Refined Source Waveform Estimation

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This source waveform estimator is chosen to minimize the power in the Noah seismogram. The basic idea is that an incorrect source waveform estimate yields a Noah seismogram which is not free from surface multiples. To the extent that such multiples add power to a seismogram, a source waveform minimizing power can be expected to be a good choice. Obviously it won't work in the occasional situation where multiples are canceling primaries. Likewise, if the source waveform estimator has too many parameters compared to the data it will begin devouring primaries as well as multiples.

The Noah seismogram $U(Z)$ has been defined as the seismogram recorded during a severe flood (SEP-1, p. 83). It is preferable to the conventional seismogram $R(Z)$ in that all free-surface multiples are absent. To deduce the Noah seismogram from the conventional seismogram it is merely necessary to equate the ratio of upcoming wave divided by downgoing wave in the presence of a free surface to that without a free surface, namely,

$$U(Z) = \frac{U(Z)}{1} = \frac{R(Z)}{1+R(Z)} = \frac{\text{up}}{\text{down}} \tag{1}$$

The Z transforms in (1) may be called the stick seismograms or the broadband seismograms because of the presumption that the source waveform was an impulse. Realistic seismograms have convolved in them some coloring waveform, say $B(Z)$. Although we will refer to $B(Z)$ as the shot waveform, it clearly incorporates such effects as recording filters. As a practical matter it might also involve some near surface attenuation, ghosting or filtering effects too. Thus, we define the colored seismograms by

$$U'(Z) = B(Z)\,U(Z) \tag{2a}$$

$$R'(Z) = B(Z)\,R(Z) \tag{2b}$$

Combining (1) and (2) we have

$$U' = B\,U = \frac{B^2 R}{B+BR} = \frac{B\,R'}{B+R'} \tag{3}$$

The main part of the calculation is the next step where we compute the partial derivative of the colored Noah seismogram U' with respect to the source waveform B . We have

$$\frac{\partial U'}{\partial B} = \frac{R'}{B+R'} - \frac{B\,R'}{(B+R')^2} = \frac{R'(B+R') - BR'}{(B+R')^2} = \frac{R'^2}{(B+R')^2} = \left(\frac{U'}{B}\right)^2 = U^2 \tag{4}$$

We intend to control the number of degrees of freedom by limiting the time duration of the shot waveform B . As a practical matter this means that we need Z transform coefficients for the members of

$$dU' = \frac{\partial U'}{\partial B}\,dB \tag{5}$$

Say

$$dB(Z) = db_0 + db_1 Z + db_2 Z^2 + \cdots \tag{6a}$$

$$dU(Z) = du_0 + du_1 Z + du_2 Z^2 + \cdots \tag{6b}$$

$$W(Z) = U(Z)^2 = w_0 + w_1 Z + w_2 Z^2 + \cdots \tag{6c}$$

In terms of Z transform coefficients (5) becomes

$$\begin{bmatrix} du'_0 \\ du'_1 \\ du'_2 \\ \\ \\ \\ \\ \end{bmatrix} = \begin{bmatrix} w_0 & & \\ w_1 & w_0 & \\ w_2 & w_1 & w_0 \\ w_3 & w_2 & w_1 \\ & w_3 & w_2 \\ & & w_3 \\ & & \end{bmatrix} \begin{bmatrix} db_0 \\ db_1 \\ \\ db_m \end{bmatrix} \tag{7}$$

Now our objective is to minimize the power in the colored Noah seismogram U' by variation of the shot waveform B. It is evident that if we take the squared magnitude of expression (3) for U', we do not get a quadratic function of B so that the stationary condition $d/dB = 0$ does not lead to linear equations. Thus, we cannot use linear least squares and we must use iterative non-linear least squares. Let $\bar{U}$ be an approximation to the correct U. Then we can minimize the length of

$$\bar{U}' + dU' = \bar{U}' + \frac{\partial \bar{U}'}{\partial B} dB \tag{8}$$

by variation of dB.

Let us write this out as an overdetermined system of simultaneous equations. For clarity in interpretation let us take the water depth to be such that $\bar{u}_1$ and $\bar{u}_2$ vanish and $\bar{u}_3$ represents the sea floor. This implies that w_0 through w_5 vanish too. For clarity let us also limit the shot waveform duration to three points. Thus,

$$
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\approx
\begin{bmatrix} 0 \\ 0 \\ 0 \\ \bar{u}'_3 \\ \bar{u}'_4 \\ \bar{u}'_5 \\ \bar{u}'_6 \\ \vdots \\ \\ \\ \\ \end{bmatrix}
+
\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ w_6 & 0 & 0 \\ w_7 & w_6 & 0 \\ w_8 & w_7 & w_6 \\ & w_8 & w_7 \\ & & w_8 \end{bmatrix}
\begin{bmatrix} db_0 \\ db_1 \\ db_2 \end{bmatrix}
\tag{9}
$$

If the method of least squares is used, then the overdetermined system (9) generates Toeplitz equations which may be solved by the Levinson recursion. Since the dimensionality of b may often be taken very small, the L_1 norm might also be easily used, giving a more robust result.

Now let us interpret the result as the iteration approaches equilibrium. Then, W tends toward the square of the broadband Noah seismogram. I like to think of it as follows: Think of the broadband Noah seismogram as being very similar to the reflection coefficient series. Instead of imagining that the initial downgoing wave was a pulse, one could imagine it as the broadband Noah seismogram. For such a downgoing wave the upcoming wave $W = U^2$ would contain multiples. These could be used to predict any possible multiples in $\bar{U}$. Note that a sea floor at u_3 generates a sea floor multiple at w_6 . Hence in the minimization (9) the parameters db_k are only trying to suppress u_t at and after the time of the first sea floor multiple.

Now the question arises whether we really want to do the minimization on (9) or whether we would like to use a weighting function or say, minimize $||U||^2$ instead of $||U'||^2$. It is certainly nice that the columns of the partial derivative matrix are broadband. Clearly, gain exponentially increasing with time might be advantageous.

Fortunately it seems that slanted coordinates are easily introduced. Then, U^2 would be replaced by the product of U at x with U at x + shift .

It is also interesting to note that unlike some of our previous source waveform estimators, the distinction between shallow water and deep water is not so sharp. Namely, explicit primary and multiple gates need not be picked, so we might find that this approach works even where water depth is less than the shot waveform duration.