Chapter 1

Introduction

For the exploration geophysicist, velocity is the key component in creating accurate images of the earth. Velocity is important in almost all phases of processing. It is used implicitly or explicitly in Normal Moveout (NMO), multiple attenuation, Dip Moveout (DMO) and migration. Velocity and the explorationist's final product, the earth image, play an important part in the work of other geoscientists. On an active oil field, seismic imaging helps target wells and monitor fluid flow (Lumley, 1995). It provides rock physicists with reservoir properties, and geostatisticians and reservoir engineers with constraints for their simulations (Mao, 1999).

With the wrong velocity, seismic events do not focus and reflectors are mispositioned. Without an accurate velocity, wells will never tie with the seismic data and directional drilling becomes much more difficult. Without an accurate velocity, seismic data could easily hinder rather than help the rock physicist (Claerbout, 1999), geostatistician, and reservoir engineer.

Obtaining an accurate velocity estimate is one of the most difficult problems in geophysics. Velocity estimation is a nonlinear, under-determined problem. The first step in all velocity estimation schemes is to make assumptions about subsurface properties. The most basic assumption is the one used by NMO, a horizontally stratified earth. A less stringent assumption, flat reflectors, was made by Toldi (1985). When the assumptions that the methods are based on are inaccurate, they fail to give satisfactory results. When this happens we are often forced to the family of methods which are generally referred to as 'tomography'.

TOMOGRAPHY

Tomography, or at least the definition of tomography I will be using in thesis, starts from the idea that there is an operator $\mathbf{T_{nl}}$ that relates slowness $\mathbf{s}$ to travel times $\mathbf{t}$,

$$\mathbf{t} = \mathbf{T_{nl}}\mathbf{s} \tag{1.1}$$

One way to think of this operator is in terms of rays. Figure 1.1 shows a simple two layered model with sources along top and left edges and receivers along all four edges. The simplest way to think of the operator $\mathbf{T_{nl}}$ is that it simply integrates the slowness along the raypath.

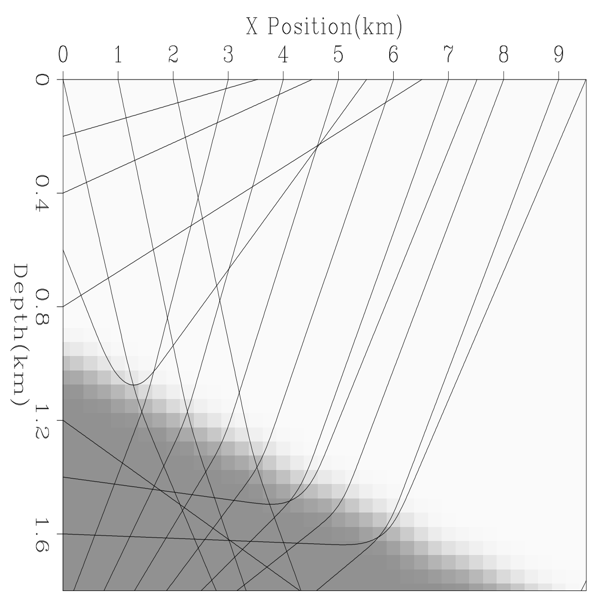

Figure 1.1: A simple example of tomography with the sources and receivers every .2 *km* in depth and every .5 *km* in x. Overlaid are the raypaths connecting the sources to the receivers. intro-cor.overlay [CR]

This simplistic view of tomography has a number of inconsistencies with the *reflection tomography problem*. The first is that we wouldn't be doing tomography if we actually knew the slowness field beforehand. What we actually have is some initial guess, $\mathbf{s_0}$, of the slowness field. If for our initial model we assume a constant slowness field and find the raypaths connecting the same source and receivers, Figure 1.2, we encounter another problem: the raypaths are significantly different. An alternate way to state this observation is that the operator is model dependent and therefore non-linear.

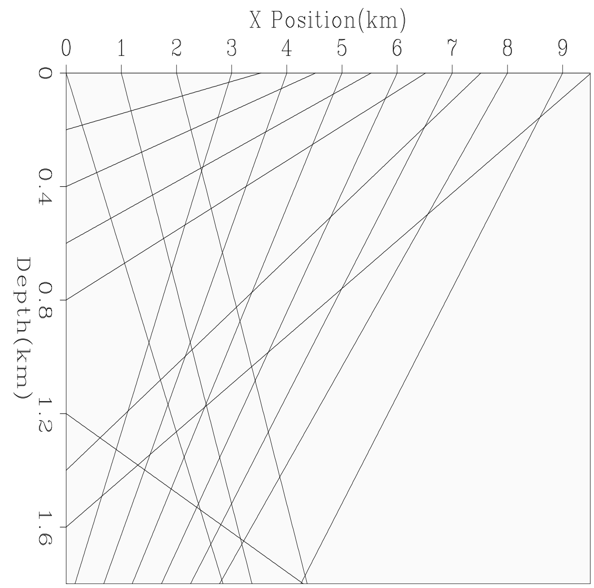

Figure 1.2: The same recording geometry as Figure 1.1, with a constant velocity field. Note that the ray paths are significantly different. intro-init.overlay [CR]

Non-linear problems

Non-linear problems are much more difficult to solve than linear problems. A method that is often successful is to make some approximations that will turn the non-linear problem into a linear problem. We can make this conversion by doing a Taylor expansion (ignoring second and higher order terms) around the initial guess at the slowness field,

$$\begin{aligned} \mathbf{t} &\approx \mathbf{T_{nl}} \mathbf{s_0} + \nabla \mathbf{T_{nl}} \Delta \mathbf{s} \\ \mathbf{t} &\approx \mathbf{t_0} + \mathbf{T_0} \Delta \mathbf{s} \\ \Delta \mathbf{t} &\approx \mathbf{T_0} \Delta \mathbf{s} \end{aligned} \tag{1.2}$$

where $\mathbf{s_0}$ is the initial guess at slowness, $\mathbf{T_0}$ is the Frechet derivative, a *linear* approximation of $\mathbf{T_{nl}}$ at $\mathbf{s_0}$, $\Delta \mathbf{s}$ is the change in slowness, $\mathbf{t_0}$ are the modeled travel times by applying $\mathbf{T_0}$ to $\mathbf{s_0}$, $\Delta \mathbf{t}$ are the differences between the modeled travel times, $\mathbf{t_0}$, and the measured travel times, $\mathbf{t}$. Reflection seismic tomography problems are too large to do direct matrix inversion so iterative methods, such as conjugate gradient, are used instead. Once we have used an iterative method to converge to an acceptable $\Delta\mathbf{s}$, we update the slowness estimate,

$$\mathbf{s_1} = \mathbf{s_0} + \Delta\mathbf{s}. \tag{1.3}$$

We can then re-linearize around this new model ($\mathbf{s_1}$), constructing a new tomography operator $\mathbf{T_1}$ and repeating the process.

Unfortunately, the linearization does not share many of the properties of a truly linear problem. First, the step length calculation that is valid in the linear problem is usually not valid in the non-linear problem. Instead, we must take into account the linearization point when calculating the step size. Second, and more importantly, the problem isn't guaranteed to converge. To see why, let's return to the simple example. The operator is based on the initial slowness estimate, which is in error. Explained in terms of rays we are back projecting along the guess at the ray trajectories, not their true trajectories. As a result we will end up back projecting the slowness changes to the wrong portions of the model space.

Figure 1.3 shows the velocity estimate after applying linearized tomography. Generally, we have moved towards the correct velocity model (Figure 1.1) but we can also see the effects of the linear approximation. In the upper right portion of Figure 1.3 the velocity has been incorrectly increased. If we look at the ray connecting ($z = 1.6, x = 0$) and ($z = 0, x = 9.5$) in Figures 1.2 and 1.1 we can begin to see why. The guess at the raypath indicates that 30% of the ray length is spent in the lower layer while in fact 50% is traveling through this high velocity layer. To account for the traveltime discrepancy, the inversion has increased the velocity in the in the upper portion of the model.

A second inconsistency is that equation (1.2) is an example of *transmission tomography* rather than *reflection tomography*. Reflection tomography attempts to invert the slowness field from raypaths that go from a known source, to a *unknown* reflection point in the subsurface, to a known receiver. Not knowing the reflection point introduces a whole new set of unknowns into the inversion. Figure 1.4 shows a synthetic anticline model. Overlaid in solid lines are the correct reflector positions and the correct raypaths to these reflector positions. The dashed lines are the reflector positions and raypaths that would be estimated using a $s(z)$ initial slowness model. Note that the guess at the reflector position is in significant error, making the guess at

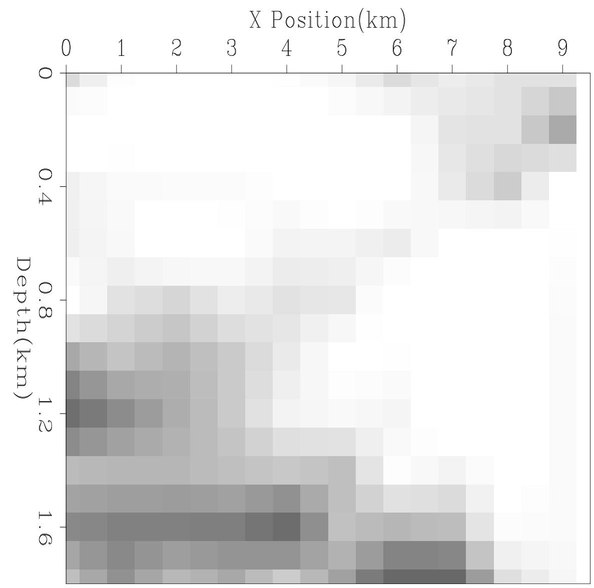

Figure 1.3: The tomography result using the slowness model and raypaths of Figure 1.2 as the initial estimates. intro-first.tomo [CR,M]

the raypaths, and the corresponding tomography operator, in greater error than its transmission tomography counterpart. In addition, traveltime error can be accounted for by shifting the reflector, changing the slowness field, or some combination of the two. The result is that reflection tomography is much more likely to get stuck in local minima and maxima than its transmission counterpart.

The possibility of converging to a local minima is well known. The general solution is to use as accurate an initial model as possible (to move the initial point as close the global minima as possible). This is usually done by first applying conventional velocity analysis methods such as semblance analysis and interval velocity estimation by Dix (Dix, 1955) or some derivation on Toldi's (1985) approach. In addition, smaller step sizes and smoother slowness changes at early iterations are used to avoid going outside the valid range of the linear approximation. Both of these approaches have potential negative consequences. A smaller step size requires a greater number of expensive non-linear iterations. In addition, conventional velocity estimation often does not provide a good initial estimate and might in fact lead us to a local maxima.

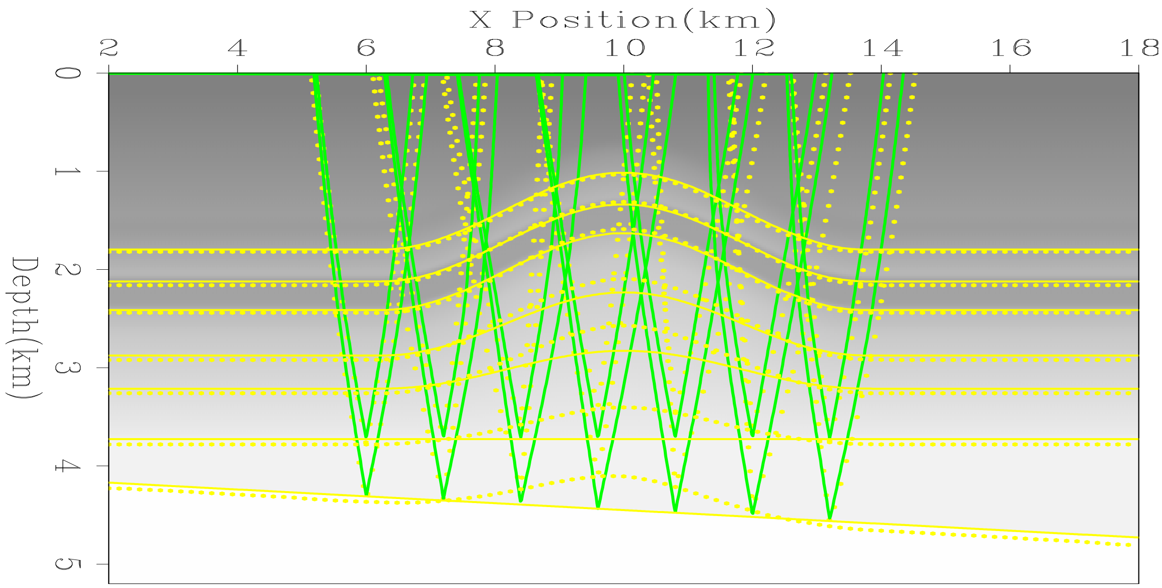

Figure 1.4: The solid lines show raypaths through the correct velocity and the correct reflector position. The dashed lines are raypaths through the initial model and the initial reflector positions. Note that the estimated raypaths have significant error, therefore the tomography operator will have significant error. intro-intro-rays [CR,M]

Tomography and null space

If the model space is a uniformally sampled Cartesian mesh or something similar, we face another problem when doing reflection tomography, its large null space. There are several reasons for this null space:

shadow zones Certain parts of the velocity model simply might not be illuminated, or illuminated poorly by the acquisition geometry. A classical example is under the edge of a salt body (Muerdter et al., 1996).

limited angle coverage The recording geometry imposes a limitation on vertical resolution. The left panel of Figure 1.5 simulates a ray-based back projection operator. Along each ray a random slowness change is back projected. The right panel of Figure 1.5 shows the Fourier response. Note how large wavenumbers in z are not well illuminated. Often this phenomenon is explained in terms of the Fourier slice theorem. The Fourier transform of a vertical projection in space is a horizontal radial profile.

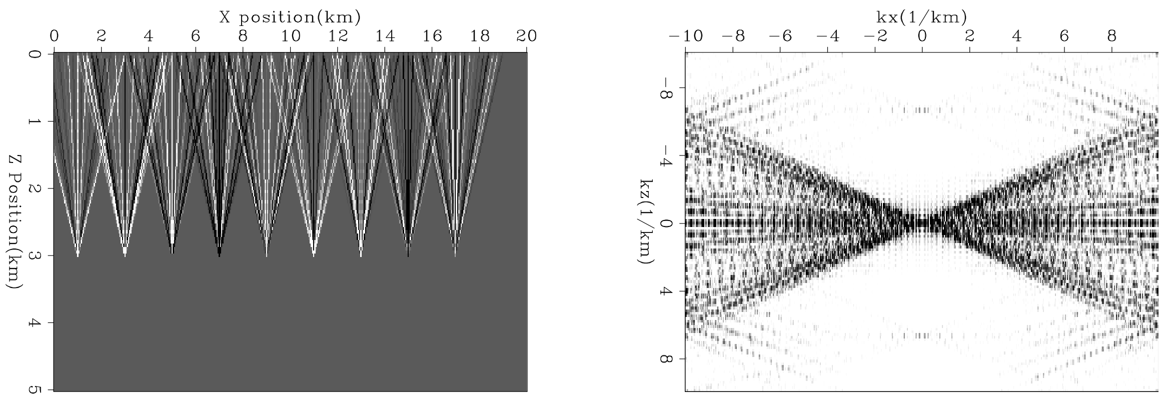

Figure 1.5: The left panel simulates a ray-based back projection operator. Along each ray a random slowness change is back projected. The right panel shows the Fourier response. intro-slice [ER,M]

resolution decreases with depth When the reflection point is unknown, normal angle reflection times do not contain any information about velocity. As the reflection angle increases, velocity discrimination becomes easier. At larger depths, given the limited surface recording geometry, the angle coverage will decrease (Figure 1.6). As a result, larger depth model components are often under- or unilluminated (Prucha et al., 1999). If we follow the Fourier analysis of Figure 1.5 we can see that the vertical resolution decreases as the reflector depth increases (Figure 1.7).

not enough equations In most reflection tomography methods, we invert a limited set of positioning errors. When doing tomography, especially 3-D tomography, it is possible to have more model points than moveout errors.

To deal with the null space of the reflection tomography operator there are two alternative paths that could be followed, reparameterization or regularization.

Reparameterization can be done by a coarser sampling of the Cartesian, but usually involves other parameterizations of the model space. It usually takes the form of either a layered model with simple velocity function (such as velocity gradient) within the layers (Kosloff et

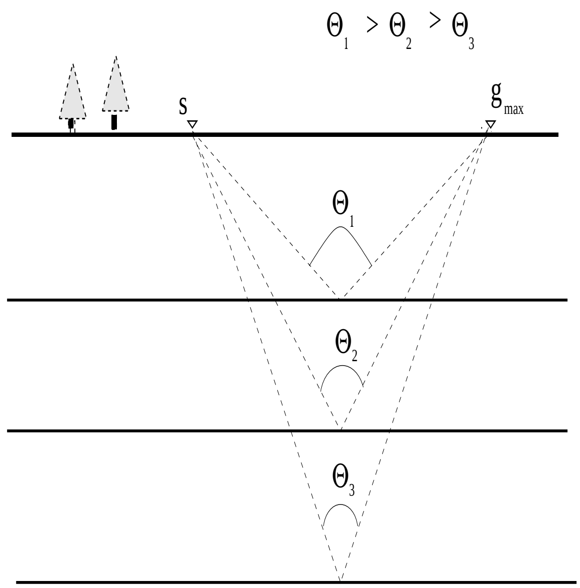

Figure 1.6: Given a fixed recording geometry (s being the source and g_{max} being the farthest receiver) as we go deeper in depth the angle range that we at each reflector decreases. intro-ray-limit [NR]

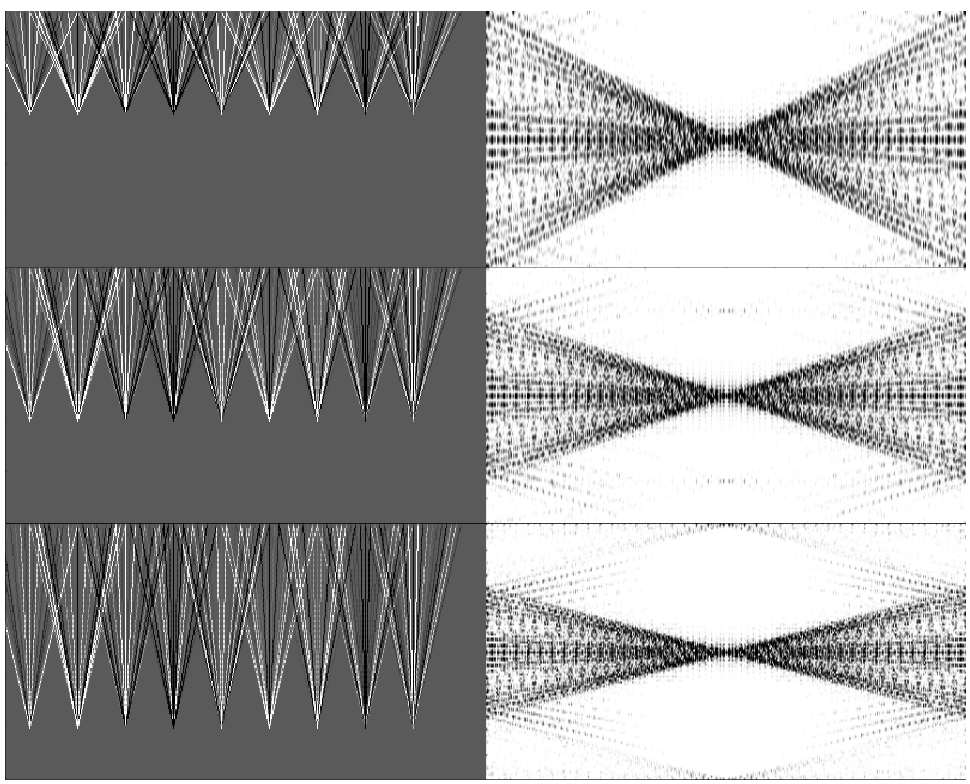

Figure 1.7: Left panel shows simulated raypaths with random energy introduced along each raypath. The right panel is the Fourier response. From top to bottom the depth of the reflector increases. intro-slice.depth [ER,M]

al., 1996) or reparameterizing the model in term of spline nodes (Biondi, 1990). The layered model approach is attractive because it allows a linking between an interpreter's geologic model (Guiziou et al., 1996). It also provides a fairly accurate description of velocity structure in regions such as the North Sea. The spline node approach can also be beneficial. It can guarantee the velocity model will be smooth (necessary for ray based methods) and with selective placement of nodes can allow less freedom in areas where the data provides less information about the velocity field. The downside of both of these approaches is that the parameterization must be chosen *a priori*. At early iterations the guess at the velocity model can be in significant error. If we do a poor job of parameterizing the model we could actually slow, or stop, the problem from converging (Delprat-Jannaud and Lailly, 1992).

The second option is regularization. In the regularization approach we add a second term to the objective function,

$$Q(\Delta\mathbf{s}) = \|\Delta\mathbf{t} - \mathbf{T_0}\Delta\mathbf{s}\|^2 + \epsilon^2\|\mathbf{A}\Delta\mathbf{s}\|^2, \tag{1.4}$$

where **A** penalizes some definition of roughness in the model. The regularization approach has its own drawbacks. The most significant is that the resulting model space may no longer has any direct connection with geology. In addition, choosing **A** is problematic and can introduce unrealistic elements into the slowness estimate.

OVERVIEW OF THESIS

The goal of this thesis is to present two different techniques that can help overcome reflection tomography's problems with nonlinearity and null space.

Tau tomography

In Chapter **??**, I introduce the concept of tau (vertical travel-time) tomography and derive the operator relating changes in slowness to moveout errors in tau space. I show how tau tomography is less sensitive to the initial guess at reflector position and slowness estimate.

Figure 1.8 shows the model of Figure 1.4 with the correct and initial reflector positions in depth (left) and tau (right).

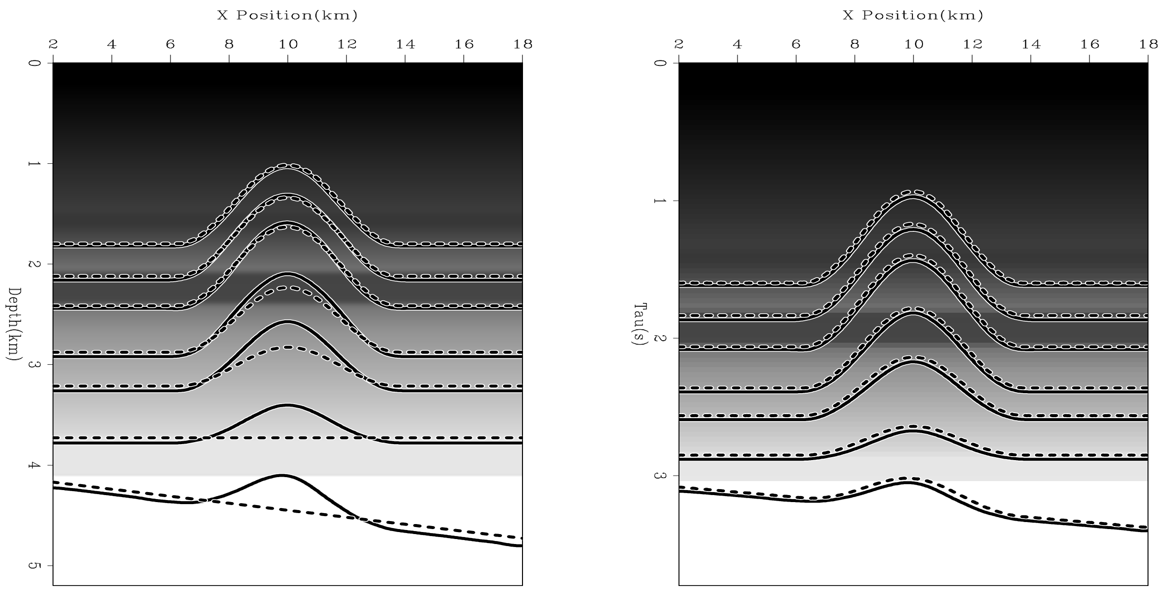

Figure 1.8: Reflector movement in depth (left) and tau (right). The solid lines are where the reflectors migrate using the initial v(z) field. The dashed lines are the correct reflector positions in tau and depth. Note that correct and initial positions are much closer in the tau case. intro-ref-movement [CR,M]

On a synthetic I show how performing tomography in tau rather than depth better constrains the slowness changes. I show that the tau tomography problem converges faster to a more reasonable result than its depth counterpart. The resulting migration is better focused and the reflectors are better positioned by doing tau rather than depth tomography.

Geologic regularization

In Chapter **??**, I introduce a new way to regularize the tomography estimate that takes into account the *a priori* estimate of geology. I start from a simple missing data problem. I compare the conventional way that geophysicists and geostatisticians characterize covariance and how they apply to a missing data problem. I show how non-stationarity makes both solutions suboptimal. I then introduce another way to approximate model covariance, a *steering filter*.

I show that a steering filter can accurately handle a non-stationary model covariance. I show how to build a steering filter with as little information as early migrated reflector positions. For example, returning to the simply two layer example (Figure 1.1), if we know the dip of the second layer in the model of Figure 1.2 we can do a better job constraining the slowness field using steering filter regularization, Figure 1.9.

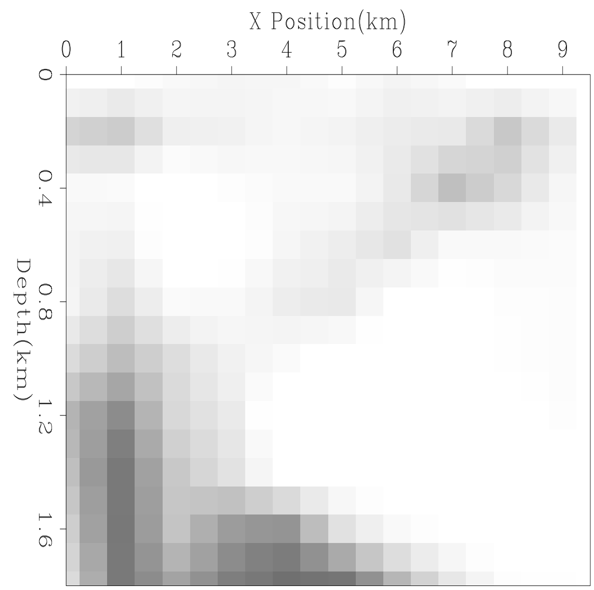

Figure 1.9: The same tomography shown in Figure 1.3 now using steering filters to regularize the velocity estimate. intro-reg.tomo [CR,M]

Using the steering filter I develop a new tomography objective function. I then compare and contrast the results of using a steering filter to regularize the tomography problem versus an isotropic regularizer in both depth and tau. I show that the most reasonable velocity model, and the best migrated image, comes from using steering filters in the tau domain.

Field tests

In Chapter **??**, I apply tau tomography with steering filter regularization to a 2-D line taken from a 3-D dataset acquired over a saltdome the North Sea. The resulting migration image is better focused with better resolution of the sediments on the salt flanks. The final velocity model still shows velocity generally following structure rather than the blobby nature common with other regularization techniques. In Chapter **??**, I apply my method in 3-D. I use the North Sea dataset introduced in Chapter **??**, concentrating on the structure above the saltdome, and improve both focusing and resolution of the migrated image.

Appendices

One of the problems with most geophysics techniques is they provide only a single answer and no measure of the uncertainty of the estimation. In Appendix **??**, I show how we can modify the way we set up the inverse problem to obtain equi-probable results. By comparing and contrasting these realizations we can better understand model variability and uncertainty.

In Appendix **??** and **??**, I extend the concepts introduced in Chapters **??** and **??** to 3-D. Appendix **??** shows how a 3-D steering filter can be constructed by cascading two 2-D filter. Finally, in Appendix **??**, I derive a 3-D tau tomography operator.

ASSUMPTIONS AND LIMITATIONS

The weakness of the method presented in this thesis is what it tries to overcome, the non-linearity of tomography. When we are far from the correct solution and/or we have an inaccurate model covariance estimate, the method is susceptible to non-convergence, or convergence to an unreasonable model.

The ray-based tomography used in this thesis is limited by the ray tracing high-frequency approximation. The high frequency approximation requires that the model varies smoothly over a wavelength. For the datasets presented in this thesis that assumption is valid except at the salt edge. In more complex environments, much more expensive wave equation tomography methods (Woodward, 1990; Biondi and Sava, 1999) have the potential to do better under these adverse conditions.

For the steering filters of Chapter **??** to be effective, they must adequately describe the model covariance. Often velocity does not follow dip, and the velocity gradient for the region would need to be used instead of the early migrated reflector position. In addition, a steering filter might not describe a complex media's covariance. For the frequencies used in reflection seismology, practice shows that velocity can be approximated as a smoothly varying function, so steering filters are a valid representation of the model's covariance.

This thesis assumes that no anisotropy exists in the data. In theory, it is not hard to incorporate anisotropy into the velocity estimation process presented in this thesis, in fact, the method presented has great potential for anisotropic estimation. The reflector stability of the tau domain method would lead to much better ability to constrain the η (Alkhalifah and Tsvankin, 1995) or equivalent anisotropic parameter. In addition, steering filters, with their ability to spread information along layers, are ideal for defining and determining anisotropic layers.

Bibliography

Alkhalifah, T., and Tsvankin, I., 1995, Velocity analysis for transversely isotropic media: Geophysics, **60**, no. 05, 1550–1566.

Biondi, B., and Sava, P., 1999, Wave-equation migration velocity analysis: SEP–**100**, 11–34.

Biondi, B., 1990, Velocity analysis using beam stacks: Ph.D. thesis, Stanford University.

Claerbout, J., 1999, Everything depends on v(x,y,z): SEP–**100**, 1–10.

Delprat-Jannaud, F., and Lailly, P., 1992, What information on the earth model do reflection travel times provide?: J. Geophysical Research, **97**, 19827–19844.

Dix, C. H., 1955, Seismic velocities from surface measurements: Geophysics, **20**, no. 1, 68–86.

Guiziou, J. L., Mallet, J. L., and Madariaga, R., 1996, 3-d seismic reflection tomography on top of the gocad depth modeler: Geophysics, **61**, no. 05, 1499–1510.

Kosloff, D., Sherwood, J., Koren, Z., Machet, E., and Falkovitz, Y., 1996, Velocity and interface depth determination by tomography of depth migrated gathers: 61, **5**, no. 1511-1523.

Lumley, D., 1995, Seismic time-lapse monitoring of subsurface fluid flow: Ph.D. thesis, Stanford University.

Mao, S., 1999, Multiple layer surface mapping with seismic data and well data: Ph.D. thesis, Stanford University.

Muerdter, D. R., Lindsay, R. O., and Ratcliff, D. W., 1996, Imaging under the edges of salt sheets: a raytracing study: 66th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 578–580.

Prucha, M. L., Biondi, B. L., and Symes, W. W., 1999, Angle-domain common image gathers by wave-equation migration: Angle-domain common image gathers by wave-equation migration:, 69th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, 824–827.

Toldi, J., 1985, Velocity analysis without picking: Ph.D. thesis, Stanford University.

Woodward, M., 1990, Wave equation tomography: Ph.D. thesis, Stanford University.