Least-squares joint imaging of multiples and primaries

Morgan Brown, Stanford University*

SUMMARY

Multiple reflections contain much subsurface reflectivity information. Exploiting it has proven elusive because of crosstalk leakage. I present a linear least-squares inversion method, LSJIMP (Least-squares Joint Imaging of Multiples and Primaries), compatible with many imaging operators, which utilizes three model regularization operators to simultaneously separate multiples from primaries, combine their information, and increase signal fidelity. I test a simple yet efficient LSJIMP implementation on 2-D and 3-D prestack field data examples and show that the method cleanly separates primaries and multiples and also uses the joint information in the events to interpolate signal in acquisition gaps.

INTRODUCTION

Multiple reflections often erect the highest barrier to the successful imaging and interpretation of marine seismic data. Despite their nuisance, however, multiples illuminate the prospect zone, and moreover, illuminate different angular ranges and reflection points than primaries. In theory and in practice, multiples provide subsurface information that primaries do not.

An important class of multiple suppression methods create a "model" of the multiples, which is subtracted from the data. Most predict multiples from primaries by "adding a multiple bounce" to the data with wavefield extrapolation (Riley and Claerbout, 1976; Morley, 1982; Berryhill and Kim, 1986; Verschuur et al., 1992). Analogous prestack multiple *imaging* methods either explicitly "remove a multiple bounce" from the data, transforming multiples into conventionally imageable pseudo-primary events (Berkhout and Verschuur, 2003; Shan, 2003), or combine the extrapolation and imaging steps (Reiter et al., 1991; Berkhout and Verschuur, 1994; Yu and Schuster, 2001; Guitton, 2002).

To use the information in the multiples, we must map multiples and primaries to a domain where they are comparable, and then combine them. The prestack image domain, and particularly, the reflection angle domain (see Sava and Fomel (2003) for a review), is an excellent choice. Imaging reduces signal to a compact form, and by arranging the signal as a function of reflection angle, we can combine multiples and primaries in a rigorous physical sense.

The semi-independent measurements provided by primaries and multiples overlap each other in one data record. In theory, averaging the multiple and primary images can improve signal fidelity and fill coverage gaps, but this encounters two problems in practice. First, unless we correct multiples for their different raypaths and additional reflections, the signal events are incomparable. Secondly, just as multiples represent noise on the primary image, primaries and higher order multiples represent noise on a first-order multiple image. Corresponding noise, or "crosstalk" events on the images are often kinematically consistent, so adding the images may actually degrade signal fidelity. Figure 2 illustrates this problem on the 2-D field data tested later. The Figure shows stacked images of the primaries and of seabed and top of salt peglegs. While the major events in all images are recognizable and consistent, crosstalk would obviously inhibit image averaging.

Separation of multiples and primaries is a prerequisite to using the information in the multiples, yet prestack separation is often expensive, and if done poorly, may bias the integration. LSJIMP simultaneously solves the separation and integration problems, as a global inversion. The model space is a collection of images, with the energy from each mode partitioned into only one image. The forward

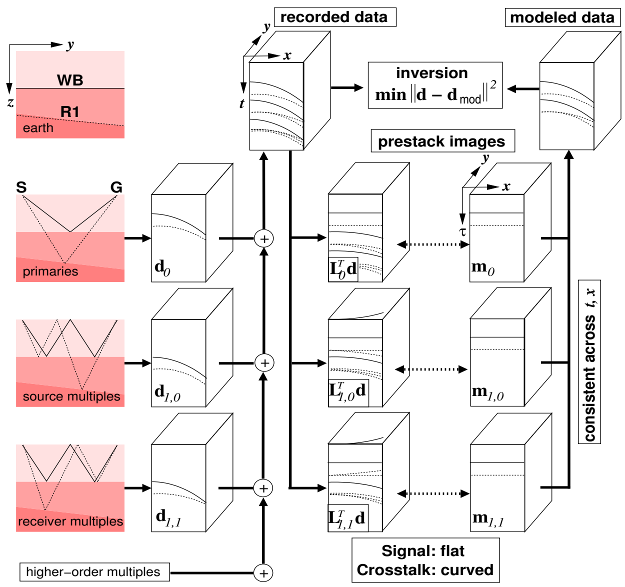

Figure 1: LSJIMP schematic. Assume: recorded data = primaries + pegleg multiples. Prestack imaging alone (e.g., $\mathbf{L}_{i,k}^T$) focuses signal events in τ/depth and offset/angle, but leaves crosstalk events. If the $\mathbf{m}_{i,k}$ contain only signal, we can fully model the data. LSJIMP suppresses crosstalk and fits the data in a least-squares sense. Model regularization operators simultaneously suppress crosstalk and increase signal fidelity. We can also add multiples from other multiple generators.

model contains amplitude corrections which ensure that the signal events in the multiple and primary images are directly comparable, in terms of both kinematics and amplitudes. Three model regularization operators exploit multiplicity between and within the images to discriminate between crosstalk and signal, combine the images, fill coverage gaps, and increase signal fidelity. Joint imaging algorithms like LSJIMP (Brown, 2002; He and Schuster, 2003) represent a generalization of least-squares imaging schemes (Kuehl and Sacchi, 2001; Prucha-Clapp and Biondi, 2002; Wang et al., 2003) which only exploit multiplicity along reflection angle to fill gaps and increase fidelity.

Brown and Guitton (2004) outline an efficient prestack modeling/imaging operator for peglegs which uses an NMO-like scheme to image peglegs and a series of relative amplitude corrections to make the angle-dependent amplitudes of the multiple and primary images comparable. I implemented LSJIMP with this operator and present successful data tests on 2-D and 3-D field data from the Gulf of Mexico. Other possible choices include Kirchhoff (Reiter et al., 1991; He and Schuster, 2003) and wave equation (Berkhout and Verschuur, 2003; Shan, 2003; Guitton, 2002) migration.

LSJIMP THEORY

LSJIMP models the recorded data as the sum of primary reflections and p orders of pegleg multiples from n_{surf} multiple generators. An i^{th}-order pegleg splits into $i+1$ legs. Denoting the primaries $\mathbf{d}_0$ and the k^{th} leg of the i^{th} order pegleg from the m^{th} multiple

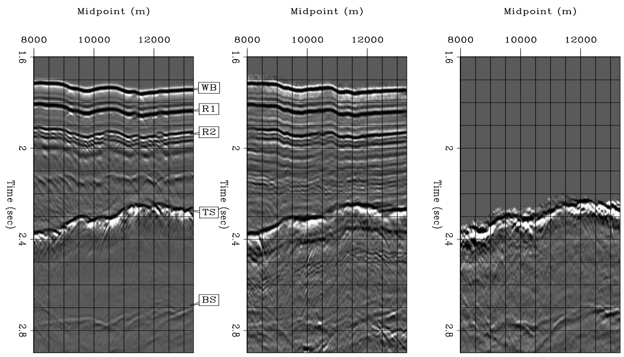

Figure 2: 2-D GoM imaging and stack. Left: Raw data. Center and right: Seabed (WB) and top of salt (TS) peglegs.

generator $\mathbf{d}_{i,k,m}$, the modeled data takes the following form:

$$\mathbf{d}_{\text{mod}} = \mathbf{d}_0 + \sum_{i=1}^{p} \sum_{k=0}^{i} \sum_{m=1}^{n_{\text{surf}}} \mathbf{d}_{i,k,m}. \tag{1}$$

In Figure 1, $p = n_{\text{surf}} = 1$. Assume we have designed imaging operators that map primaries and multiples to directly comparable (kinematics and angle-dependent amplitudes) signal events. We can similarly denote the modeling operators (adjoint to imaging) for primaries and peglegs as $\mathbf{L}_0$ and $\mathbf{L}_{i,k,m}$, respectively, and the images of the primaries and peglegs as $\mathbf{m}_0$ and $\mathbf{m}_{i,k,m}$, respectively. Rewriting equation (1), we have:

$$\mathbf{d}_{\text{mod}} \;=\; \mathbf{L}_0 \mathbf{m}_0 + \sum_{i=1}^{p} \sum_{k=0}^{i} \sum_{m=1}^{n_{\text{surf}}} \mathbf{L}_{i,k,m} \mathbf{m}_{i,k,m} \;=\; \mathbf{L}\mathbf{m} \tag{2}$$

The LSJIMP method optimizes the primary and multiple images, $\mathbf{m}$, by minimizing the ℓ_2 norm of the difference between the recorded data, $\mathbf{d}$, and the modeled data, $\mathbf{d}_{\text{mod}}$:

$$\min_{\mathbf{m}} \; \|\mathbf{d} - \mathbf{L}\mathbf{m}\|^2. \tag{3}$$

Nemeth et al. (1999) solved a similar problem to jointly image primaries and non-multiple coherent noise. Guitton et al. (2001) modeled primaries and multiples with nonstationary prediction-error filters, but cast the problem purely in terms of separation.

REGULARIZATION OF THE LSJIMP PROBLEM

Minimization (3) is under-determined for most choices of $\mathbf{L}_0$ and $\mathbf{L}_{i,k,m}$, implying infinitely many solutions. Crosstalk leakage is a symptom of the problem. For instance, $\mathbf{L}_0$ maps residual first-order multiple energy in $\mathbf{m}_0$ to the position of a first-order multiple in data space. Minimization (3) alone cannot distinguish between crosstalk and signal.

I devise discriminants between crosstalk and signal, and use them to derive three model regularization operators which choose the set of primary and multiple images which are optimally free of crosstalk. Moreover, these operators exploit signal multiplicity–within and between images–to increase signal fidelity, fill coverage gaps, and combine multiple and primary information. The operators are applied at one midpoint location only, so the discussion assumes that the $\mathbf{m}_{i,k,m}$ are functions of zero-offset traveltime and offset (τ, x), but not midpoint, though they could equivalently be parameterized by depth and reflection angle.

Regularization 1: Differencing between images
By design, corresponding signal events on all $\mathbf{m}_{i,k,m}$ are flat with offset and comparable in amplitude. Conversely, corresponding crosstalk events on any two images usually have different residual moveout, the magnitude which is generally small at near offsets, but larger at far offsets and in the presence of subsurface complexity (Brown, 2004). At fixed (τ, x), the difference between two $\mathbf{m}_{i,k,m}$ should be small where there is signal, but large where there is crosstalk. LSJIMP endeavors to combine information from the multiple and primary images by averaging. Differencing between images enforces a degree of consistency between images and provides a systematic framework for averaging images.

Regularization 2: Differencing across offset
After imaging, signal events on all $\mathbf{m}_{i,k,m}$ are flat in offset, while crosstalk events usually have residual curvature, especially at far offsets. Provided that the amplitude-versus-offset (AVO) response of the signal changes slowly with offset, the difference (in offset) between adjacent samples of any $\mathbf{m}_{i,k,m}$ will be small where there is signal, but large where there is crosstalk.

Regularization 3: Crosstalk penalty weights
Given a signal estimate, we can model the expected crosstalk events on each $\mathbf{m}_{i,k,m}$, and build a diagonal weight to penalize crosstalk. The signal estimate may come from a nonlinear iteration (Brown, 2004) or, more crudely, from $\mathbf{L}_0^T \mathbf{d}$. Denoting the signal estimate $\mathbf{z}_0$, then

$$\mathbf{z}_{i,k,m} = \mathbf{L}_{i,k,m} \mathbf{z}_0, \tag{4}$$

is a model of the k^{th} leg of the i^{th} order multiple from the m^{th} multiple generator. To simulate the total crosstalk in $\mathbf{m}_{i,k,m}$, we apply $\mathbf{L}_{i,k,m}$ to all multiple model panels $\mathbf{z}$ (except $\mathbf{z}_{i,k,m}$) and sum:

$$\mathbf{c}_{l,n,q} \quad = \quad \sum_{j=l_0}^{p} \sum_{k=0}^{j} \sum_{m=1}^{n_{\text{surf}}} \mathbf{L}_{l,n,q}^T \mathbf{z}_{j,k,m}, \tag{5}$$
$$\text{where } \; k \neq n,\; m \neq q \text{ and } \; l_0 = \begin{cases} 1 & \text{if } l = 0, \\ l & \text{otherwise} \end{cases}$$

$\mathbf{c}_{i,k,m}$ is an estimate of the crosstalk in $\mathbf{m}_{i,k,m}$. I compute $\mathbf{c}_{i,k,m}$'s envelope to build a diagonal weight function, $\mathbf{w}_{i,k,m}$, which in the regularization is applied directly to $\mathbf{m}_{i,k,m}$. $\mathbf{w}_{i,k,m}$ will overlap (and damage) signal events to some extent, but the other two regularization operators will exploit the signal's flatness and self-consistency between images to compensate with redundant information from other images and offsets.

THE COMBINED LSJIMP PROBLEM

To solve the regularized LSJIMP problem, we supplement minimization (3) with the three model regularization operators:

$$\min_{\mathbf{m}} \; \|\mathbf{L}\mathbf{m} - \mathbf{d}\|^2 \;+\; \epsilon_1^2 \|\mathbf{r}_m^{[1]}\|^2 \;+\; \epsilon_2^2 \|\mathbf{r}_m^{[2]}\|^2 \;+\; \epsilon_3^2 \|\mathbf{r}_m^{[3]}\|^2. \tag{6}$$

$\mathbf{r}_m^{[1]}$, $\mathbf{r}_m^{[2]}$, and $\mathbf{r}_m^{[3]}$ are the model residuals corresponding to differencing across image, differencing across offset, and crosstalk penalty weighting, respectively. Scalars ϵ_1, ϵ_2, and ϵ_3 balance the relative weight of the three model residuals with the data residual. I use the conjugate gradient method for minimization (6).

FIELD DATA RESULTS

WesternGeco released 2-D data from Mississippi Canyon, Gulf of Mexico, for multiple suppression testing. Figure 3 shows that the data contain a variety of strong surface-related multiples which hamper interpretation. I tested a particular LSJIMP implementation (Brown and Guitton, 2004) on 750 CMPs of the data, modeling only first-order multiples from the four labeled multiple generators.

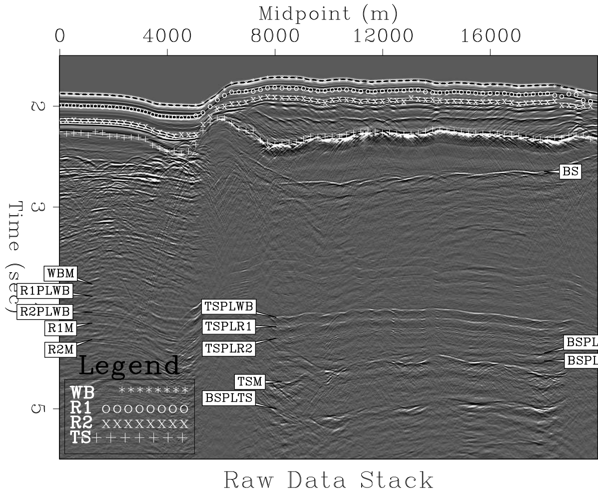

Figure 3: Stacked 2-D GoM data, after t^2 gain. Picks denote the four multiple generators (seabed - WB, R1, R2, and top of salt - TS). Naming convention for pure multiples: (*reflector*)M and for pegleg multiples: (*target*)PL(*multiple generator*), e.g., BSPLWB.

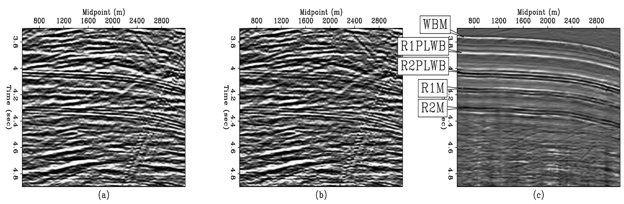

Figure 4: Zoom (CMP 500-3200 m) of 2-D GoM stack before and after LSJIMP, with t^2 gain. (a) Raw data; (b) Estimated primaries ($\mathbf{m}_0$); (c) Difference. Events labeled as in Figure 3.

Twenty conjugate gradient iterations on 28 CPUs (1.3 Ghz P-3) of a Linux cluster required 3 hours to run.

Figures 4 and 5 show stacks of the raw data, LSJIMP estimated primaries, and the difference. Figure 4 comes from a sedimentary basin. A variety of strong peglegs are removed without badly damaging the updipping primaries. Figure 5 comes from over the tabular salt body. Shallow multiples are cleanly separated from the data, while the salt-related multiples are only partially separated, due to limitations of the simple imaging operator used.

Figures 6 and 7 show prestack LSJIMP results at two CMP locations. Panels (c), (d), (g), and (h) show the estimated total first order pegleg from the seabed, R1, R2, and top of salt, respectively. The modeled data (panel (e)) is the sum of the estimated primaries (panel (b)) and panels (c), (d), (g), and (h). The residual error (panel (f)) is the difference between the input data and modeled data.

Figure 6 comes from the sedimentary basin, where the multiples exhibit simple moveout behavior. From the small amount of correlated residual energy, we note that LSJIMP preserves primaries and models most multiples in the data. Figure 7 comes from over the salt. Both the relatively simple shallow multiples and the more complex, visibly split salt-related multiples are modeled and separated fairly well. In both cases, notice that the data's missing near offset information has been interpolated by the method.

CGG acquired a 3-D speculative survey in the Gulf of Mexico's Green Canyon, a region characterized by sedimentary "minibasins" and complex salt bodies (AAPG, 1998). I process a subset (192 by 14 CMP locations) from one such minibasin, which contains nontrivial crossline dip ($> 3°$). Brown and Guitton (2004) note that the data's crossline offset axis can be ignored, leaving a four-dimensional data cube, which strongly speeds up LSJIMP's performance. The 3-D example required roughly eight hours of run time on the same machine (20 iterations).

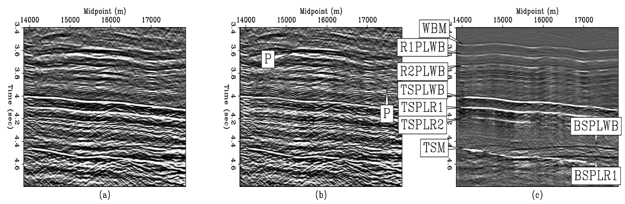

Figure 5: Zoom (CMP 13850-17850 m) of 2-D GoM stacked section before and after LSJIMP, with t^2 gain. (a) Raw data; (b) Estimated primaries ($\mathbf{m}_0$); (c) Difference. Events labeled as in Figure 3. Subsalt primary labeled "P".

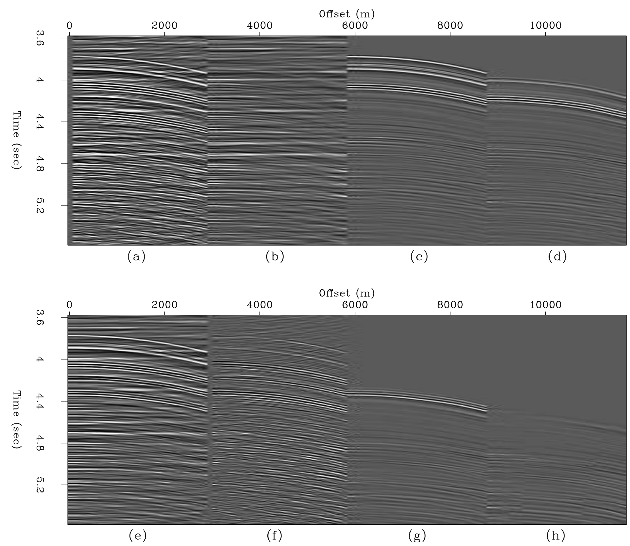

Figure 6: 2-D GoM CMP 55 (1440 m) before and after LSJIMP. Panels defined in text. All panels NMO'ed, windowed from 3.5 to 5.5 seconds, and gained with t^2.

Figure 8 zooms into the multiple-infested zone before and after LSJIMP. Stacking mostly suppresses multiples, but from the difference panel, note that LSJIMP nontheless subtracts much remaining multiple energy without seriously harming primaries. The timeslice on the 3-D cube transects a strong seabed pegleg; it shows up prominently on the raw data stack and the difference panel, but is largely absent from the LSJIMP estimated primaries stack.

Figure 9 shows LSJIMP's performance on a CMP gather (note absence of crossline offset). Note strong multiples at τ of around 4.3 seconds in the raw data (panel (a)), and strong primaries under the multiple curtain. The LSJIMP estimated primaries (panel (b)) are effectively free of multiples, and moreover, since the data residual (panel (f)) contains little correlated energy, we have preserved the primaries and effectively modeled the data's important multiples (panels (c) and (d)).

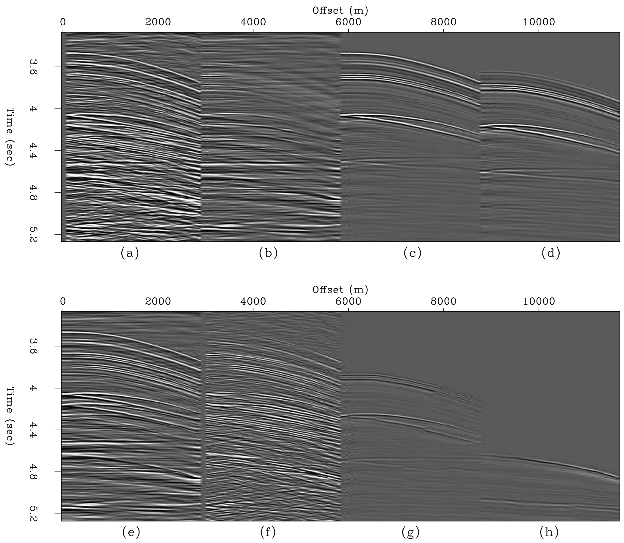

Figure 7: 2-D GoM CMP 344 (9150 m) before and after LSJIMP. Panels defined in text. All panels NMO'ed, windowed from 3.5 to 5.5 seconds, and gained with t^2.

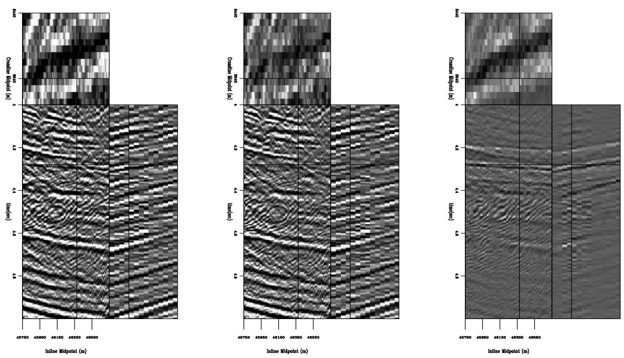

Figure 8: Zoom of stacked subset of CGG 3-D data before and after LSJIMP. All panels windowed in time from 4.0 to 5.0 seconds. Left: Raw data stack. Center: Stack of estimated primary image, $\mathbf{m}_0$. Right: Stack of the subtracted multiples.

CONCLUSIONS

I introduced the LSJIMP method, a least-squares inverse problem, to simultaneously separate and combine the multiples and primaries in the prestack image space. Using only a simple choice of multiple imaging operator, I demonstrated good separation results on 2-D and 3-D prestack field data examples. As future computing power increases, allowing the use of more capable imaging operators, the method shows great promise to exploit the potentially great amount of information provided by multiples in complex regions.

ACKNOWLEDGEMENT

I thank WesternGeco and CGG for releasing the 2-D and 3-D data, respectively, shown in this abstract.

REFERENCES

AAPG, 1998, Gulf of mexico petroleum systems: AAPG Bulletin, **82**, no. 5.

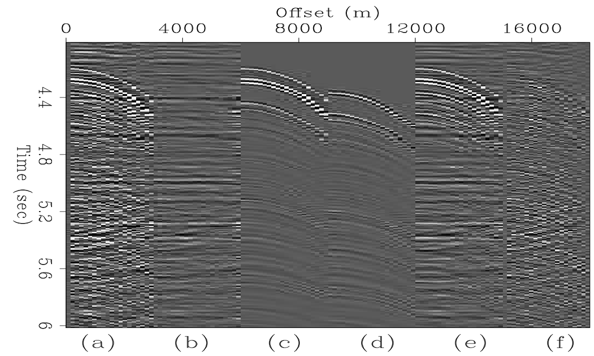

Figure 9: LSJIMP results on individual midpoint location (CMPx=100,CMPy=4). All panels NMO'ed, with t^2 gain.

Berkhout, A. J., and Verschuur, D. J., 1994, Multiple technology: Part 2, migration of multiple reflections: 64th Ann. Internat. Mtg, 1497–1500.

Berkhout, A. J., and Verschuur, D. J., 2003, Transformation of multiples into primary reflections:, *in* 73rd Ann. Internat. Mtg Soc. of Expl. Geophys.

Berryhill, J. R., and Kim, Y. C., 1986, Deep-water peglegs and multiples - Emulation and suppression: Geophysics, **51**, no. 12, 2177–2184.

Brown, M., and Guitton, A., 2004, Efficient prestack modeling and imaging of pegleg multiples: SEG Exp. Abstracts, submitted.

Brown, M., 2002, Least-squares joint imaging of primaries and multiples: 72nd Ann. Internat. Mtg, 890–893.

Brown, M. P., 2004, Least-squares joint imaging of multiples and primaries: Ph.D. thesis, Stanford University.

Guitton, A., Brown, M., Rickett, J., and Clapp, R., 2001, Multiple attenuation using a t-x pattern-based subtraction method: 71st Ann. Internat. Mtg, 1305–1308.

Guitton, A., 2002, Shot-profile migration of multiple reflections: 72nd Ann. Internat. Mtg, 1296–1299.

He, R., and Schuster, G., 2003, Least-squares migration of both primaries and multiples:, *in* 73rd Ann. Internat. Mtg Soc. of Expl. Geophys.

Kuehl, H., and Sacchi, M., 2001, Generalized least-squares DSR migration using a common angle imaging condtion: 71st Ann. Internat. Mtg, 1025–1028.

Morley, L., 1982, Predictive multiple suppression: Ph.D. thesis, Stanford University.

Nemeth, T., Wu, C., and Schuster, G. T., 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, no. 1, 208–221.

Prucha-Clapp, M., and Biondi, B., 2002, Subsalt event regularization with steering filters: 72nd Ann. Internat. Mtg, 1176–1179.

Reiter, E. C., Toksoz, M. N., Keho, T. H., and Purdy, G. M., 1991, Imaging with deep-water multiples: Geophysics, **56**, no. 07, 1081–1086.

Riley, D. C., and Claerbout, J. F., 1976, 2-D multiple reflections: Geophysics, **41**, no. 04, 592–620.

Sava, P., and Fomel, S., 2003, Angle-domain common-image gathers by wavefield continuation methods: Geophysics, **68**, no. 3, 1065–1074.

Shan, G., 2003, Source-receiver migration of multiple reflections:, *in* 73rd Ann. Internat. Mtg Soc. of Expl. Geophys.

Verschuur, D. J., Berkhout, A. J., and Wapenaar, C. P. A., 1992, Adaptive surface-related multiple elimination: Geophysics, **57**, no. 09, 1166–1177.

Wang, J., Kuehl, H., and Sacchi, M. D., 2003, Least-squares wave-equation avp imaging of 3D common azimuth data:, *in* 73rd Ann. Internat. Mtg Soc. of Expl. Geophys.

Yu, J., and Schuster, G., 2001, Crosscorrelogram migration of IVSPWD data: 71st Ann. Internat. Mtg, 456–459.