Regularized least-squares inversion for 3-D subsalt imaging

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SUMMARY

Obtaining seismic images of the subsurface near and beneath salt is very difficult due to seismic energy that is lost by propagating outside of the survey area or becoming evanescent at salt boundaries (poor illumination). We demonstrate an iterative regularized least-squares inversion for imaging that helps to compensate for illumination problems. We show the use of a regularization operator that acts to regularize amplitudes along reflection angles (or equivalent offset ray parameters) to compensate for the sudden, large amplitude changes caused by poor illumination. This regularization operator has the effect of filling in the gaps created in the reflection angle range due to the lost seismic energy. We discuss the use of this regularization operator in an iterative least-squares inversion scheme to improve imaging for a poorly illuminated 3-D seismic dataset. We introduce a new type of joint inversion problem that will allows us to simultaneously invert two or more datasets. For this iterative inversion scheme, we design a different regularization operator that allows us to share illumination information between images created from the different datasets involved. This regularized inversion allows us to create images that share the illumination information from the different datasets.

INTRODUCTION

In an ideal world, a 3-D seismic survey would have infinite extents and dense shot and receiver grids over the entire x-y plane. This would provide the best illumination possible everywhere in the subsurface. In our world, our limited source-receiver geometries allow energy to leave the survey and the density of our shot and receiver arrays depends on the equipment available. For 3-D surveys, the geometry leads to limited azimuth ranges dependent on the direction in which the survey is shot. The limited extents of the 3-D survey will allow seismic energy to escape as it is redirected by subsurface structures, such as salt bodies. These illumination problems make it very difficult to properly image the subsurface near and beneath salt bodies.

Migration techniques try to put the energy recorded in our seismic data back where it belongs in the subsurface. When such energy has not been recorded, due to its escape from the survey area or becoming evanescent (as it may at salt boundaries), the migrated image will have shadow zones (Muerdter et al., 1996) where the signal is weak or non-existent. Even improved migration methods such as downward continuation migration that generates angle-domain common image gathers (Prucha et al., 1999) cannot properly image such areas.

To improve imaging in areas with poor illumination, we can use migration and its adjoint process in a conjugate-gradient minimization that performs least-squares inversion. Unfortunately, the "missing" data that escaped from the survey can render this process unstable, as it is one reason that the iterative inversion problem may have a null space. The inversion can be stabilized through the addition of regularization (Tikhonov and Arsenin, 1977) that allows us to impose some type of regularization on the resulting image. This regularization can take many forms. In this abstract, we will discuss two possibilities: *geophysical regularization* in which amplitudes are regularized along reflection angles for every point in the subsurface (Prucha and Biondi, 2002; Kuehl and Sacchi, 2001) and *dual stacked image regularization* in which two images of the same subsurface volume with different illumination patterns are used to regularize each other (Clapp, 2003).

We will begin by formulating our scheme for regularized inversion for 3-D seismic data. We will demonstrate this algorithm using *geophysical regularization* on a complex 2-D synthetic and discuss its use on a real 3-D dataset from the Gulf of Mexico. Finally, we will explain the potential use of *dual stacked image regularization*.

REGULARIZED INVERSION

Downward continuation migration can be thought of as an operator and therefore used in a conjugate-gradient inversion. We choose a regularized least-squares inversion in which we minimize this objective function:

$$min\{Q(\mathbf{m}) \;=\; ||\mathbf{Lm}-\mathbf{d}||^2+\epsilon^2\;||\mathbf{Am}||^2\}. \qquad (1)$$

Here, $\mathbf{d}$ is the input data and $\mathbf{m}$ is the image obtained through inversion. $\mathbf{L}$ is a linear operator, which may be a migration operator such as downward continuation migration (Prucha et al., 1999), or in the case of 3-D imaging it can be common azimuth migration (CAM) (Biondi and Palacharla, 1996). $\mathbf{A}$ is a regularization operator. ϵ controls the strength of the regularization.

The first part of equation (1) can be thought of as the "data fitting goal", meaning that it is responsible for making a model that is consistent with the data. The second part is the "model styling goal", meaning that it allows us to impose some idea of what the model should look like using the regularization operator $\mathbf{A}$. The model styling goal helps to keep the solution from diverging due to the null space during the inversion.

We can reduce the necessary number of iterations by preconditioning the model. This incorporates the preconditioning transformation $\mathbf{m}=\mathbf{A}^{-1}\mathbf{p}$ (Fomel and Claerbout, 2003) into equation (1). $\mathbf{A}^{-1}$ is obtained by mapping the multi-dimensional regularization operator $\mathbf{A}$ to helical space and applying polynomial division (Claerbout, 1998).

The choice of regularization operator $\mathbf{A}$ depends on the information we have in the data and our expectations for the resulting model. We know that poor illumination will result in holes in the angle-domain common image gathers (ADCIGs) that are produced by the migration operator, as seen in the migration result shown in Figure 1. This is the result of downward continuation migration on a complex 2-D synthetic, showing a common ray parameter slice on the left and a common image gather on the right. Areas affected by poor illumination are indicated by ovals. To reduce the effects of this poor illumination, our regularization operator can be designed to penalize large changes in amplitude along the reflection angle (or equivalent offset ray parameter) axis (Prucha and Biondi, 2002). We refer to this operator as *geophysical regularization*. The use of this regularization scheme is demonstrated in a 2-D synthetic example by Figure 2. Figure 2 shows the result of three iterations of geophysically regularized inversion with model preconditioning, with the same areas circled as in Figure 1. In the common ray parameter slice shown as the left panel, the shadow zone beneath the salt edge is much cleaner and the events are easier to see. We have also improved the events beneath the salt body itself. In the common image gather shown as the right panel, the holes in events indicated with ovals on the migration result have been significantly filled in by the regularization in the inversion result. The same type of improvement can be expected for the 3-D case. We will discuss the use of *geophysical regularization* for a 3-D real problem in the next section. In a later section of this abstract, we will discuss the

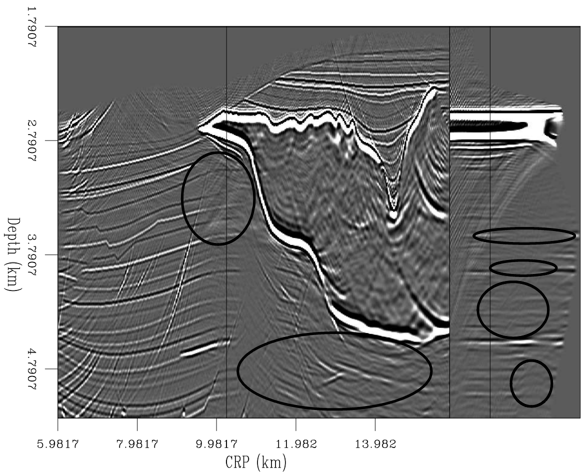

Figure 1: Result of 2-D downward continuation migration with angle-domain common image gathers. The left side shows a common ray parameter slice, the right side shows a common image gather taken from CRP=10.2337. The ovals indicate areas where poor illumination is affecting the image.

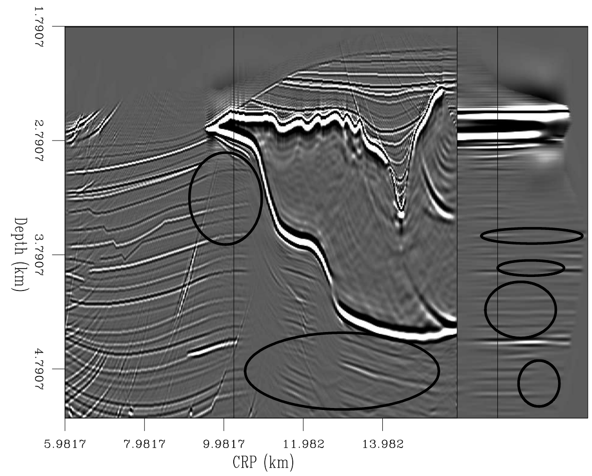

Figure 2: Result of 3 iterations of geophysically regularized inversion with model preconditioning. The left side shows a common ray parameter slice, the right side shows a common image gather taken from CRP=10.2337. The ovals indicate the same areas shown in Figure 1. Note that these areas are cleaner in the common ray parameter slice, with better continuity along the events, particularly underneath the salt body. In the common image gather, the holes in events seen in the migration result have begun to fill in.

use of a different regularization operator that is based on the idea of inverting two different 3-D datasets to obtain images that share their illumination information.

BP GULF OF MEXICO DATASET

To experiment with our 3-D regularized least-squares inversion, we required a 3-D dataset with poor illumination and an accurate velocity model. Fortunately, BP and ExxonMobil were able to provide us with such a dataset. The BP Gulf of Mexico dataset is a real deepwater dataset over a typical GOM salt structure. A flattened cube showing the velocity model can be seen in Figure 3. The velocities are believed to be accurate and range from a water velocity of 1.5 km/s to a salt velocity of 4.54 km/s. The data itself has an inline common midpoint (CMP) spacing of .025 km, a crossline CMP spacing of .025 km, and an inline offset range from .3 km to 9.4 km sampled every .05 km.

To obtain an idea of the illumination problems faced with this dataset, we ran a zero offset migration on the 3-D dataset (Figure 4). In this result, there are well defined shadow zones beneath the salt edges. These shadow zones are clearly the effect of poor illumination caused by the salt body.

In order to run regularized inversion on this 3-D dataset, we must select an appropriate linear operator $\mathbf{L}$ and regularization operator $\mathbf{A}$ for the objective function in equation (1). In this case, we choose to make $\mathbf{L}$ a common azimuth migration (CAM) operator. This means that we will only be handling offsets in the inline direction. Therefore, the regularization operator $\mathbf{A}$ can simply be the *geophysical regularization* previously discussed, acting along the inline offset ray parameter axis of the image. As seen in the 2-D example, this regularization will help to reduce artifacts and fill in the shadow zones along the ray parameter axis.

JOINT INVERSION

Although *geophysical regularization* does help to compensate for poor illumination, it is still limited to the information we can squeeze out of the regularized inversion as it is performed on one dataset. When dealing with 3-D surveys, we have another option. In most areas of particular interest, multiple 3-D surveys are shot, often in different directions. Some studies have been done comparing strike direction and dip direction surveys, which can be considered to be a special case of any two surveys shot in different directions over the same area. O'Connell et al. (1993) found that for many CMP-based processes, strike direction datasets had advantages over dip directions. Etgen and Regone (1998) showed that there are differences in multiple attenuation and illumination between strike and dip direction surveys. For these reasons, we can expect that the images produced by inverting two or more datasets shot over the same subsurface volume will contain different illumination information. We demonstrate these differences with the synthetic 2.5-D dataset represented by the velocity model in Figure 5. This is the North Sea part of the Amoco 2.5-D Carpathian Mountains over the North Sea synthetic. We have created two datasets over this velocity model, one "shot" in the dip direction and one "shot" in the strike direction. The results of common azimuth migration of these two datasets are in Figures 6 and 7. Some areas where the differences in illumination are clear have been circled. In this section, we will explain how we might use regularized inversion to "share" this illumination information among the different models by jointly inverting the available datasets.

To simplify the equations necessary to explain how joint inversion may be accomplished, let us rewrite the objective function in equation (1):

$$\begin{aligned} \mathbf{0} &\approx \mathbf{Lm} - \mathbf{d} \\ \mathbf{0} &\approx \epsilon \mathbf{Am}. \end{aligned} \tag{2}$$

Here we have split up the two "fitting goals" described earlier. We are still minimizing the same least-squares objective function using conjugate-gradient iterations, we have just rewritten the equation to simplify its form. Since we are now dealing with two datasets and we wish to obtain two models, we can replicate these fitting goals to obtain two models from two datasets:

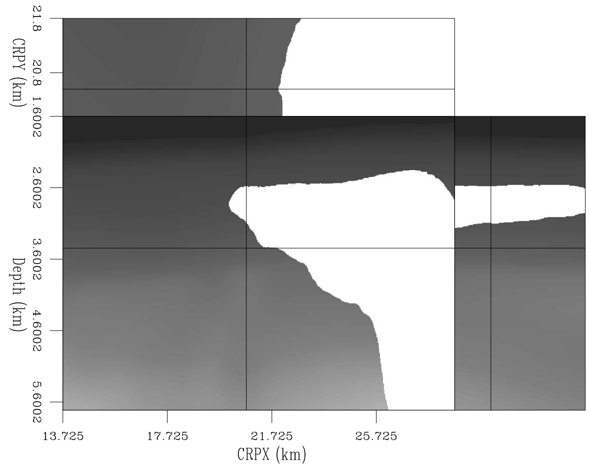

Figure 3: Velocity model for the BP Gulf of Mexico dataset. It is shown as a flattened cube, with the slices taken from the lines seen on each face of the cube.

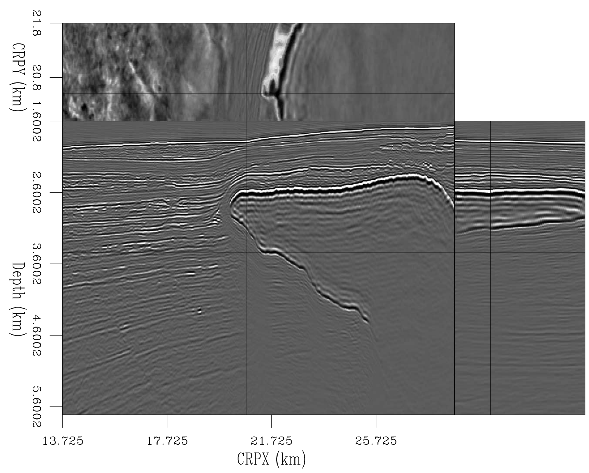

Figure 4: Zero offset migration for the BP Gulf of Mexico dataset. It is shown as a flattened cube, with the slices taken from the lines seen on each face of the cube. Note the two shadow zones extending beneath the salt edge.

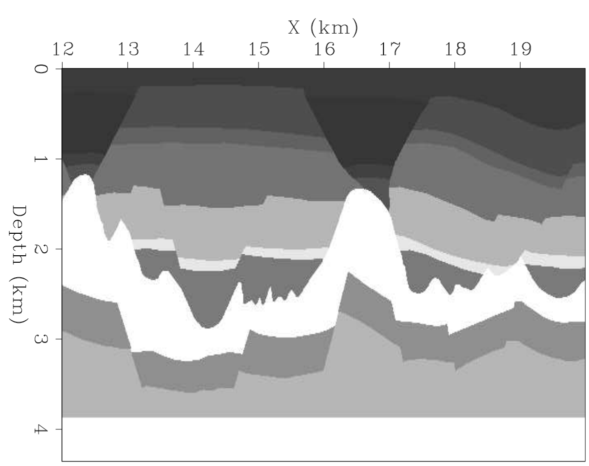

Figure 5: Velocity model for the Amoco 2.5-D North Sea dataset.

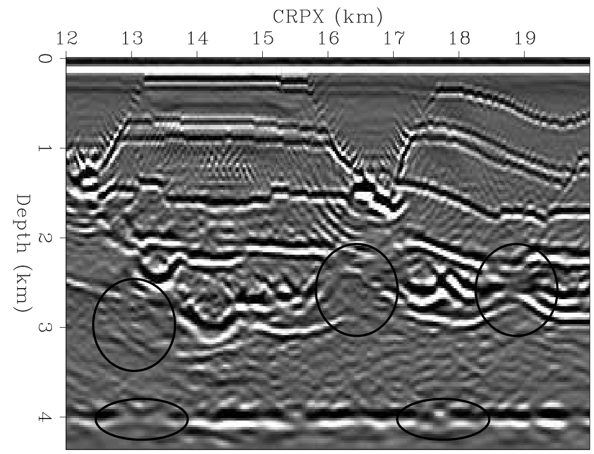

Figure 6: CAM result for the Amoco 2.5-D North Sea dataset "shot" in the dip direction. The ovals indicate particular areas that have different illumination from the strike direction result.

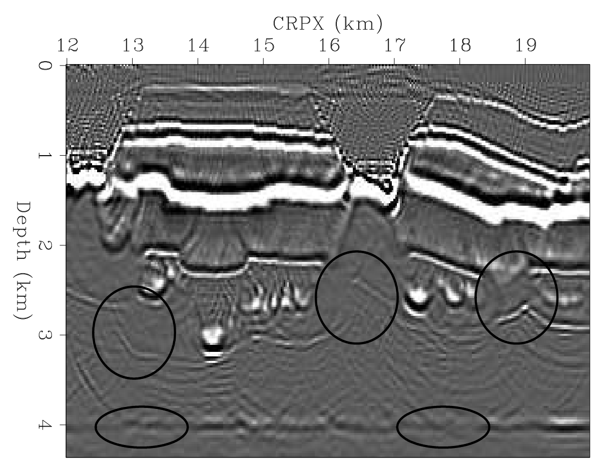

Figure 7: CAM result for the Amoco 2.5-D North Sea dataset "shot" in the strike direction. The ovals indicate particular areas that have different illumination from the dip direction result.

$$
\begin{aligned}
\mathbf{0} &\approx \begin{bmatrix} \mathbf{L_1} \\ \mathbf{L_2} \end{bmatrix} \begin{bmatrix} \mathbf{m_1} \\ \mathbf{m_2} \end{bmatrix} - \begin{bmatrix} \mathbf{d_1} \\ \mathbf{d_2} \end{bmatrix} \\
\mathbf{0} &\approx \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \begin{bmatrix} \mathbf{A_1} \\ \mathbf{A_2} \end{bmatrix} \begin{bmatrix} \mathbf{m_1} \\ \mathbf{m_2} \end{bmatrix}.
\end{aligned} \tag{3}
$$

In these fitting goals, $\mathbf{L_1}$ and $\mathbf{L_2}$ are 3-D linear imaging operators such as a common azimuth migration operator that relates the individual models, $\mathbf{m_1}$ and $\mathbf{m_2}$, to the individual datasets $\mathbf{d_1}$ and $\mathbf{d_2}$. The regularization operators $\mathbf{A_1}$ and $\mathbf{A_2}$ are individually applied to the two different models and should be designed to compensate for poor illumination. In most cases, these regularization operators will be similar to the *geophysical regularization* previously discussed. However, these expanded fitting goals do not take advantage of the fact that we are imaging the same areas of the subsurface using two different datasets. We need an additional fitting goal that will allow us to regularize the models based on each other.

Our proposed scheme for jointly inverting two datasets shot over the same area is based on regularization between stacks of the models. This inversion can be expressed in terms of three fitting goals that combine the two datasets:

$$
\begin{aligned}
\mathbf{0} &\approx \begin{bmatrix} \mathbf{L_1} \\ \mathbf{L_2} \end{bmatrix} \begin{bmatrix} \mathbf{m_1} \\ \mathbf{m_2} \end{bmatrix} - \begin{bmatrix} \mathbf{d_1} \\ \mathbf{d_2} \end{bmatrix} \\
\mathbf{0} &\approx \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \begin{bmatrix} \mathbf{A_1} \\ \mathbf{A_2} \end{bmatrix} \begin{bmatrix} \mathbf{m_1} \\ \mathbf{m_2} \end{bmatrix} \\
\mathbf{0} &\approx \epsilon_3 \, [\mathbf{S_1} - \mathbf{E}\mathbf{S_2}] \begin{bmatrix} \mathbf{m_1} \\ \mathbf{m_2} \end{bmatrix}.
\end{aligned} \tag{4}
$$

The third fitting goal is the key to regularizing the illumination between the two models. $\mathbf{S_1}$ and $\mathbf{S_2}$ are stacking operators for the two models. $\mathbf{E}$ is a cross-equalization operator that compensates for differences in amplitudes and wavelets between the two models. $\mathbf{E}$ can be created from the migration result of each dataset (Rickett et al., 1997; Rickett and Lumley, 1998). The ϵs are weights that are used to balance the strengths of the different fitting goals.

These fitting goals will result in two models that have been regularized to help fill in areas that are illuminated differently by the two surveys. Either of these models should have better illumination than the result of migration of the individual datasets. Ideally, if both datasets have the same angular coverage, the stacks of the models should be the same, but the information along the ray parameter axes will be different. This difference is due to the fact that the ray couples for each survey are traveling through different media. The directions of the surveys will cause the rays to have different illumination problems and encounter other problems such as anisotropy, attenuation, and velocity contrasts that may cause evanescence. The three fitting goals (5) will allow us to minimize the poor imaging of either individual dataset.

CONCLUSIONS

We have explained the use of iterative regularized least-squares inversion for imaging. We demonstrated the use of a geophysical regularization operator for filling in poorly illuminated areas and discussed its application to a 3-D seismic imaging problem. We also introduced a different regularization operator, called *dual stacked image regularization*, that allows us to share illumination information between images created from datasets shot in different directions over the same subsurface volume. The use of these regularization operators is an integral part of using iterative least-squares inversion to improve imaging in poorly illuminated areas.

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