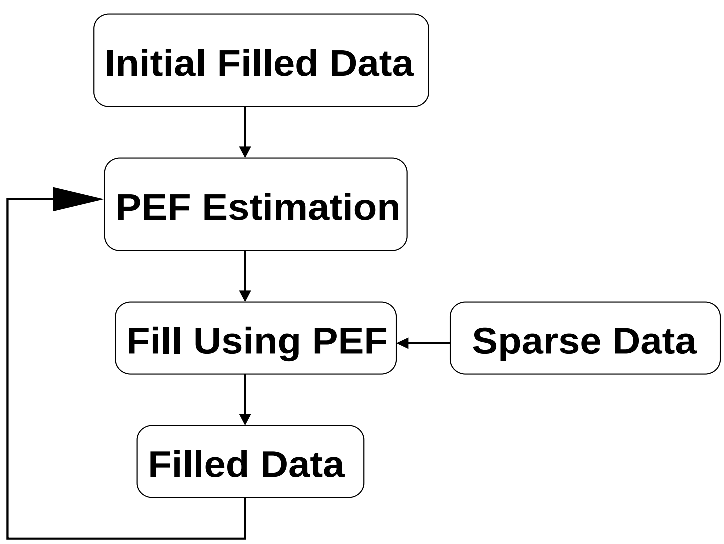

Figure 5: Flow chart for nonlinear PEF estimation. bill1-nlflow [NR]

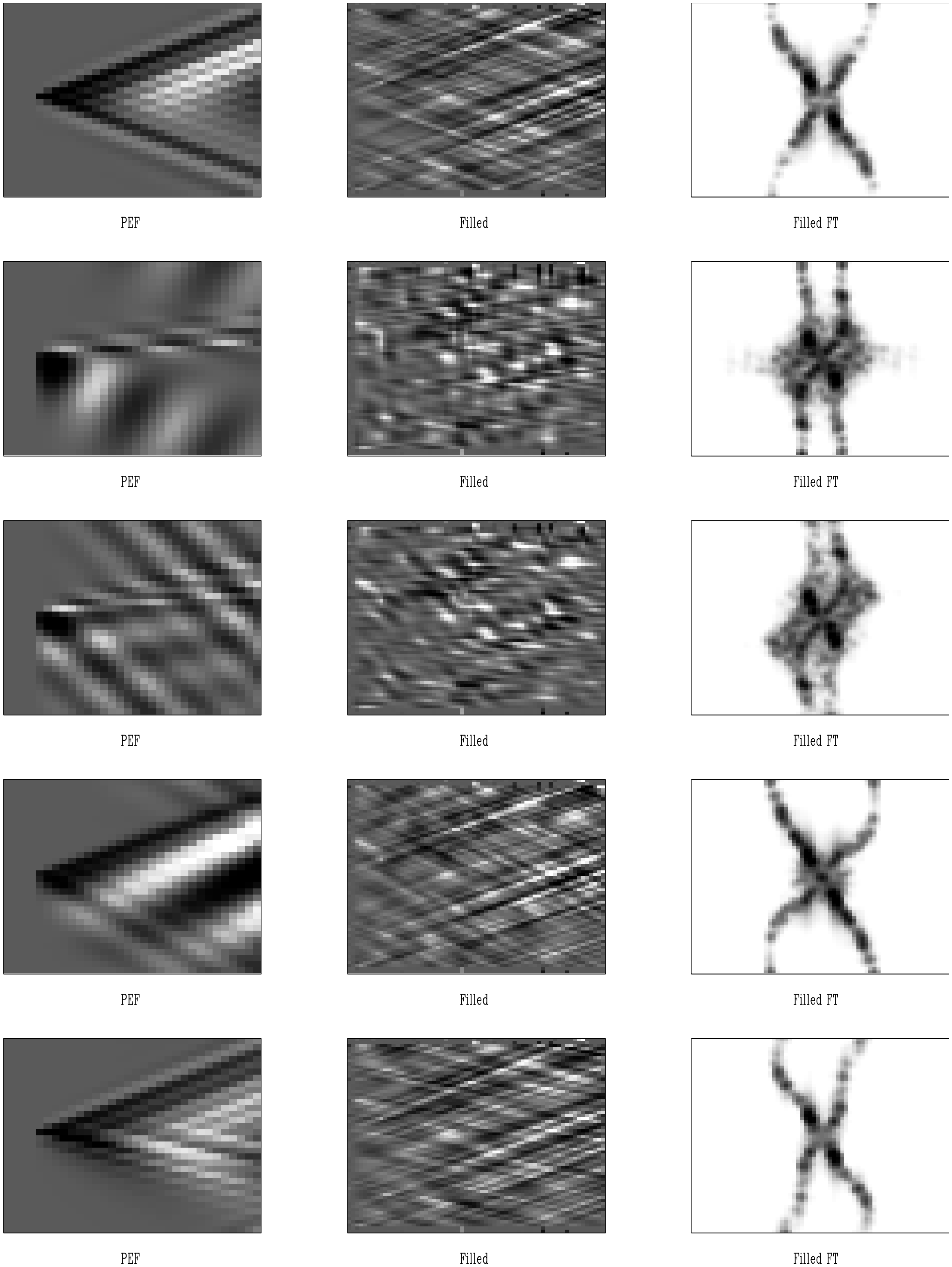

Figure 6: Nonlinear PEF estimation with various starting guesses. From left to right: inverse impulse response of PEF; filled data; envelope of Fourier Transform of filled data, all after 10 nonlinear iterations. From top to bottom: filled with PEF obtained from original data; filled with Laplacian; filled with single scale (32x32) PEF, filled with second single scale (20x20) PEF, filled with multiscale PEF. bill1-nlfigure [ER,M]

the PEF is calculated from the fully sampled data, and is then used as a starting guess in the nonlinear estimation. The multiscale result was obtained without knowing the solution, while the second single scale result was obtained by comparing various results with the original filled data, and selecting the most similar.

CONCLUSIONS

The estimation of a PEF with conventional methods for this example was impossible. Single scale PEF estimation can provide a good solution, but small changes in scale can create very different final solutions, especially when used as the starting guess for a nonlinear problem. Multiscale PEF estimation overcomes the variability in scales, and produces a result which is not perfect, but gives a robust result that requires less prior knowledge.

At this point, there are several avenues that can still be explored, such as examining the differences between normalized linear interpolation and binning while regridding, using weighting functions during both the multiscale PEF estimation and the nonlinear estimation, and developing better automated methods of choosing appropriate scales for the estimation.

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