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Short Note

Revisiting the half-derivative filter

Marie Prucha[1]

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INTRODUCTION

A primary goal in designing an iterative optimization method for inversion is to minimize the number of iterations necessary for convergence. Often, we tackle this by applying an appropriate filter.

When conjugate gradient (CG) methods are applied to a problem like velocity transform in 2-D, the principal artifact faced is the half-integration waveform (Claerbout, 1993, 1995). Logically, the best filter to suppress a half-integration waveform is the half-derivative filter. In practice, filters like the half-derivative filter have been reported as causing CG residuals to diverge (Lumley, 1994). However, when recently revisiting this approach, I encountered no similar obstacle to convergence.

For this work, I applied the half-derivative filter to a CG inversion used for velocity transformations for both a synthetic and a real case. I experimented with the order of the weighting functions in an attempt to improve the rate of convergence. The results in all of the cases showed that the half-derivative filter did not cause divergence in any of the cases and improved the convergence rate beyond that of CG inversion without the half-derivative filter.

THEORY

The origin of the half-derivative filter lies in the simple operation of causal integration (Claerbout, 1993). With each pass of causal integration, we are actually convolving the signal in the time domain with a scaled ramp function which is equivalent to multiplication in the frequency domain with the inverse of frequency. This can be expressed as:

$$t^{n-1}\, step(t) \;=\; FT\left(\frac{1}{(-i\omega)^n}\right) \tag{1}$$

[1]**email:** marie @ sep.Stanford.edu

In two dimensions, the principal artifact that will affect our velocity transform occurs at $n = \frac{1}{2}$ (Claerbout, 1995). This leads to

$$\frac{1}{\sqrt{t}}\, step(t) \;=\; FT\left(\frac{1}{\sqrt{-i\omega}}\right) \;. \tag{2}$$

To compensate,we need to apply $FT(\sqrt{-i\omega})$, which, recalling that:

$$\frac{d}{dt} \;=\; FT(-i\omega) \;, \tag{3}$$

we can obtain by the formula

$$\frac{d}{dt}\frac{1}{\sqrt{t}}\, step(t) \;=\; FT(\sqrt{-i\omega}) \;. \tag{4}$$

So, to repair the principal artifact of 2-D hyperbola summation, we need to apply this filter – the half-derivative filter.

PRACTICE

The half-derivative filter belongs to a family of filters known as rho filters. Past attempts (Lumley, 1994) to apply these filters to a CG inversion have exhibited divergence after the first few iterations, despite the fact that CG iteration should never diverge when the operator is linear and the gradient is non-zero. Nevertheless, I attempted CG inversion of the system:

$$d \;=\; \mathbf{HV}m \;, \tag{5}$$

where $d =$ data, $H =$ half-derivative filter, $V =$ velocity transform operator, and $m =$ model. This was applied to the CMP gathers seen in Figures 2 (synthetic) and 1 (real). These two figures also both show the residuals converging, the velocity scans resulting from the CG iterations, and the forward-modeled data. In an attempt to improve the rate of convergence, I re-ordered the equation as:

$$d \;=\; \mathbf{VH}m \tag{6}$$

This also allowed convergence. Figures 3 and 4 show the convergence rate of the second system (6) to be better than that of the first system (5) and both of these are superior to the convergence speed for the trial with no half-derivative filter. It is interesting to note that the residuals decrease more and the half-derivative filter has a greater effect for the real CMP gather than for the synthetic CMP gather. This is most likely a result of dispersion from the finite-difference method used to create the synthetic data.

CONSIDERATION OF PAST EXPERIMENTS

Lumley (1994) observed that rho filters caused divergence during CG iteration. Although he did not use the actual half-derivative filter, his results indicated that the half-derivative filter

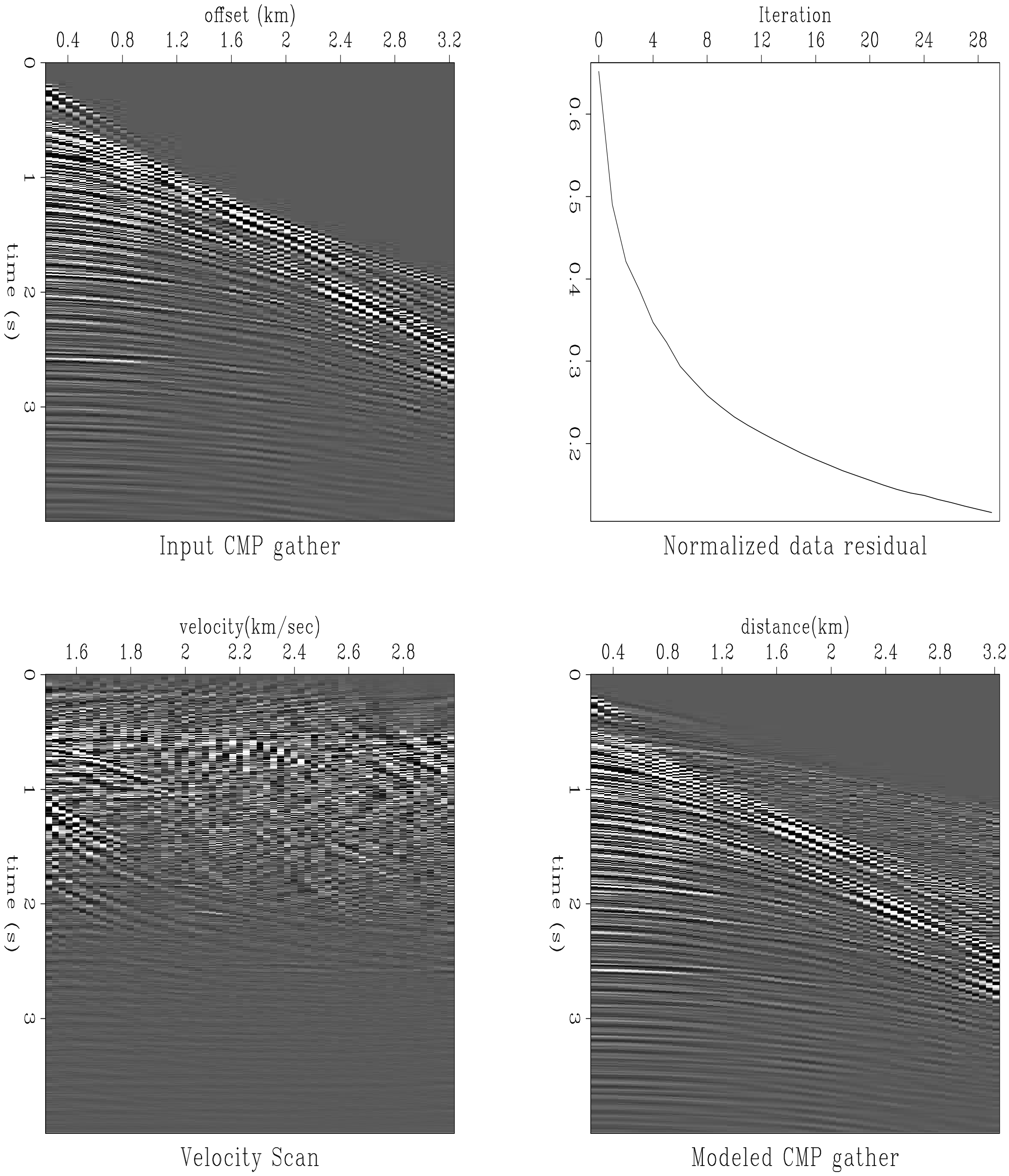

Figure 1: Top left: Real CMP gather. Top right: Residual power versus iteration. Bottom left: Representation of CMP gather in velocity space. Bottom right: Inversion of bottom left. marie1-real.vel [ER]

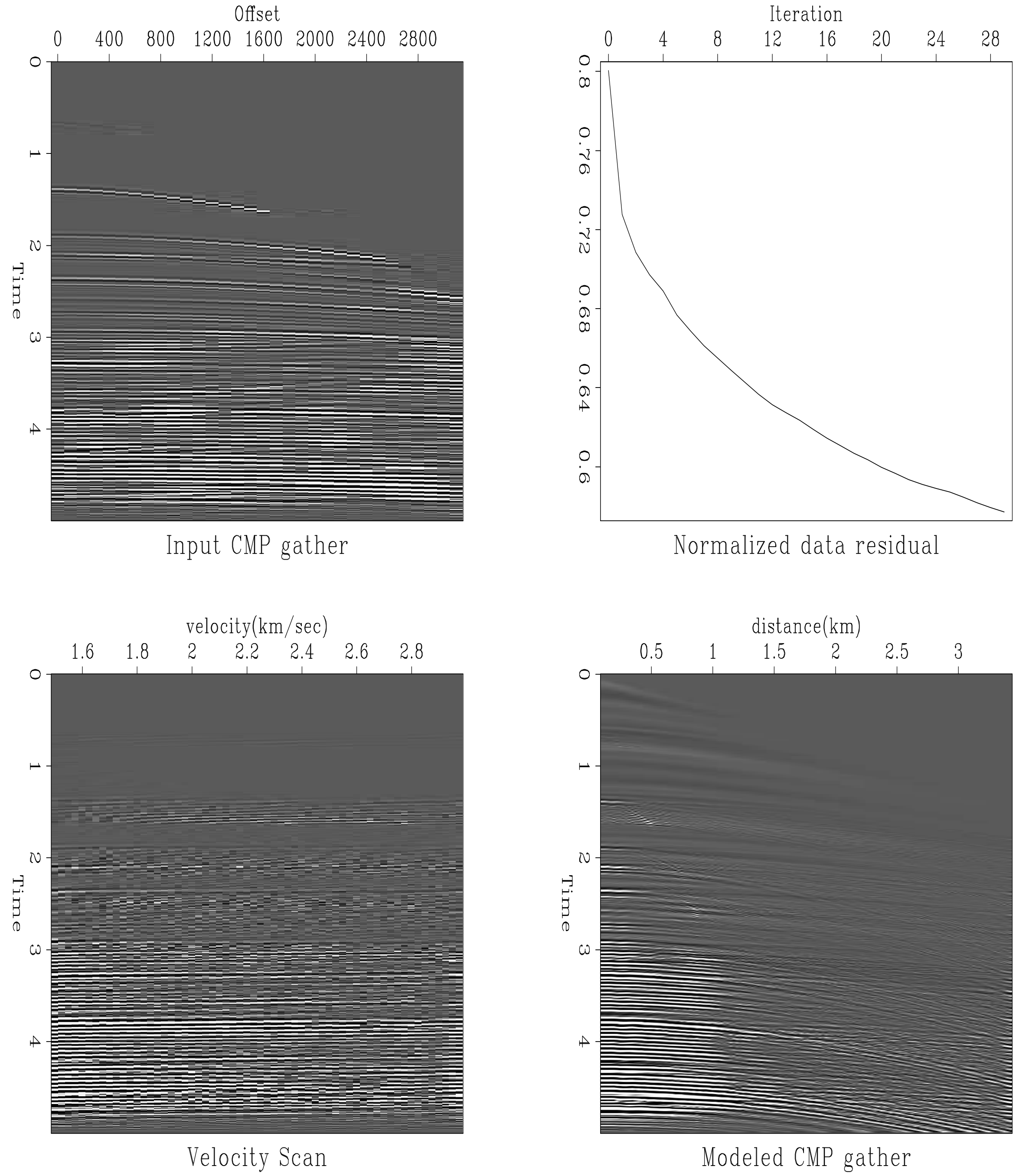

Figure 2: Top left: Synthetic CMP gather. Top Right: Residual power versus iteration. Bottom left: Representation of CMP gather in velocity space. Bottom right: Inversion of bottom left. marie1-elf.vel [ER]

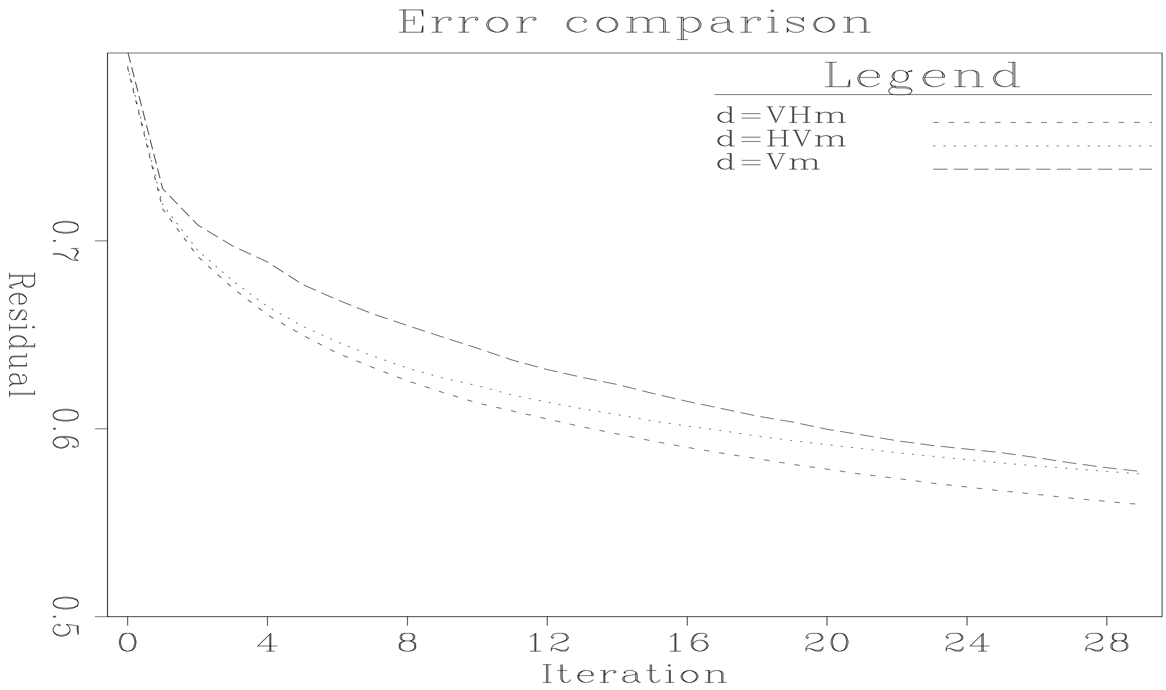

Figure 3: Comparison of rates of convergence for the synthetic data. While there is relatively little difference in the magnitudes of the final residuals, the two best rates are obtained with the half-derivative filter. marie1-elf.residual-compare [ER]

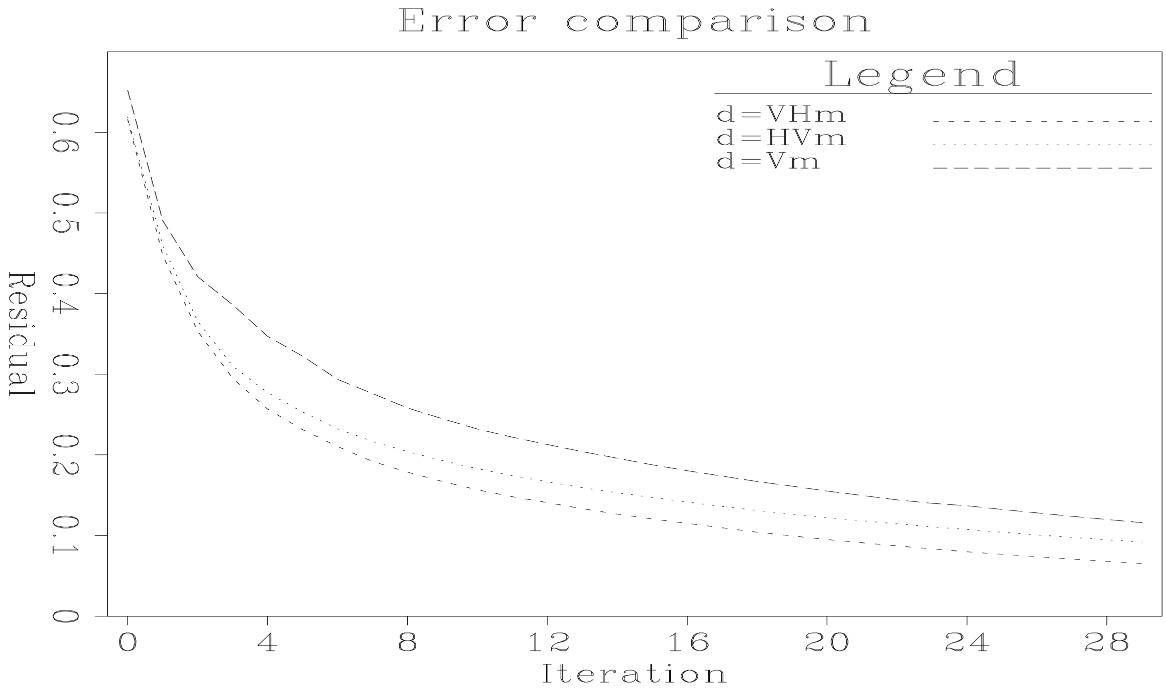

Figure 4: Comparison of rates of convergence for the real data. The best rate is obtained for the system $d = VHm$. marie1-real.residual-compare [ER]

should cause even more divergence than the rho filters he did use. While not yet confirmed, these results may have been due to the CG algorithm he used. Lumley used the CG method described by Hestenes and Steifel (1952) which can be unstable (Fomel, 1996). The results in this paper were obtained with the CG method described by Claerbout (1986).

CONCLUSION

I have shown that the half-derivative filter may be used effectively in CG velocity-transform inversion. The rate of convergence for the system $d = VHm$ was slightly faster than that for $d = HVm$ but both of these systems had better convergence rates than a system that did not use the half-derivative filter at all.

ACKNOWLEDGEMENTS

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