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Seismic velocity decrement ratios for regions of partial melt near the core-mantle boundary

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ABSTRACT

For regions of partial melt in the lower mantle, both compressional and shear wave velocities decrease monotonically with increasing melt volume fraction. It has been observed that regions close to the core-mantle boundary thought to contain partial melt have a velocity decrement ratio (relative change in shear velocity over relative change of compressional velocity) of about 3. This is certainly a high value for this ratio and arguments based on effective medium theories have been given to show that such values are predicted for partial melt systems. The present work confirms the value of 3 near the core-mantle boundary. It is shown, furthermore, that the velocity decrement ratio can be estimated without detailed knowledge of, or assumptions about, the microstructure of the partial melt system by using Gassmann's equation of poroelasicity together with some reasonable assumptions about the change (or lack of change) of the density and bulk modulus for lower mantle pressure and temperature conditions.

INTRODUCTION

A number of recent papers (*Williams and Garnero*, 1996; *Revenaugh and Meyer*, 1997; *Wen and Helmberger*, 1998a; 1998b) have shown that the ratio of seismic velocity decrements $d\ln v_s/d\ln v_s$ (where v_s is the shear velocity, v_p is the compressional velocity) is approximately equal to 3 in ultralow velocity zones near the the core-mantle boundary (see *Young and Lay* (1987) for a review of CMB issues). Changes in both numerator and denominator are negative but the ratio has been found to be on the order of 3, and because this value is so high it is generally argued that these results provide evidence of partial melt in these regions. The rock physics analyses used in these papers are generally based on classical effective medium theories such as those reviewed by *Watt et al.* (1976). The main problem with such analyses is that the results tend to be quite sensitive to the assumed microstructure of the partial melt system [see *Williams and Garnero* (1996)] and therefore may not be truly representative of the system being studied. I will give a different derivation here of the velocity decrement ratio that highlights the key assumptions that must be made to arrive at this ratio. This approach shows how general and insensitive to microstructure the *ratio* really is for partial melt systems,

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and shows furthermore how to analyze deviations from the assumptions made. The methods presented may also be extended to permit estimates of changes not only of the ratio but also of the two seismic velocities themselves. A procedure for doing so is outlined at the end of the paper.

SEISMIC VELOCITIES FOR PARTIAL MELT SYSYTEMS

The velocities, v_p and v_s, are related to the bulk (K) and shear (μ) moduli and the density (ρ) of the system by the well-known relations:

$$v_p = \sqrt{\left(K + \frac{4}{3}\mu\right)/\rho}, \tag{1}$$

and

$$v_s = \sqrt{\mu/\rho}. \tag{2}$$

These results are usually derived for elastic systems, but are also valid for poroelastic systems when the frequencies are small enough, and seismic frequencies are virtually always small enough.

Velocity analysis

It is not hard to show that the bulk and shear moduli can both be assumed to be decreasing functions of the volume fraction of partial melt. When there is no melt, the solid material constants are K_g for the purely solid (or grain) bulk modulus and μ_g for the purely solid shear modulus. As solid transforms into melt, the melt volume fraction is ϕ and the remaining solid volume fraction is $1-\phi$. General relations for the changing elastic constants for small to modest values of ϕ are

$$K_{sat} = K_g(1-c_1\phi), \tag{3}$$

and

$$\mu_{sat} = \mu_g(1-c_2\phi). \tag{4}$$

The new symbols used here are K_{sat} for bulk modulus of solid containing pores saturated with melt, μ_{sat} for shear modulus of solid containing pores saturated with melt, and c_1 and c_2 are nonnegative, dimensionless parameters. (If we were to do perturbation theory for small ϕ around the solid limit, then these parameters would be constant, independent of ϕ. But, we will instead use a more rigorous approach based on Gassmann's equation (*Gassmann*, 1951) and arrive at exact results for c_1 that incorporate ϕ dependence and are therefore valid for a much wider range of values than would be possible using perturbation theory.) If in addition we make the assumption which is commonly made about these systems [see *Williams and*

Garnero (1996)] that the melt density is approximately the same as that of the solid material, then we have the additional formula for changes in density

$$d\rho \simeq 0. \tag{5}$$

$$d\ln v_p = \frac{1}{2} d\ln\left(K_{sat} + \tfrac{4}{3}\mu_{sat}\right) \simeq -\frac{1}{2}\frac{c_1 K_g + c_2 \frac{4}{3}\mu_g}{K_g + \frac{4}{3}\mu_g}\phi, \tag{6}$$

and

$$d\ln v_s = \frac{1}{2} d\ln\mu_{sat} \simeq -\frac{1}{2}c_2\phi. \tag{7}$$

To simplify the expression in (6) further, we can make use of the well-known approximation that

$$\frac{v_p}{v_s} \simeq 2. \tag{8}$$

(We relax this strong assumption later in the paper.) Substituting (1) and (2) into (8) shows that

$$K_g \simeq \frac{8}{3}\mu_g, \tag{9}$$

which when substituted into (6) shows that

$$d\ln v_p \simeq -\frac{1}{6}(2c_1 + c_2)\phi. \tag{10}$$

Gassmann's equation

How does the constant c_1 depend on the fluid bulk modulus in a region of partial melt? The analysis usually quoted for addressing this problem in regions of partial melt have normally used some type of classical effective medium theory, which is most appropriately used for systems in which the inclusions are both disconnected and of small volume fraction ϕ. However, partial melt systems in the upper mantle are generally believed to be dominated by connected tubes of melt lying along grain edges (*Waff and Bulau*, 1979; *Mavko*, 1980; *Toramaru and Fujii*, 1986). When the fluid is in pressure-temperature equilibrium with its surroundings, it therefore makes sense to consider Gassmann's equations (*Gassmann*, 1951; *Berryman*, 1995) from the theory of poroelasticity for the system. This approach is particularly appealing for this problem because, except for an assumption of fluid connectedness, Gassmann's equations do not depend explicitly on the microgeometry, and this simplification should permit universal behavior to be predicted by the resulting theory.

Gassmann's equation for fluid substitution is often written to emphasize the change in saturated bulk modulus K_{sat} from that of the drained bulk modulus K_{dr}. The well-known result is

$$K_{sat} = K_{dr} + \frac{\alpha^2}{(\alpha - \phi)/K_g + \phi/K_f}, \tag{11}$$

where ϕ is the fluid-saturated porosity, K_g is the solid or grain material bulk modulus, K_f is the bulk modulus of the saturating fluid (the melt for this application), and

$$\alpha = 1 - \frac{K_{dr}}{K_g} \tag{12}$$

is the Biot-Willis (or effective stress) parameter. Formula (11) can be rearranged to emphasize how the saturated bulk modulus changes as the value of K_f deviates from the value of the solid bulk modulus K_g. The result is

$$K_{sat} = K_g(1 - c_1\phi), \tag{13}$$

where

$$c_1 = \frac{K_g/K_f - 1}{1 + (\phi/\alpha)(K_g/K_f - 1)}. \tag{14}$$

An important observation follows easily from (14). If the fluid bulk modulus satisfies $K_f \equiv K_g$, then $c_1 \equiv 0$ and the saturated bulk modulus is the same as that of the solid material. This is a definite prediction of Gassmann's formula. This is *not* a surprising result however, because it is also a quite general prediction of homogenization theory (for example, the well-known Hashin-Shtrikman bounds (Hashin and Shtrikman, 1963) also degenerate to a constant value when the constituents have the same moduli). If $K_f = K_g$ then the bulk modulus is actually uniform throughout the medium. The point of (14) is that it shows in addition how to compute deviations from this case when $K_f \neq K_g$ but $K_f \simeq K_g$. The result is independent of the details of the geometry of the melt system as long as the melt is connected (percolating) throughout the volume.

Predicted decrement ratio

To complete the analysis we need one more fact or approximation. We have assumed that the density of the melt differs little from that of the solid. The Birch-Murnaghan equations (*Birch*, 1938; 1952; *Anderson*, 1989) show that the bulk moduli of solid systems change in a predictable way as a function of the changing density. A similar result for simple liquids known as Rao's rule (*Rao*, 1941) also shows that the bulk modulus of many pure (*i.e.*, single constituent) liquids is also a simple function of the density. Based on these results, if the density of the melt is the same as that of the surrounding solid, then we expect the bulk modulus of the melt to differ very little (on the order of a few per cent) even though the shear modulus has dropped from a finite value to zero. We have shown in the preceding paragraph that, if the fluid inclusions have the same bulk modulus as the solid, then $K_{sat} = K_g$ and $c_1 = 0$.

We expect this approximation to have the same level of validity as the approximation that the density is constant (which is to say that we think of it as a reasonable first approximation). With this approximation substituted into (10), we find that

$$\frac{d\ln v_s}{d\ln v_p} \simeq 3. \tag{15}$$

This is correct as long as $c_2 \neq 0$. But the shear modulus will necessarily decrease as the melt fraction increases, since, for fixed overall shear distortion, less shear energy can be stored in the system, and this implies that $c_2 > 0$. Computing the actual value of c_2 requires a model of the microstructure, but the result (15) shows that the precise value c_2 is not required to obtain the desired result in (15), since it cancels out of the final formula.

EXTENSION, EXAMPLES, AND EXPERIMENT

If the actual ratio v_p/v_s differs significantly from (8), then we can repeat the calculation using the parametrization

$$\frac{v_p}{v_s} = 2\sqrt{1-\delta}, \tag{16}$$

where δ is a small number on the order of $\delta \simeq 0.5$ or less. The general result then becomes

$$\frac{d\ln v_s}{d\ln v_p} \simeq \frac{3v_p^2}{4v_s^2} = 3(1-\delta), \tag{17}$$

showing, for example, that solid material $v_p/v_s \simeq 1.4$ or 1.7 implies a decrement ratio $\simeq$ 1.5 or 2.2, respectively. A plot of these results is shown in Figure 1, where various models (*Doornbos and Mondt*, 1979; *Dziewonski and Anderson*, 1981; *Kennett and Engdahl*, 1991) of the velocities at the core-mantle boundary are used to provide specific examples of the predictions obtained using this approach. For comparison, the value anticipated for olivine at 2 GPa in the upper mantle is also plotted to show that the results obtained are very close to those of *Mavko* (1980), who used much more detailed model calculations to arrive at the result. (*Mavko* (1980) found, using various assumed microstructures and a self-consistent effective medium approach, that the expected change was about 10% in shear velocity and about 5% in compressional velocity, giving a decrement ratio of about 2 – which compares favorably with the result 2.2 obtained here for olivine.) Table 1 lists the special values used for the plot.

To see how these results compare with the observations, consider the plots of *Revenaugh and Meyer* (1997) showing that, for the most credible models of lower mantle deviations from IASP91, the seismic velocity decrement can lie in the range from 2 to 5, with the most likely value being approximately equal to 3.

TABLE 1. Seismic v_p/v_s ratios and predicted velocity decrement ratios at the core-mantle boundary for some standard earth models, and olivine at 2 GPa.

Earth Model	v_p/v_s	$d\ln v_s/d\ln v_p$
PEMC-L01	1.9105	2.738
PREM	1.8866	2.669
IASP91	1.8751	2.637
Material		
Olivine	1.72	2.219

CONCLUSIONS

This analysis has shown that a seismic velocity decrement ratio of about 3 is expected in partial melt systems at the core mantle boundary based on an elementary use of Gassmann's equations (*Gassmann*, 1951) and some simple assumptions about small changes in density and bulk modulus near the CMB.

Corrections to these results can also be computed using the formulas presented. If the density changes significantly, then (6) and (7) must be modified using (1) and (2). Rao's rule (*Rao*, 1941) can then be used to obtain estimates of K_f, and these results substituted into (14). Effective medium theory (*Watt et al.*, 1976; *Berryman*, 1995) is required in these more precise calculations to determine K_{dr} for use in (12) and again in (14), and similarly to determine c_2 for the shear modulus.

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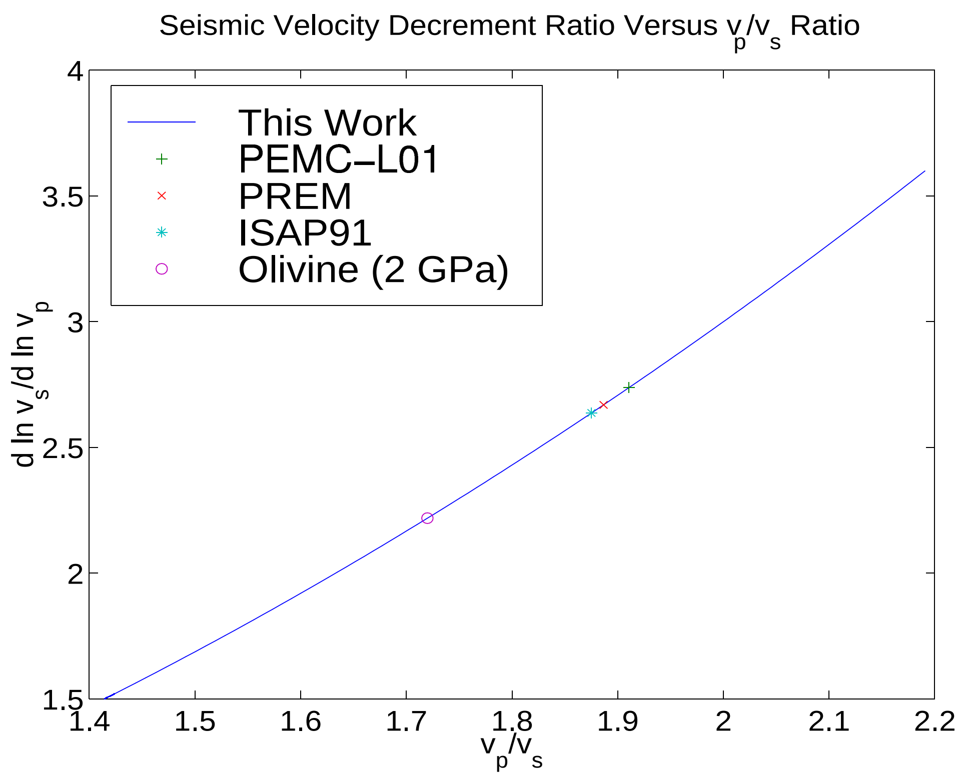

Figure 1: Plot of the dependence of the theoretically predicted seismic velocity decrement ratio on the v_p/v_s ratio. Specific points shown correspond to values of v_p/v_s for the lower mantle just above the core-mantle boundary for various models: PEMC-L01 (*Doornbos and Mondt*, 1979), PREM (*Dziewonski and Anderson*, 1981) and IASP91 (*Kennett and Engdahl*, 1991). For comparison, the result for olivine at 2 GPa with $v_p/v_s \simeq 1.72$, which would be typical of upper mantle conditions, is also shown. jim3-various [NR]

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