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Short Note

On Stolt common-azimuth residual migration

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INTRODUCTION

This short note presents the extension of the velocity-independent Stolt residual migration introduced in a previous report (Sava, 1999) to common-azimuth (CA) data. The derivation is very similar to its 2-D equivalent and retains the important feature of being independent of any assumption about the velocity used to obtain the image being residually migrated.

Residual migration can be used for both image improvement and velocity analysis. As was previously shown (Biondi and Sava, 1999), we can use the differences between a given image and an improved version of it in wave-equation velocity analysis. Since full 3-D prestack velocity analysis using the wave equation is still not within reach owing to its huge computational cost, the common-azimuth approximation seems to be the proper technology usable for 3-D datasets. It is therefore necessary to devise the appropriate residual migration tools for common-azimuth data.

In the first part of this short note I present the derivation of the common-azimuth Stolt residual migration equations, followed in the second part by a simple synthetic example.

THEORY

Common-azimuth data represent subsets of 3-D datasets that have been recorded or transformed to zero cross-line offsets ($h_y = 0$). Stolt constant velocity migration for common-azimuth data involves the use of the following dispersion relation (Biondi and Palacharla, 1995):

$$k_{zx} = \frac{1}{2}\sqrt{\left(\frac{\omega}{v}\right)^2 - (k_{mx} - k_{hx})^2} + \frac{1}{2}\sqrt{\left(\frac{\omega}{v}\right)^2 - (k_{mx} + k_{hx})^2}, \qquad (1)$$

where the depth wavenumber of the common-azimuth dataset (k_z) is

$$k_z = \sqrt{k_{zx}^2 - k_{my}^2}, \qquad (2)$$

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and the midpoint and offset wavenumbers are defined as $\mathbf{k_m} = k_{m\,x}\mathbf{x} + k_{m\,y}\mathbf{y}$ and $\mathbf{k_h} = k_{h\,x}\mathbf{x} + k_{h\,y}\mathbf{y}$.

We can write equations (2) and (1) for a given reference velocity (v_0) as

$$k_{z\,x\,0} = \frac{1}{2}\sqrt{\left(\frac{\omega}{v_0}\right)^2 - (k_{m\,x} - k_{h\,x})^2} + \frac{1}{2}\sqrt{\left(\frac{\omega}{v_0}\right)^2 - (k_{m\,x} + k_{h\,x})^2}, \tag{3}$$

and

$$k_{z\,0} = \sqrt{{k_{z\,x\,0}}^2 - {k_{m\,y}}^2}. \tag{4}$$

The goal of common-azimuth Stolt residual migrations is to obtain k_z (equations 2 and 1) from $k_{z\,0}$ (equations 4 and 3).

If we express the frequency ω from equation (3) as

$$\omega^2 = v_0^2 \frac{\left[{k_{z\,0}}^2 + {k_{m\,y}}^2 + {k_{h\,x}}^2\right]\left[{k_{z\,0}}^2 + {k_{m\,y}}^2 + {k_{m\,x}}^2\right]}{{k_{z\,0}}^2 + {k_{m\,y}}^2}$$

and introduce it in equation (1), we obtain the common-azimuth residual migration equations

$$\left\{ \begin{aligned} k_{z\,x} = & \tfrac{1}{2}\sqrt{\tfrac{v_0^2}{v^2}\tfrac{\left[{k_{z\,0}}^2 + {k_{m\,y}}^2 + {k_{h\,x}}^2\right]\left[{k_{z\,0}}^2 + {k_{m\,y}}^2 + {k_{m\,x}}^2\right]}{{k_{z\,0}}^2 + {k_{m\,y}}^2} - (k_{m\,x} - k_{h\,x})^2} \\ & + \tfrac{1}{2}\sqrt{\tfrac{v_0^2}{v^2}\tfrac{\left[{k_{z\,0}}^2 + {k_{m\,y}}^2 + {k_{h\,x}}^2\right]\left[{k_{z\,0}}^2 + {k_{m\,y}}^2 + {k_{m\,x}}^2\right]}{{k_{z\,0}}^2 + {k_{m\,y}}^2} - (k_{m\,x} + k_{h\,x})^2} \\ k_z = & \sqrt{{k_{z\,x}}^2 - {k_{m\,y}}^2} \end{aligned} \right. \tag{5}$$

Note that for 2-D prestack data, [2] equations (5) reduce to the 2-D prestack form (Sava, 1999):

$$\begin{aligned} k_z = & \tfrac{1}{2}\sqrt{\tfrac{v_0^2}{v^2}\tfrac{\left[{k_{z\,0}}^2 + {k_{h\,x}}^2\right]\left[{k_{z\,0}}^2 + {k_{m\,x}}^2\right]}{{k_{z\,0}}^2} - (k_{m\,x} - k_{h\,x})^2} \\ & + \tfrac{1}{2}\sqrt{\tfrac{v_0^2}{v^2}\tfrac{\left[{k_{z\,0}}^2 + {k_{h\,x}}^2\right]\left[{k_{z\,0}}^2 + {k_{m\,x}}^2\right]}{{k_{z\,0}}^2} - (k_{m\,x} + k_{h\,x})^2} \end{aligned},$$

which, furthermore, reduces to the well-known Stolt 2-D post-stack residual migration equa-

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[2]

$$s_y = g_y \Leftrightarrow k_{m\,y} = 0$$

$$k_z \equiv k_{z\,x}$$

$$k_{z\,0} \equiv k_{z\,x\,0}$$

tion (Stolt, 1996) [3]:

$$k_z = \sqrt{\frac{v_0^2}{v^2}\left[{k_z}_0^2 + {k_m}_x^2\right] - {k_m}_x^2}.$$

As for the 2-D prestack data, the common-azimuth residual migration is velocity independent; that is, we need not make any assumption about the actual values of the velocities for the reference and improved migration, but only about their ratio. In this way, we can take an image and residually migrate it without knowing what velocity model has been used to image it in the first place.

EXAMPLE

In this section, I present a synthetic example of Stolt migration and residual migration for common-azimuth data. The model is represented by a set of four plane reflectors at different depths, with different lateral dimensions and varying reflectivity (Figure 1).

Figure 2 shows the result of Stolt modeling using the true velocity $v = 3.0$ km/s. Next, I apply Stolt migration with a velocity of $v_0 = 3.6$ km/s (Figure 3). Since the velocity is not the true one used to generate the data, the image is not focused correctly at the three original reflectors.

Finally, I use Stolt common-azimuth residual migration as described in the preceding section and with a velocity ratio $v_0/v = 1.2$ to get the image in Figure 4. Now the original reflectors are well-focused at their original locations.

CONCLUSIONS

In this short note, I have presented the extension of the velocity independent Stolt residual migration to common-azimuth data. Equations (5) can be used to obtain images corresponding to new velocities defined by a given ratio to the original velocity.

Such a residual migration method inherits the advantages and disadvantages of Stolt residual migration. Some of the most notable characteristics are:

- Speed: Because CA Stolt residual migration is essentially a Stolt migration, its cost is comparable to that of any Stolt migration.
- Constant velocity: Stolt migration assumes constant velocity. In theory, CA residual Stolt migration assumes constant velocity, too. In practice, however, if we residually

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[3]

$$v k_z = \sqrt{v_0^2 {k_z}_0^2 + k_{mx}\left(v_0^2 - v^2\right)}$$

$$\omega = \sqrt{\omega_0^2 + k_{mx}\left(v_0^2 - v^2\right)}$$

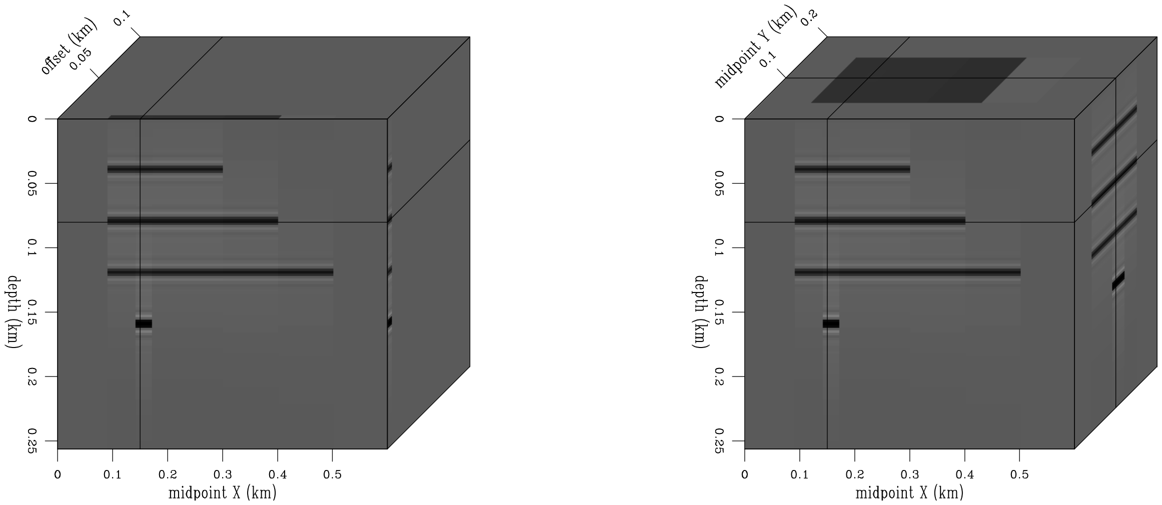

Figure 1: The reflectivity model. The right panel represents the zero-offset view in the common-azimuth space, while the left panel represents the prestack view for the inline selected on the right panel. $\boxed{\text{paul2-model}}$ [ER]

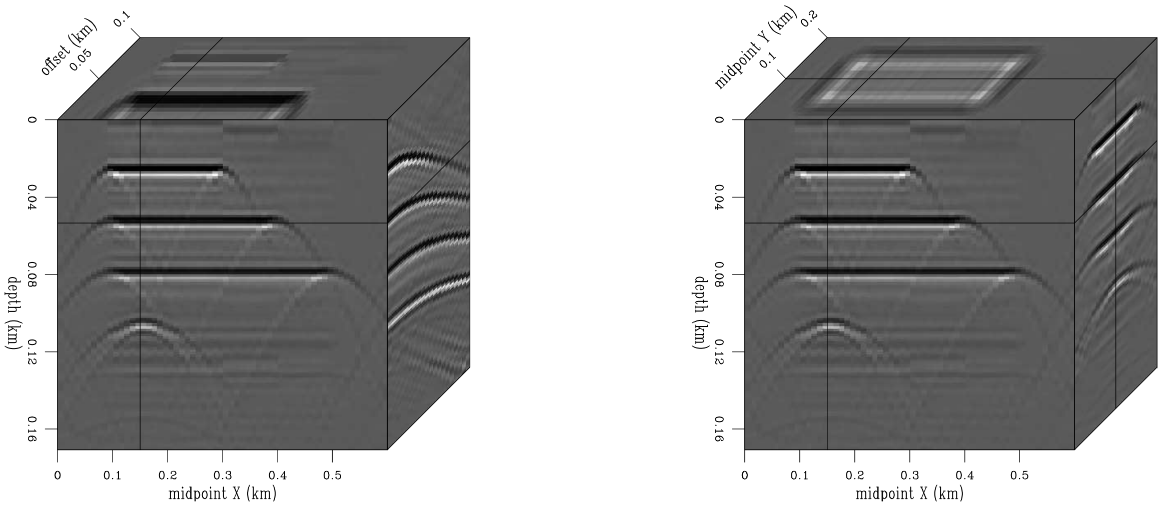

Figure 2: The data generated using Stolt common-azimuth modeling. The velocity used for modeling is $v = 3.0$ km/s. The right panel represents the zero-offset view in the common-azimuth space, while the left panel represents the prestack view for the inline selected on the right panel. $\boxed{\text{paul2-data}}$ [ER]

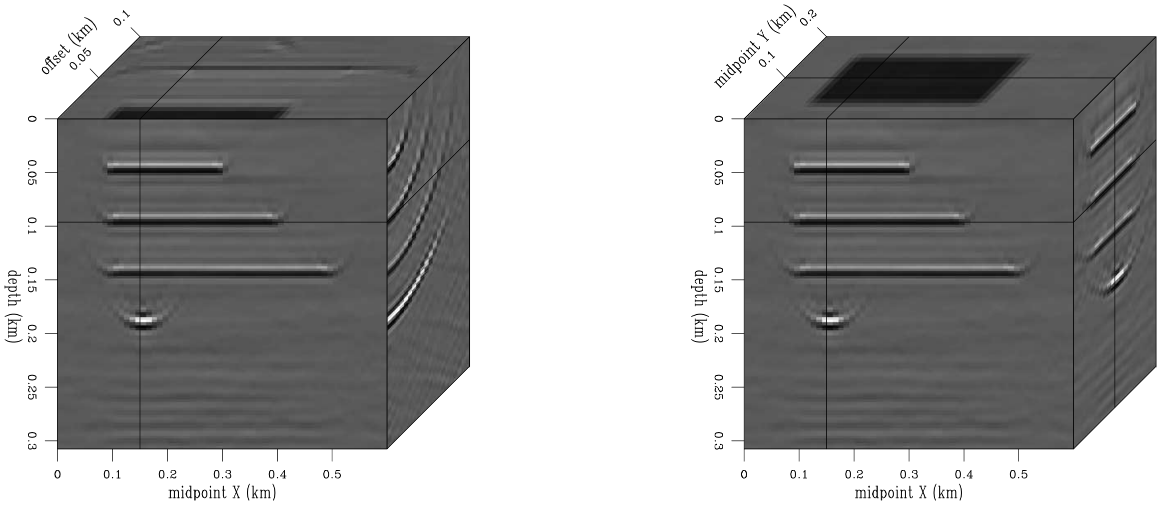

Figure 3: The image obtained using Stolt common-azimuth migration. The velocity used for migration is $v_0 = 3.6$ km/s. Since $v_0 > v$, the image is clearly over-migrated. The right panel represents the zero-offset view of the common-azimuth data, while the left panel represents the prestack view for the inline selected on the right panel. paul2-mig [ER]

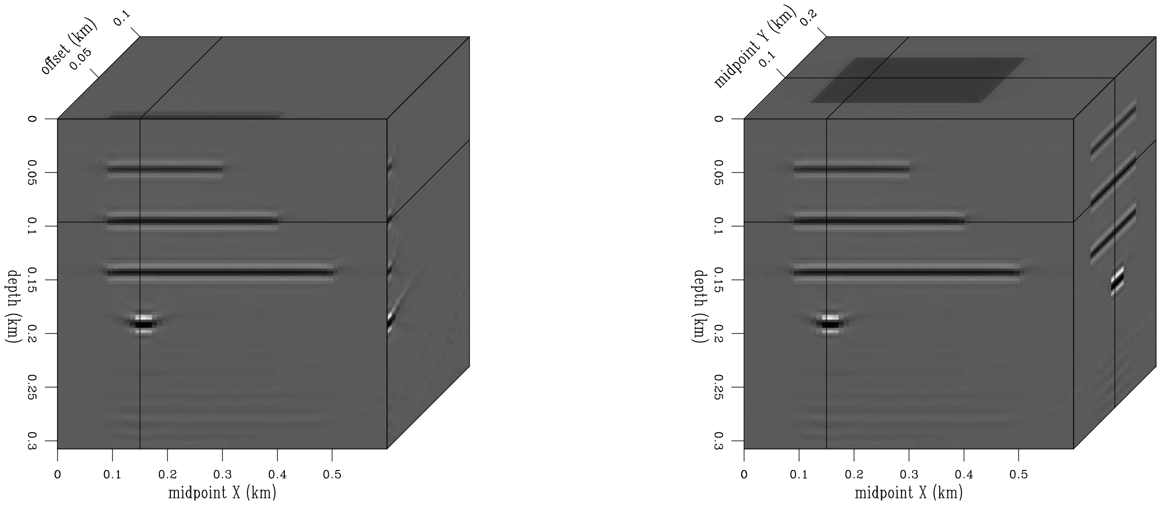

Figure 4: The image obtained using Stolt common-azimuth residual migration using velocity ratio $v_0/v = 1.2$. The image is nicely focused at zero-offset. The right panel represents the zero-offset view of the common-azimuth data, while the left panel represents the prestack view for the inline selected on the right panel. paul2-rst [ER]

migrate an image migrated with a variable velocity, we will obtain another image that has a corresponding variable velocity. In this case, however, the actual relationship between the velocities before and after residual migration is still not clarified and remains a subject for further research.

- Artifacts: Since Stolt CA residual migration is a frequency-domain method, it is susceptible to the same interpolation and wrap-around artifacts as the usual Stolt migration. Nevertheless, any technique used to reduce the Stolt artifacts (Levin, 1994; Fomel, 1998) can also be used for CA residual migration with the same efficiency.

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