Chapter 1

Introduction

Many functions in mathematics are reversible: for example, if you add seven to a number, you can then subtract seven to recover the original number. Other functions are not reversible: for example, if you multiply a number by zero, there is nothing you can do to recover the original value. In general, functions are only reversible if there is a one-to-one mapping between their inputs and outputs.

If I take a number and square it, I can then try and reverse the process by taking a square root. Unfortunately, however, an ambiguity exists over the original sign of the number. Since both positive and negative numbers have the same square, the squaring process is not reversible.

In this thesis, I am interested in taking square roots not of single numbers, but of multi-dimensional wavefields and operators. The wavefields are very large, containing tens of millions of pixels, and the operators represent wave-propagation through very complicated structures. However, a fundamental problem remains the same: when taking the square root of nine, do we choose three, or minus three?

1.1 Spectral factorization

Calculating the autocorrelation of a function is similar to calculating the square of a number. However, rather than just losing information about the sign, we lose information about the function's phase. Rather than there being two numbers that both have the same square, there is an infinite family of functions that share the same autocorrelation. This leaves us with a much larger ambiguity.

The simplest way to resolve the ambiguity is to set the phase to zero. Unfortunately, this solution results in *acausal* functions — energy starts to arrive before time zero. This is somewhat non-intuitive, and inappropriate for most physical systems, as it implies a ball moves before it's kicked.

A second alternative to resolve the ambiguity is to insist on a causal function, but one whose energy is packed as close to time zero as possible. This is known as the *minimum-phase* solution. As well as being causal and relatively compact in time, minimum-phase functions have another interesting property: the inverse of a minimum-phase function is also minimum phase, and hence causal. Because of these properties, it turns out that many physical systems fit the minimum phase model.

There is only one possible minimum-phase function with a given autocorrelation, and spectral factorization is the problem of determining that unique function.

One-dimensional solutions to the spectral factorization problem are well known: for example, Claerbout (1992) describes several approaches, including the Kolmogorov algorithm (Kolmogorov, 1939) which I briefly review in Chapter 2. For multi-dimensional signals, however, the problem itself is less clear: what exactly constitutes a causal function in multi-dimensional spaces?

1.1.1 Multi-dimensional spectral factorization on a helix

Claerbout (1998b) describes the isomorphic process by which multi-dimensional functions can be mapped into equivalent one-dimensional functions. The process depends on the concept of the helical boundary conditions, and is best illustrated by Figure 1.1, which shows a small five-point filter on a two-dimensional space, being mapped into an equivalent one-dimensional filter.

Figure 1.1: Illustration of helical boundary conditions mapping a two-dimensional function (a) onto a helix (b), and then unwrapping the helix (c) into an equivalent one-dimensional function (d). Figure by Sergey Fomel. int-helix [NR]

Under such a transformation, the concepts of causality and minimum-phase become clear. One-dimensional spectral factorization algorithms can be directly applied to the multi-dimensional helical functions.

1.2 Applications of multi-dimensional spectral factorization

This thesis describes three applications of multi-dimensional spectral factorization that are relevant to exploration geophysics.

1.2.1 Spectral factorization of seismic wavefields

The first part (Chapter 2) covers the multidimensional spectral factorization of passive seismic wavefields. I show that calculating the 3-D spectral factorization of a passive seismic wavefield amounts to estimating the medium's 3-D impulse response. Helioseismic dopplergrams provide a high-quality passive seismic dataset that has uniquely dense spatial coverage. This enables me to compare the solar impulse response from derived by spectral factorization with that derived by cross-correlation. The spectral factorization results contain significantly higher bandwidth, both temporally and spatially. I then show it is possible to derive active-source seismograms from terrestrial passive seismic data by crosscorrelating and factorizing data collected during a small passive seismic experiment conducted in Long Beach, CA.

1.2.2 Spectral factorization of partial differential equations

In the second part, I use multidimensional spectral factorization to facilitate solving multi-dimensional partial differential equations (PDEs) with application to exploration seismology. As an illustration of the methodology, in Chapter 3, I show how the Helmholtz operator can be factored into two filters, which then can used to propagate waves. With more practical interests in mind, in Chapter 4, I apply the methodology to the problem of implicit finite-difference modeling and migration by wavefield extrapolation. Chapter 5 describes how the method can be applied in areas where the velocity varies laterally.

1.2.3 Spectral factorization of linear modeling operators

Most industrial-strength geophysics involves filtering the recorded data with the adjoint of the physical process that created it. If the true earth model is $\mathbf{m}$ and $\mathbf{A}$ is the physical forward modeling operator, then the image we compute is $\mathbf{A}'\mathbf{A}\mathbf{m}$. In the final part of this thesis, I form approximations to the $\mathbf{A}'\mathbf{A}$ operators associated with prestack depth migration that are diagonal in physical space. Since the approximations are diagonal, I can easily compute two factors simply by taking the square root of their diagonal elements. These factors are also diagonal, and hence easily invertible. These diagonal factors can then be applied directly to the migrated image to produce an image whose amplitude more closely resembles those of the true earth model. Alternatively, the factors can be applied in concert with the original operator, to produce a new dimensionless composite operator, which is more easily invertible with iterative linear solvers.

In Chapter 6, I discuss how to calculate the shot-illumination cheaply during shot-profile migration. I then show that for sparse-shot geometries with dense receiver coverage this weighting function can completely compensate for illumination problems on flat events.

Lastly, in Chapter 7, I compare alternative methods of computing appropriate diagonal model-space and data-space weighting functions appropriate for generic linear operators, and discuss how model-space and data-space weights can be calculated and applied simultaneously.

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