Robust reflection tomography in the time domain

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SUMMARY

The convergence of conventional reflection tomography is often uncertain when the starting velocity function is too far from the correct one. The time-domain reflection tomography that we present in this paper is more robust than conventional depth-domain reflection tomography. Our new tomographic method avoids some of the instabilities of conventional depth-domain tomography, by solving for a velocity model and reflector geometry defined in vertical-traveltime domain. These instabilities are often caused by the coupling between the velocity function and the depth-mapping of reflectors. Time-domain tomography keeps the distinction, which is typical of time processing, between the velocity function that best focuses the data and the velocity function that correctly maps the reflectors in depth.

Time-domain reflection tomography is based on a new eikonal equation that is derived by a transformation of the conventional eikonal equation from depth coordinates (z, x) into vertical-traveltime coordinates (τ, ξ). The transformed eikonal enables the computation of reflections traveltimes independent of depth-mapping. This separation allows the focusing and mapping steps to be performed sequentially even in the presence of complex velocity functions, that would otherwise require "depth" migration.

We compute the solutions of the transformed eikonal equation by solving the associated ray tracing equations. The application of Fermat's principle leads to the expression of linear relationships between perturbations in traveltimes and perturbations in focusing velocity. We use this linearization, in conjunction with ray tracing, for the time-domain tomographic estimation of focusing velocity.

INTRODUCTION

Velocity has a dual role in reflection-seismic imaging. It is needed to focus the data through migration, and to map the reflectors in depth by converting arrival times into depths. These two imaging goals are often conflicting. The velocity function that best focuses the data is not necessarily the velocity that performs the correct depth mapping.

The *focusing* velocity is the one that best predicts the relative delays between reflections originated at the same point in the subsurface and recorded at different offsets and midpoints. We can measure these relative delays and try to estimate the focusing velocity by solving an inverse problem. On the contrary, the *mapping* velocity mostly affects the absolute delays of the reflections. If we do not know the depth of the reflectors, we cannot estimate the mapping velocity from reflection data. To estimate mapping velocity, we need other sources of information, such as well data and a priori geological information. It would be natural to keep the distinction between focusing and mapping velocity estimating the focusing velocity from the recorded data, but that is not the current practice in performing depth imaging.

The distinction between focusing and mapping velocity is routinely taken into account when the data are imaged in the time domain, and is one of the source of robustness of the time-imaging procedure. In time imaging, the data are first focused by determining stacking and/or root-mean-square (RMS) velocities, then map-migrated to depth along the image rays using an appropriate mapping velocity (Hubral, 1977; Larner et al., 1981). A related advantage of time imaging is that both the reflectors and the focusing velocity are mapped in time and not in depth. Consequently the deeper parts of the model are only marginally sensitive to velocity variations in the shallower parts of the model. Unfortunately all these useful features of time imaging are currently not available when strong lateral velocity variations make prestack depth migration necessary to focus the data. This shortcoming of depth migration is not intrinsic, but it is imposed by limitations in current depth-migration and tomography methods. In this paper we present a time-domain methodology for both imaging the data and estimating velocity valid in arbitrarily inhomogeneous media. The method is based on a coordinate transformation from depth to two-way vertical traveltime applied to the eikonal equation. We demonstrate that under fairly mild assumptions on the relation between focusing and mapping velocities, the traveltimes computed using the transformed eikonal are only functions of the focusing velocity. As a result, we call the transformed eikonal equation the *focusing eikonal*. The focusing eikonal provides the analytical foundations necessary to apply to depth-migration problems the same robust two-steps imaging procedure that is possible in time imaging.

We present a time-domain reflection tomography method that is similar to conventional depth-domain reflection tomography (Stork and Clayton, 1991), but with an important difference: both the reflectors and the velocity function are mapped in time and not in depth. Therefore, time-domain tomography avoids the instability induced by vertical shifts of both the reflectors and velocity function when the velocity is perturbed in a shallower layer. This instability often forces the application of reflection tomography in a layer-stripping procedure, although layer stripping is known to be less accurate than global tomography. Global tomography has the advantage over layer stripping of using the reflections from all layers at the same time. In contrast, in time-domain tomography the reflector geometry and the positioning in depth of velocity anomalies are only weakly dependent on the velocity of the overburden. This decoupling makes the relationship between the focusing velocity and the reflection traveltimes more linear and thus improves the robustness of global tomography and makes layer stripping unnecessary.

The example that we present shows that time-domain reflection tomography converges more robustly, and to a more accurate solution, than the equivalent tomography in the depth domain (Clapp et al., 1998). In particular, while the first few non-linear iterations of depth-domain tomography required layer stripping to prevent divergence, time-domain tomography converged towards the desired solution starting from the first non-linear iteration.

FOCUSING EIKONAL EQUATION

To derive the focusing eikonal equation we apply a coordinate transformation from depth z to two-ways vertical traveltime τ to the eikonal of the acoustic wave equation. The eikonal equation for the arrival time t of high-frequency acoustic waves is

$$V_m\left(z,x\right)^2\left[\frac{\partial t\left(z,x\right)}{\partial z}\right]^2+V_f\left(z,x\right)^2\left[\frac{\partial t\left(z,x\right)}{\partial x}\right]^2=1, \qquad (1)$$

where V_m and V_f are respectively the **mapping velocity** and the **focusing velocity**. Because we are interested to analyze the effects of focusing and mapping velocities on reflection traveltimes, we kept the V_f and V_m distinguished in equation (1). Although this equation is valid for a general elliptical anisotropic medium, in this paper we focus on isotropic media. Alkhalifah et al. (1998). discuss the focusing eikonal for a general transversely isotropic media with a vertical axis of symmetry

The mapping between the depth (z, x) and the vertical traveltime (τ, ξ) domain are defined by the following transformation of coordinates:

$$\tau\left(z,x\right) \quad = \quad \int_0^z \frac{2}{V\left(z',x\right)}dz' \qquad (2)$$

$$\xi(z,x) = x. \tag{3}$$

This transformation implies the following relationships between the partial derivatives of the traveltime that appear in the eikonal equation (1):

$$\frac{\partial t}{\partial z} = \frac{\partial t}{\partial \tau}\frac{2}{V_m(z,x)} \tag{4}$$

$$\frac{\partial t}{\partial x} = \frac{\partial t}{\partial \xi} + \frac{\partial t}{\partial \tau}\int_0^z \frac{\partial}{\partial x}\left[\frac{2}{V_m(z',x)}\right]dz' = \frac{\partial t}{\partial \xi} + \frac{\partial t}{\partial \tau}\sigma_m. \tag{5}$$

Substituting these partial derivatives in the eikonal equation (1) we derive the **focusing eikonal** equation

$$4\left[\frac{\partial t(\tau,\xi)}{\partial \tau}\right]^2 + V_f^2\left[\frac{\partial t(\tau,\xi)}{\partial \xi} + \sigma_m\frac{\partial t(\tau,\xi)}{\partial \tau}\right]^2 = 1. \tag{6}$$

The focusing eikonal depends directly from the the focusing velocity but only indirectly from the mapping velocity, through the **differential mapping factor** σ_m. Furthermore, because σ_m is the vertical integral of the horizontal derivative of V_m, when V_m is assumed to be proportional to V_f with a constant of proportionality that is only function of depth; that is,

$$V_m(z,x) = \alpha(z)\,V_f(z,x), \tag{7}$$

the focusing eikonal becomes independent from V_m. This property is easily demonstrated by performing the change of variable from z to τ defined in equation (2) in the integral that defines σ_m in equation (5). This result demonstrates that, as long as the condition of equation (7) is satisfied, reflection data can be focused without knowledge of the mapping velocity, and thus that the focusing step and the mapping step can be performed sequentially.

Notice that in an horizontally stratified medium the focusing eikonal becomes the eikonal for an elliptical anisotropic medium with normalized vertical “velocity” equal to 2. If the velocity is laterally varying, neglecting σ_f is equivalent to neglecting the thin-lens term in finite-difference time migration (Hatton et al., 1981). Raynaud and Thore (1993) used this approximation to trace rays in the τ domain.

Ray tracing in (τ,ξ)

The solutions to the focusing eikonal can be computed using current methods for solving the standard eikonal, either directly by modern eikonal solvers (Sethian and Popovici, 1997), or by ray tracing. We chose a ray tracing solution, because for reflection tomography is handier to have rays than traveltime maps.

To derive the ray-tracing system for the focusing eikonal we begin by writing its associated Hamiltonian as a function of the ray parameters p_τ and p_ξ,

$$H(\tau,\xi,p_\tau,p_\xi) = \frac{1}{2}\left\{4p_\tau^2 + V_f^2\left[p_\xi + \sigma_f p_\tau\right]^2\right\}. \tag{8}$$

The associated ray-tracing equation are:

$$\frac{d\xi}{dt} = V_f^2\left(p_\xi + \sigma_f p_\tau\right)$$
$$\frac{d\tau}{dt} = V_f^2\sigma_f\left(p_\xi + \sigma_f p_\tau\right) + 4p_\tau \tag{9}$$
$$\frac{dp_\xi}{dt} = -\left[\left(p_\xi + \sigma_f p_\tau\right)^2 V_f\frac{\partial V_f}{\partial \xi} + V_f^2 p_\tau\left(p_\xi + \sigma_f p_\tau\right)\frac{\partial \sigma_f}{\partial \xi}\right]$$
$$\frac{dp_\tau}{dt} = -\left[\left(p_\xi + \sigma_f p_\tau\right)^2 V_f\frac{\partial V_f}{\partial \tau} + V_f^2 p_\tau\left(p_\xi + \sigma_f p_\tau\right)\frac{\partial \sigma_f}{\partial \tau}\right].$$

Rays can be traced in (τ,ξ) by solving the ray-tracing equations in (10) by a standard ODE solver. The appropriate initial conditions for the ray parameters p_τ and p_ξ when the source is at (τ_0,ξ_0) and the take-off angle is θ_τ are:

$$p_{\tau_0} = \frac{\cos\theta_\tau}{2}$$
$$p_{\xi_0} = \left[\frac{\sin\theta_\tau}{V(\tau_0,\xi_0)} - \sigma_f(\tau_0,\xi_0)\,p_{\tau_0}\right]. \tag{10}$$

To test the accuracy of our derivations we numerically solved the ray tracing equations (10) for a heterogeneous velocity function, and compared the results with a ray-tracing solution of the standard eikonal equation. As expected, τ-rays map exactly into z-rays, for all velocity fields. Figure 1 and Figure 2 show an example of the ray field when the velocity function is a Gaussian-shaped negative velocity anomaly superimposed onto a constant velocity background. Notice that the focusing eikonal handles correctly the caustic and wavefront triplication below the anomaly. Figure 3 shows the effects of neglecting the differential mapping factor σ_f. It shows the τ-rays computed setting σ_f to zero, and remapped into (z,x). The wavefronts are distorted compared to the true wavefronts shown in Figure 1

REFLECTION TOMOGRAPHY IN (τ,ξ)

To perform reflection tomography, in addition to ray tracing, we need to compute the gradient of traveltimes with respect to the velocity function. In this section we derive the traveltime gradients for τ-rays. The derivation is straightforward and is based on Fermat principle applied to the τ-rays.

The transformation of variables defined in equations (2) and (3) implies the following relationships between the differential quantities (dz,dx) and $(d\tau,d\xi)$.

$$dz = \frac{V_m}{2}d\tau - \frac{V_m\sigma_f}{2}d\xi \tag{11}$$
$$dx = d\xi. \tag{12}$$

Applying this transformations to the expression of the time increment along a z-ray, leads to the equivalent expression for the the time increment along a τ-ray,

$$dt = \sqrt{\frac{dz^2}{V_m^2} + \frac{dx^2}{V_f^2}} = \sqrt{\left(\frac{d\tau - \sigma_f d\xi}{2}\right)^2 + S_f^2 d\xi^2}, \tag{13}$$

where S_f is the focusing slowness. The first derivative of the time increment dt with respect to the focusing slowness is given by

$$\frac{d(dt)}{dS_f} = \frac{\widetilde{S}_f d\xi^2}{\sqrt{\left(\frac{d\tau - \widetilde{\sigma}_f d\xi}{2}\right)^2 + \widetilde{S}_f^{\,2} d\xi^2}}\frac{d\widetilde{S}_f}{dS_f}$$

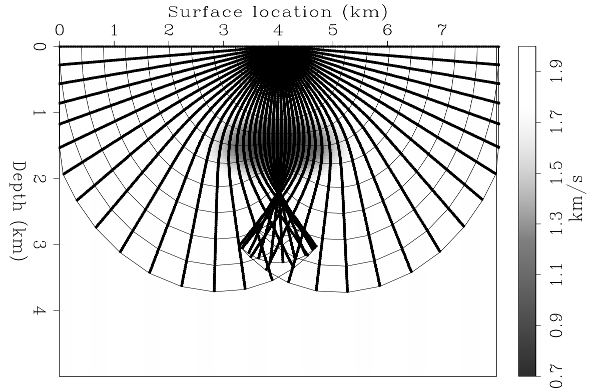

Figure 1: Ray field in (z, x) domain with a negative velocity anomaly in constant background.

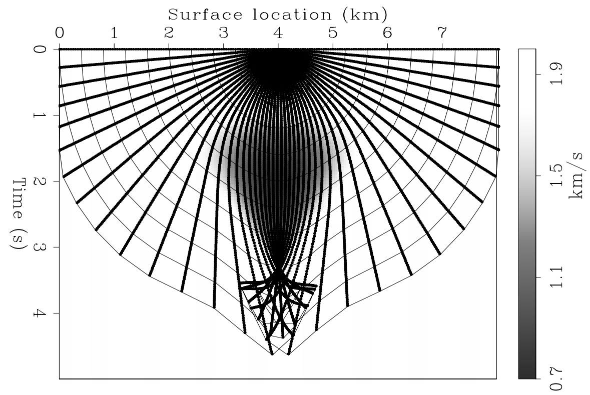

Figure 2: Ray field in (τ, ξ) domain with a negative velocity anomaly in constant background. This ray field maps exactly into the one shown in Figure 1.

$$- \frac{\left(d\tau - \widetilde{\sigma}_f d\xi\right) d\xi}{4\sqrt{\left(\frac{d\tau - \widetilde{\sigma}_f d\xi}{2}\right)^2 + \widetilde{S}_f{}^2 d\xi^2}} \frac{d\widetilde{\sigma}_f}{dS_f}$$

$$= \frac{\widetilde{S}_f d\xi^2}{\widetilde{dt}} \frac{d\widetilde{S}_f}{dS_f} - \frac{\left(d\tau - \widetilde{\sigma}_f d\xi\right) d\xi}{4\widetilde{dt},} \frac{d\widetilde{\sigma}_f}{dS_f}, \quad (14)$$

where the tildes on the variables indicate that they are evaluated along the raypath.

Applying Fermat principle, the first order perturbations in the traveltimes Δt caused by perturbations in slowness ΔS_f are given by the following integral evaluated along the unperturbed raypath τ-ray,

$$\Delta t = \int_{\tau-\mathrm{ray}} \frac{d\,(dt)}{dS_f} \Delta S_f \;\; dl, \quad (15)$$

where dl is the path-length increment. Notice that the τ-ray is not stationary in the (z, x) domain, but that the term in equation (14) that includes $\widetilde{\sigma_f}$ takes into account the perturbation of the raypath in the (z, x) domain.

Tracking reflectors movements

One of the most challenging problems of reflection tomography is to

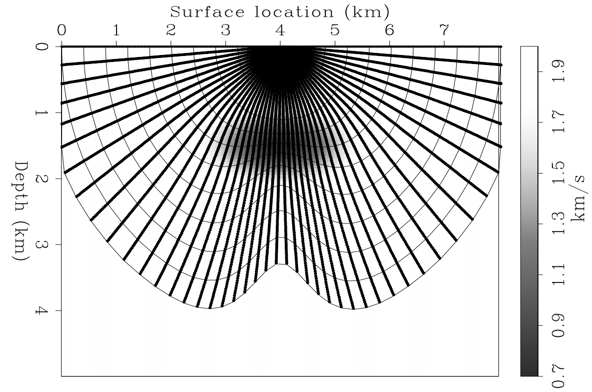

Figure 3: Ray field in (z, x) domain computed assuming the differential mapping factor σ equal to zero. This ray field is different than the one shown in Figure 1.

track correctly the movement of reflectors caused by changes in velocity. Usually the reflectors are parametrized independently from velocity and large reflectors movement can cause instability in the inversion process. One of the advantages of (τ, ξ) tomography over (z, x) tomography is that reflectors move less in the (τ, ξ) than in the (z, x) domain, and that they move more consistently with the velocity function. In time-domain tomography the vertical component of the reflector movement is automatically taken into account by the linearization introduced in equation (14). In contrast, (z, x) tomography needs an additional term to take into account the reflectors movements. In the examples shown in this paper we used the following adaptation of the expression presented by Stork and Clayton (1991) to correct for the residual mapping in (z, x) tomography:

$$\Delta t = \Delta z \left(p_{z\Downarrow} + p_{z\Uparrow}\right), \quad (16)$$

where Δz is the reflector vertical movement, while $p_{z\Downarrow}$ and $p_{z\Uparrow}$ are respectively the vertical ray parameters of the incident and reflected rays at the reflection point. To be consistent in the comparison between (τ, ξ) domain tomography and (z, x) domain tomography, we computed Δz as a residual mapping along the vertical path. A more accurate reflector movement could be computed by performing a residual map migration of the zero-offset arrivals. This residual migration term could be added to both (τ, ξ) domain tomography and (z, x) domain tomography.

TOMOGRAPHY IN (τ, ξ) VS. TOMOGRAPHY (Z, X)

We compare the performances of time-domain tomography with the performances of depth-domain tomography on the estimation of the velocity model shown in Figure 4 (depth domain) and Figure 5 (time domain). We used the traveltimes of the reflections from the four reflectors superimposed as solid lines onto the velocity models. At each iteration the reflector geometry is estimated by map-migrating the true zero-offset arrivals assuming the velocity model obtained by the previous iteration. The reflector geometry corresponding to the starting velocity model (layered medium) are shown by the dashed lines superimposed onto the velocity models. Notice that the starting reflection geometry for time-domain tomography is almost undistinguishable from the correct one. In contrast, in the depth-domain the starting reflectors show a significant pull-up. Because the velocity estimation problem is poorly constrained we regularized the tomographic inversion by imposing a smoothness constrain on the model. In particular, we applied the inverse of a Laplacian as smoothing function, within the inversion framework described by Clapp et al. (1998).

Figure 6 shows the perturbations from the initial model estimated by

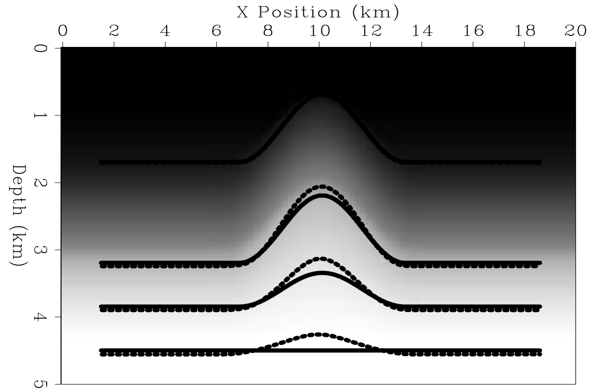

Figure 4: Correct velocity model and reflector geometries in depth.

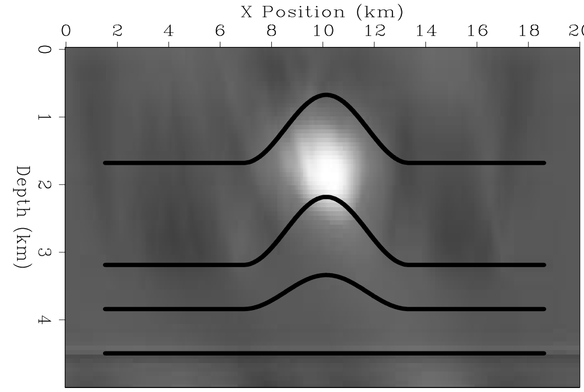

Figure 6: Velocity perturbations estimated by depth-domain tomography.

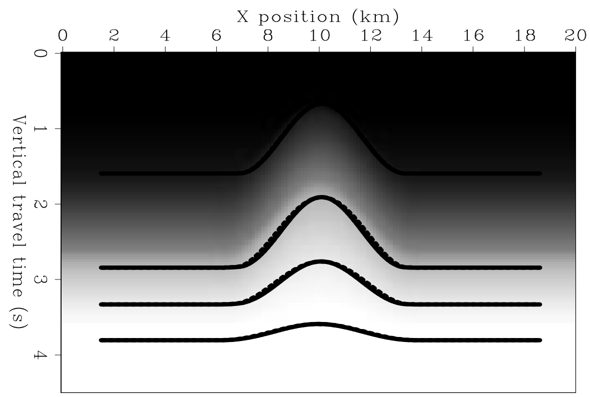

Figure 5: Correct velocity model and reflector geometries in time.

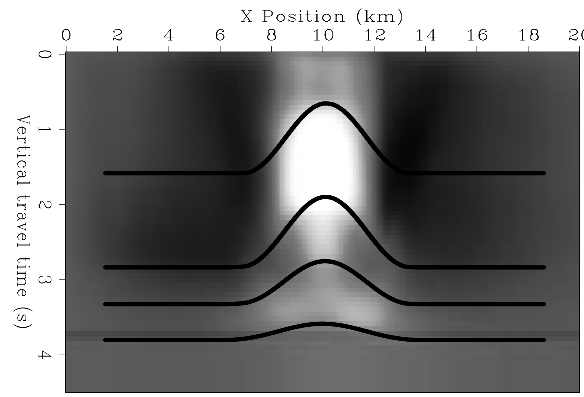

Figure 7: Velocity perturbations estimated by time-domain tomography.

depth-domain tomography after few non-linear iterations. Layer stripping was required to prevent depth-domain tomography from diverging. Figure 7 shows the results of the time-domain tomography after just one non-linear iteration. It compares very favorably with Figure 6. The time-domain tomography convergeed toward the correct solution in one single iteration, without the need of applying layer stripping.

CONCLUSIONS

We introduced a new tomographic method for estimating velocity functions and reflectors geometry in the time domain. A comparison between conventional depth-domain tomography and the new time-domain tomography on a simple estimation problem indicates that reflection tomography in the time domain converges faster and more robustly than the equivalent tomography in the depth domain. Reflection tomography in the time domain is made straightforward by our derivation of the focusing eikonal equation. We showed that the focusing eikonal equation exactly models the traveltimes in a heterogeneous medium parametrized by the vertical traveltime τ in place of depth z. The solutions of the focusing eikonal can be efficiently computed by solving the associated ray tracing equations.

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