Regularizing velocity estimation using geologic dip information

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SUMMARY

Standard tomography schemes suffer from slow convergence and tend to create isotropic features in velocity that are unreasonable when applying other criteria such as geologic feasibility. These isotropic features are due in large part to using symmetric regularization operators. By replacing these symmetric operators with operators that tend to spread information along structural dip we can generate more geologically reasonable velocity models. We can find the inverse of these space-varying, anisotropic operators by forming them in helix 1-D space and performing polynomial division. These inverse operators can be used as a preconditioner to a standard tomography problem, significantly improving convergence speed compared to the typical, regularized inversion problem.

INTRODUCTION

When attempting to do inversion we are constantly confronted with the problem of slow convergence. Claerbout and Nichols(1994) suggested using a preconditioner to speed up convergence. Unfortunately it is often difficult to find an appropriate preconditioner and/or the preconditioner is so computationally expensive that it negates the savings gained by reducing the number of iterations (Claerbout, 1997). Claerbout (1998) proposed designing helicon-style operators to provide a method to find stable inverses, and potentially, appropriate preconditioners (Fomel et al., 1997).

In addition, geophysical problems are often under-determined, requiring some type of regularization. Unfortunately the simplest, and most common, regularization techniques tend to create isotropic features when we would often prefer solutions that follow trends. This problem is especially prevalent in velocity estimation. The result obtained through many inversion schemes produces a velocity structure that geologists (whose insights are hard to encode into the regression equations) find unreasonable. Fortunately, there are often other sources of information that can be encoded into the regularization operator that allow the inversion to be guided towards a more appealing result. Dip information, easily obtainable from stack, migrated section, or regional geologic trends, is just such an information source. We create regularization operators, and by the helix methodology preconditioners, built from this dip information. We use these preconditioners to quickly produce geologically reasonable velocity models that still fit the geophysical data.

Here we create small, space-variant, plane-wave annihilators filters built from dip information. We use the inverse of these filters to form a preconditioner to steer the inversion. We show this methodology applied to two different types of problems. In the first example we use a preconditioner, built from steering filters, to interpolate well-log information along dip. In the second example we use the preconditioner in a standard tomography problem and attempt to estimate a synthetic anticline velocity model.

THEORY/MOTIVATION

Regularization

In general, geophysical problems are under-constrained. To obtain pleasing results we impose some type of regularization criteria such as limiting solutions to large singular values (Clapp and Biondi, 1995) or minimizing different solution norms (Nichols, 1994). A more operator oriented approach is to minimize the power out of a regularization operator ($\mathbf{A}$) applied to the model ($\mathbf{m}$), giving us the typical regularized inversion goals:

$$\mathbf{d} \approx \mathbf{C}\mathbf{m} \qquad (1)$$
$$\mathbf{0} \approx \mathbf{A}\mathbf{m}$$

where $\mathbf{C}$ is the mapping operator from model ($\mathbf{m}$) to data ($\mathbf{d}$). $\mathbf{A}$'s spectrum will be the inverse of $\mathbf{m}$, so to produce a smooth $\mathbf{m}$, we need a rough $\mathbf{A}$ (Claerbout, 1997). The regularization operator can take many forms, in order of increasing complexity:

Laplacian operator (∇^2) The symmetric nature of the Laplacian leads to isotropic features in the model.

Steering filters Simple plane wave annihilation filters which tend to orient the model at some a priori chosen direction. These filters can be simple two point filters, Figure 1, to larger filters that sacrifice compactness for more precise dip annihilation.

1	-.5
	-.5

Figure 1: An example of steering filter. In this case preference is given to slopes at 22.5 degrees from horizontal.

Prediction Error Filters (PEF) Like steering filters apply preferential smoothing directions. Unlike steering filters, are not limited to a single dip or chosen a priori, rather are estimated from the data itself (Schwab et al., 1996).

For tomography problems steering filters are the most attractive alternative. In tomography the data space (traveltimes) and model space (velocity) are different, making prediction error filter estimation difficult. In addition, we usually have a stack, migrated section, or preconceived notions of reflector dip readily available and easily translatable to steering filters.

Preconditioning

Another important consideration when solving inverse problems is convergence speed. The size of most geophysical problems make direct matrix inversion methods impractical. For linear problems an attractive alternative is the family of iterative conjugate gradient methods. Unfortunately, the operators used in seismic reflection problems are usually computationally expensive, so we must limit the number of iterations required to obtain a "reasonable" solution. Part of the reason for slow convergence is that our regularization operator, $\mathbf{A}$, is ill-conditioned, significantly increasing the number of iterations required for convergence. One way to reduce the number of iterations is by reformulating the problem in terms of some new variable ($\mathbf{p}$) with a preconditioning operator ($\mathbf{B}$), that produces desired shapes in model space. Ideally we can think of $\mathbf{B}$ as being something close to $\mathbf{A^{-1}}$ from equation (2). This would allow us to change the traditional regularized inversion goals (2) to

$$\mathbf{d} \approx \mathbf{C}\mathbf{B}\mathbf{p} \qquad (2)$$
$$\mathbf{0} \approx \epsilon\mathbf{p}$$

Where ($\mathbf{B}$) is the preconditioning operator, ($\mathbf{p}$) is the preconditioned variable and

$$\mathbf{m} = \mathbf{B}\mathbf{p}. \qquad (3)$$

In theory this formulation should be significantly faster because the fitting goal $\mathbf{0} \approx \epsilon\mathbf{p}$ is basically free. The problem comes in finding and applying $\mathbf{B}$.

Helix transform
So how to we obtain $\mathbf{B}$? We have three general requirements:

- it produces relatively smooth (by some criteria) results;
- it spreads information quickly;
- and it is computationally inexpensive.

By defining the operators via the helix method (Claerbout, 1998) we can meet all of these requirements. The helix concept is to transform *N*-Dimensional operators into 1-D operators to take advantage of some well understood properties of 1-D functions. In this case we utilize the ability to construct stable inverses from simple, causal filters through polynomial division. If $\mathbf{A}$ is a small roughening operator, $\mathbf{B}$ is a large smoothing operator. But, because we are applying polynomial division we get the effect of the usually computationally expensive $\mathbf{B}$ at the cost of the inexpensive $\mathbf{A}$.

Steering Filters
Plane waves with a given slope on a discrete grid can be predicted (destroyed) with compact filters (Schwab et al., 1996). Inverting such a filter by the helix method, we can create a signal with a given arbitrary slope extremely quickly. If this slope is expected in the model, the described procedure gives us a very efficient method of preconditioning the model estimation problem, fitting goal (**??**).

How can a plane prediction (steering) filter be created? On the helix surface, the plane wave $A(t,x) = f(t - px)$ translates naturally into a periodic signal with the period of $T = N_t + \sigma$, where N_t is the number of points on the t trace, and $\sigma = \frac{p \triangle x}{\triangle t}$, where σ is the plane slope, [1] and $\triangle x$ and $\triangle t$ correspond to the mesh size. If we design a filter that is two columns long (assuming the columns go in the t direction), then the *plane prediction* problem is simply connected with the *interpolation* problem: to destroy a plane wave, shift the signal by T, interpolate it, and subtract the result from the original signal. Therefore, we can formally write

$$\mathbf{A} = I - \mathbf{S}(\sigma) \ , \tag{4}$$

where $\mathbf{A}$ denotes the steering filter, $\mathbf{S}$ is the shift-and-interpolation operator, and $\mathbf{I}$ is the identity operator.

Different choices for the operator $\mathbf{S}$ in (4) produce filters with different length and prediction power. A shifting operation corresponds to the filter with the Z-transform $\Sigma(Z) = Z^T$, while the operator $\mathbf{S}$ corresponds to an approximation of $\Sigma(Z)$ with integer powers of Z. One possible approach is to expand $\Sigma(Z)Z^{-N_t}$ using the Taylor series around the zero frequency ($Z = 1$). For example, the first-order approximation is

$$S_1(Z) = Z^{N_t}\,(1 + \sigma(Z-1)) = (1-\sigma)Z^T + \sigma Z^{T+1} \ , \tag{5}$$

which corresponds to linear interpolation and leads in the two-dimensional space to the steering filter $\mathbf{A}$ of the form

1	$-\sigma$
	$\sigma - 1$

(6)

By applying polynomial division we simulate the inverse of $\mathbf{A}$, $\mathbf{B}$. Figure 2 shows the result of applying polynomial division using (6) as the filter. Note how the small annihilation filter acts as a large smoothing filter when applying polynomial division.

[1] In computational physics, the dimensionless number σ is sometimes referred to as the CFL (Courant, Friedrichs, and Lewy) number (Sod, 1985).

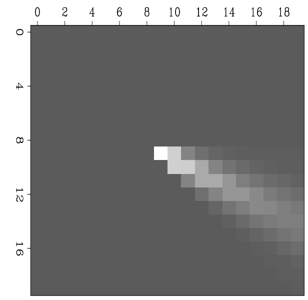

Figure 2: The impulse response of a steering filter oriented 22.5 degrees from horizontal.

Space variable filters
Steering filters are effective in spreading information along a given direction, but are limited to a single dip. If it is inappropriate to apply a single smoothing direction to the entire model there are two general courses of action:

Patching (Claerbout, 1992; Crawley, 1998) Redefine the problem into a series of problems, each on a small subset of the data (patches) where a dip stationarity assumption is valid. Then recombine these patches to produce the final output. Unfortunately, determining subsets of the data where the stationarity condition is satisfied is difficult. In addition, recombining the various patches is not straightforward. In problems like tomography we face yet another limitation; how to effectively combine the global tomography problem, with the local patching/regularization problem.

Space varying filters Filters that vary with location but are spatially smooth. In many ways this is the a more appealing approach. In general, space varying filters require a high level of spatial smoothness to avoid artifacts. Steering filters are unidirectional, so we can simply smooth the scalar dip field rather than dealing with the problem of smoothing the multi-component filter field (issues of filter stability can quickly arise when using simplistic smoothing schemes.) In addition, because we are applying polynomial division produced inverse filters we achieve a higher level of smoothness automatically. Each inverse filter spreads information over large, overlapping regions at each iteration.

The ease of creating and smoothing steering filters, in addition to the difficulty in combing the global tomography with the local patching problem led us to choose the second option for this paper.

WELL LOG/DIP INTERPOLATION

To illustrate the effectiveness of this method imagine a simple interpolation problem. Following the methodology of (Fomel et al., 1997) we first bin the data, producing a model $\mathbf{m}$, composed of known data $\mathbf{m}_k$ and unknown data $\mathbf{m}_u$. We have an operator $\mathbf{J}$ which is simply a diagonal *selector* (selects where the model can change) operator with zeros at known data locations and ones at unknown locations. We can write $\mathbf{m}_k$ and $\mathbf{m}_u$ in terms of $\mathbf{m}$ and $\mathbf{J}$,

$$\begin{aligned} \mathbf{m}_k &\approx (\mathbf{I}-\mathbf{J})\mathbf{m} \\ \mathbf{m}_u &\approx \mathbf{J}\mathbf{m} \end{aligned} \tag{7}$$

where $\mathbf{I}$ is the identity matrix. We have the preconditioning operator $\mathbf{B}$, which applies polynomial division using the helix methodology.

Our fitting goals can be written as

$$\begin{aligned} \mathbf{m}_k &\approx (\mathbf{I} - \mathbf{J})\mathbf{B}\mathbf{p} \\ 0 &\approx \epsilon \mathbf{p} \end{aligned} \tag{8}$$

So the only question that remains is what to use for $\mathbf{B}$, or more specifically $\mathbf{B}^{-1}$, $\mathbf{A}$.

For this experiment we create a series of well logs by sub-sampling a 2-D velocity field. We use as our a priori information source, reflector dips, to build the steering filters, and thus the operator $\mathbf{A}$. For this synthetic test we pick the dips from our "goal", the left portion of Figure 3. We define regions in which we believe each of these dips to be approximately correct, and smooth the overall dip field (right portion of Figure 3).

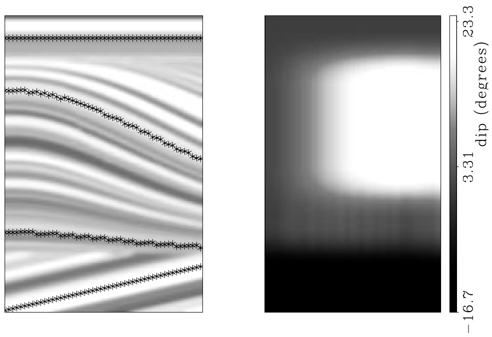

Figure 3: Left, a synthetic seismic section with four picked reflectors indicated by '*'; right; the dip field constructed from the picked reflectors.

For a test, we simulate nine well logs along the survey (Figure 4). We use equation (8) as our fitting goals and a conjugate gradient solver to estimate $\mathbf{p}$. Within 12 iterations we have a satisfactory solution (Figure 4). If you look closely, you can still see the well locations, but in general the solution converges quickly to something close to the correct velocity field (Figure 3).

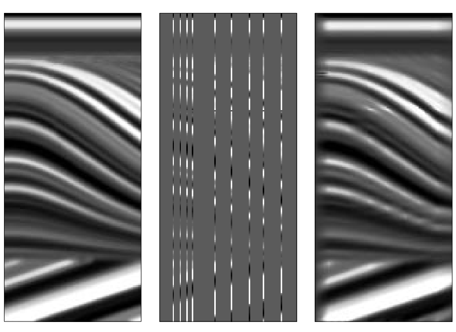

Figure 4: Left, correct velocity field; middle, well subset selected as input; right, velocity field resulting from interpolation.

REGULARIZATION OF A STANDARD TOMOGRAPHY PROBLEM

For a more realistic test we applied the steering filter methodology to a standard tomography problem. We started with a simple anticline velocity model, Figure 5 and a dip field (Figure 6) following the reflector geometry.

We formulated the tomography problem as

$$\begin{aligned} \delta \mathbf{t} &\approx \mathbf{T}\mathbf{p} \\ \mathbf{0} &\approx \epsilon \mathbf{A}\mathbf{m}. \end{aligned} \tag{9}$$

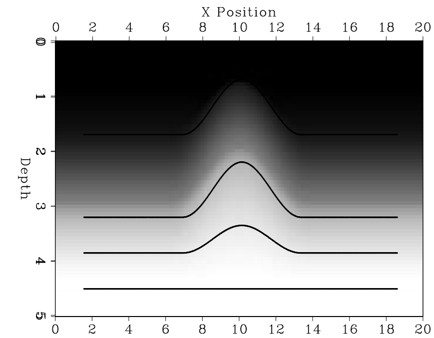

Figure 5: Reflector position superimposed over correct velocity model.

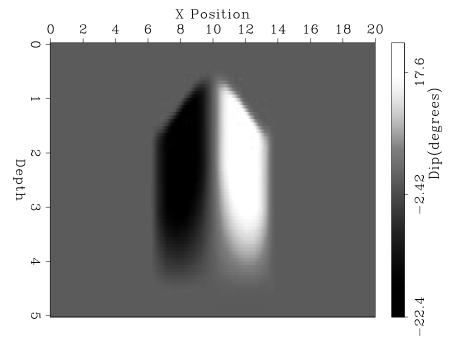

Figure 6: Dip field used to create the steering filters.

Which can be rewritten using the same preconditioning logic as,

$$\begin{aligned} \delta \mathbf{t} &\approx \mathbf{T}\mathbf{B}\mathbf{p} \\ \mathbf{0} &\approx \epsilon \mathbf{p}. \end{aligned} \tag{10}$$

Where:

$\mathbf{T}$ is our tomography operator, in this case a simple back projection operator that also accounts for reflector movement;

$\mathbf{B} = \mathbf{A}^{-1}$ our are steering filters;

$\delta \mathbf{t}$ is the difference between the traveltimes $(\mathbf{t_c})$ through the true slowness model to the true reflector position and the modeled traveltimes $(\mathbf{t_i})$ through the current slowness model and current guess at reflector location;

$\mathbf{p}$ is the preconditioned variable; and

$\delta \mathbf{s} = \mathbf{B}\mathbf{p}$ is the change in the slowness model.

Ray bending and reflector movement make this problem non-linear. As a result we must add an outer loop, updating the slowness model,

$$\begin{aligned} \mathbf{s_{i+1}} &= \mathbf{s_i} + \mathbf{B}\mathbf{p} \\ &= \mathbf{s_i} + \delta \mathbf{s}, \end{aligned} \tag{11}$$

after every outer-loop iteration and reestimating the reflector position through map-migration.

For this test we started with an initial $v(z)$ velocity model, with velocity errors up to $230\ m/s$, Figure 7. We then attempted to recover the correct velocity model by a layer-stripping approach (the result down to the second reflector is shown in Figure 8. The difference between the initial model and the new model, Figure 9, clearly illustrates that the velocity perturbations follow structural dip. For comparison we performed the same sequence using the inverse of the Laplacian as the preconditioner. Not surprisingly, we get much more isotropic features, Figure 10, which are inconsistent with our conception of geology.

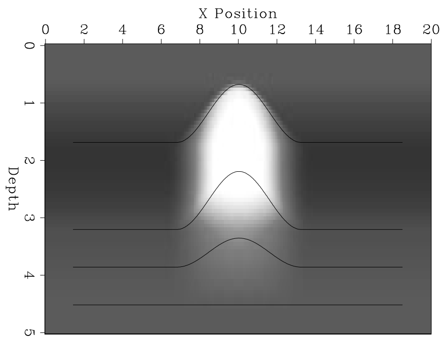

Figure 7: Initial error in velocity.

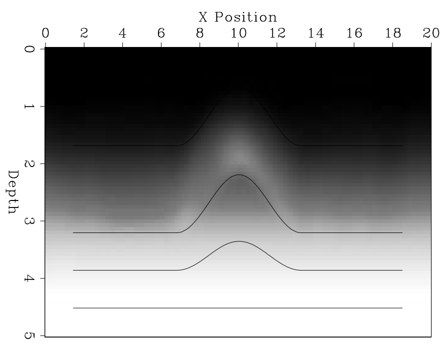

Figure 8: Reflector position overlaying our velocity model using the first two reflectors.

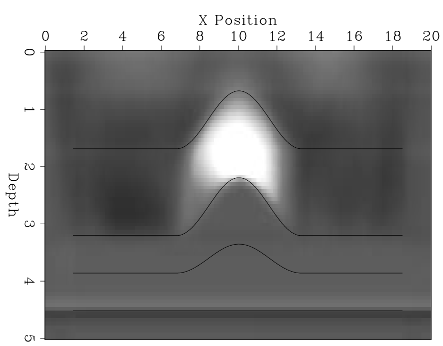

Figure 9: Change in our velocity model estimate using traveltimes from the first two reflectors. Note how the velocity perturbation follow the anticline.

CONCLUSIONS

We show that we can quickly converge to geologic consistent velocity models by reformulating the standard isotropic, regularized inversion problem into a preconditioned inversion problem using anisotropic smoothers oriented along dip. This method holds promise in finally bringing geologist's insights, while still honoring geophysicist's data, into the velocity estimation problem.

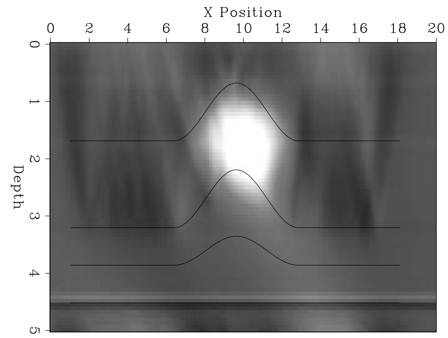

Figure 10: Change in our velocity model using traveltimes from the first two reflectors. Note how the velocity perturbation are isotropic rather than following geology.

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