Interval velocity estimation using edge-preserving regularization

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SUMMARY

We test two edge-preserving forms of model regularization in a least-squares implementation of Dix formula that leads to interval velocities with sharp edges in the (τ, x) plane. This property of the interval velocity may be desirable in geologic environments with abrupt changes in velocity, like carbonate layers, salt bodies, or strong faulting.

INTRODUCTION

Interval velocity estimation is a central problem in reflection seismology (Claerbout, 1999). Without an estimate of seismic velocities, we would be unable to transform prestack seismic data into an interpretable image. Advanced velocity estimation techniques (Clapp, 2001; Biondi and Sava, 1999) have been developed to estimate interval velocity in complex geological environments, though the cost of these methods is often considerable.

In the early stages of prospect evaluation, an inexpensive interval velocity estimate is often desired. The Dix equation (Dix, 1952) analytically inverts root-mean-square (RMS) velocity for interval velocity as a function of time. In addition to many physical shortcomings (assumption of a stratified $v(z)$ earth), Dix inversion suffers from numerical problems that lead to poor velocity estimates. Dix inversion is unstable when RMS velocities vary rapidly, and may produce interval velocities with unreasonably large and rapid variations. For this reason, the problem is often cast as a least-squares problem, which is regularized in time with a differential operator to penalize rapid velocity variations and to produce a smooth result (Clapp et al., 1998).

While temporal velocity smoothness may often be justified from a geological point of view, in some cases however, it can change abruptly (e.g., carbonate layers, salt bodies, strong faulting). In these situations we desire a regularization technique that yields smooth velocities while preserving sharp geologic interval velocity contrasts. In addition, no pre-defined boundaries should be supplied.

In this paper we present two automatic edge-preserving regularization methodologies for the least-squares implementation of Dix formula. Both methods use iterative reweighted least-squares (IRLS). The first method imposes a Cauchy distribution of the model parameters to allow a “spiky” or “sparse” model residual, which leads to a “blocky” velocity model. The second uses an isotropic edge detector, the gradient magnitude, in a nonlinear scheme to compute a measure of the edges of the model. These edges are used as a model residual weights (Clapp et al., 1998; Lizarralde and Swift, 1999).

DIX EQUATION AS A LEAST-SQUARES PROBLEM

The Dix equation states the nonlinear relationship between root-mean-square (RMS) velocity and interval velocity.

$$V^2(\tau) = \frac{1}{\tau} \sum_i v_i^2 \Delta\tau_i \tag{1}$$

Where V is the RMS velocity, v is the interval velocity and τ is the vertical traveltime.

However, this equation is linear in the square of the velocities. An example of the linear solution of the problem is given by Clapp et al. (1998). They apply a preconditioned least squares optimization to invert Dix equation, with spatial smoothness constraints.

To get the interval velocities the least-squares problem is stated as a minimization problem where the quadratic function to minimize is

$$Q(\mathbf{u}) = \|\mathbf{W}(\mathbf{d} - \mathbf{Cu})\|^2, \tag{2}$$

which is equivalent to the least-squares fitting goal:

$$\mathbf{W}(\mathbf{Cu} - \mathbf{d}) \approx \mathbf{0}, \tag{3}$$

and where $\mathbf{u}$ is the unknown vector of squared interval velocities, $\mathbf{d}$ is the known data, a vector of squared RMS velocities multiplied by the vertical traveltime, $\mathbf{C}$ is the causal integration operator, and $\mathbf{W}$ is a data residual weighting function, which is proportional to our confidence in the RMS velocity picks.

Fitting goal (3) is notoriously unstable to high frequency variations in RMS velocity, and moreover, is under-determined in the sense that only strong reflections really qualify as “data”. Therefore, Clapp et al. (1998) supplement the system with a regularization term which penalizes wiggliness. In our case we use a first order derivative operator, but as we will see later, other rougheners can be used:

$$\mathbf{W}(\mathbf{Cu} - \mathbf{d}) \approx \mathbf{0}, \tag{4}$$

$$\epsilon_\tau \mathbf{D}_\tau \mathbf{u} \approx \mathbf{0}, \tag{5}$$

$$\epsilon_x \mathbf{D_x} \mathbf{u} \approx \mathbf{0}, \tag{6}$$

where $\mathbf{D}_\tau$ and $\mathbf{D_x}$ are first-order finite-difference derivatives in time and midpoint, respectively, and the scalars ϵ_τ and ϵ_x balance the relative importance of the two model residuals with the data residual.

BLOCKY MODELS

In hard rock environments like carbonates, velocities tend to be homogeneous for intervals, with abrupt discontinuities at changes in lithology. There, the desire for a blocky interval velocity model is well-justified.

In the following sections we introduce two schemes to weight the model residuals in equations (5) and (6) to obtain sharp edges in the estimated $\mathbf{u}$.

Edge preserving regularization with the Cauchy norm

Imagine that after solving (4)-(6), the model residuals in equations (5) and (6) consist of spikes separated by relatively large distances. Then the estimated interval velocity $\mathbf{u}$ would be piecewise smooth with jumps at the spike locations, which is what we desire. However in solving (4)–(6) we use the least-squares criterion – minimization of the ℓ_2 norm of the residual. Large spikes in the residual tend to be be attenuated. To do this, the solver smoothes the velocity across the spike location.

It is known that the ℓ_1 norm is less sensitive to spikes in the residual (Claerbout and Muir, 1973; Darche, 1989; Nichols, 1994). ℓ_1 norm minimization makes the assumption that the residuals have an exponential distribution, a “long-tailed” distribution relative to the Gaussian distribution assumed by the ℓ_2 norm inversion. Here we compute nonlinear model residual weights which force a Cauchy distribution, another long-tailed distribution which approximates an exponential distribution (Youzwishen, 2001).

Our method consists of recomputing the weights at each non linear iteration, solving small piecewise linear problems. The IRLS algorithms converge if each minimization reaches a minimum for a constant weight. We perform the following non linear iterations: starting with the weights $\mathbf{Q_\tau}^0 = \mathbf{Q_x}^0 = \mathbf{I}$, at the k^{th} iteration the algorithm solves

$$\begin{aligned} \mathbf{W}(\mathbf{Cu}^k - \mathbf{d}) &\approx \mathbf{0} \\ \epsilon_\tau \mathbf{Q_\tau}^{k-1}\mathbf{D_\tau u}^k &\approx \mathbf{0} \\ \epsilon_x \mathbf{Q_x}^{k-1}\mathbf{D_x u}^k &\approx \mathbf{0} \end{aligned} \tag{7}$$

where

$$\mathbf{Q_\tau}^{k-1} = \frac{1}{\left[1 + \left(\frac{\mathbf{D_\tau u}^{k-1}}{\alpha_\tau}\right)^2\right]^{\frac{1}{2}}}, \tag{8}$$

$$\mathbf{Q_x}^{k-1} = \frac{1}{\left[1 + \left(\frac{\mathbf{D_x u}^{k-1}}{\alpha_x}\right)^2\right]^{\frac{1}{2}}}, \tag{9}$$

and $\mathbf{u}^k$ is the result of the k^{th} nonlinear iteration, $\mathbf{Q_\tau}^{k-1}$ and $\mathbf{Q_x}^{k-1}$ are the $(k-1)^{th}$ diagonal weighting operators, $\mathbf{D_\tau}$ and $\mathbf{D_x}$ are the first order derivatives in time and midpoint, $\mathbf{I}$ is the identity matrix, the scalars α_τ and α_x are the trade-off parameters controlling the discontinuities in the solution, and the scalars ϵ_τ and ϵ_x balance the relative importance of the two model residuals.

Edge preserving regularization with the gradient magnitude

In the previous section we changed the norm of the minimization problem to prevent the roughener from smoothing over edges in the model. In this section we shift from a statistical to a more mechanical approach to attain the same goal.

To preserve the edges of the model Clapp et al. (1998) propose adding a weight that de-emphasizes the model residual at the geologic edges. Lizarralde and Swift (1999) implement a similar approach for the inversion of VSP data for interval velocity. This approach requires human intervention for reflector picking. We want to design a weight which automatically de-emphasizes edges in the model residual.

The 2-D gradient magnitude is a good isotropic edge-detection operator that can be used to calculate the diagonal weights. By using the gradient magnitude we can iteratively obtain sharp edges.

We perform the following non linear iterations: starting with $\mathbf{Q}_{|\nabla|}{}^0 = \mathbf{I}$, at the k^{th} iteration the algorithm solves

$$\begin{aligned} \mathbf{W}(\mathbf{Cu}^k - \mathbf{d}) &\approx \mathbf{0}, \\ \epsilon \mathbf{Q}_{|\nabla|}{}^{k-1}\nabla^{\mathbf{2}}\mathbf{u}^k &\approx \mathbf{0}, \end{aligned} \tag{10}$$

where

$$\mathbf{Q}_{|\nabla|}{}^{k-1} = \frac{1}{1 + \frac{|\nabla \mathbf{u}^{k-1}|}{\alpha}}, \tag{11}$$

and $\mathbf{u}^k$ is the result of the k^{th} nonlinear iteration, $\mathbf{Q}_{|\nabla|}{}^{k-1}$ is the $(k-1)^{th}$ diagonal weight operator, $|\nabla|$ is the gradient magnitude, $\nabla^{\mathbf{2}}$ is the Laplacian operator, $\mathbf{I}$ is the identity matrix, the scalar α is the trade-off parameter controlling the discontinuities in the solution, and the scalar ϵ balances the relative importance of model and data residuals.

REAL DATA RESULTS

We tested both inversion methods on 125 CMP's from a 2-D prestack dataset acquired in the Gulf of Mexico. This data is suitable for using Dix equation, since the main reflectors are flat. The area is heavily faulted which may imply some lateral velocity variations with sharp edges to preserve.

First, we performed conventional stacking velocity analysis on each CMP gather, and then used an auto-picker to pick the maximum stacking power that corresponds to the best stacking velocity at each CMP location. In these section we assume the stacking velocity to be equivalent the RMS velocity. The value of the stacking power at the auto-picked RMS velocity was used as a quality measure of the data, and used as the data residual weight ($\mathbf{W}$) in equations (4), (7), and (10). Figures 1,2, and 3 show a particular CMP gather, the auto-picked RMS velocity, and a stack of the CMP's.

We hand-picked five faults, displayed as "o" symbols on figure 3. Since our regularization schemes are meant to allow velocity discontinuities at faults and other lithologic boundaries, seeing some expression of the faults on the estimated velocity panels is a crucial proof of concept.

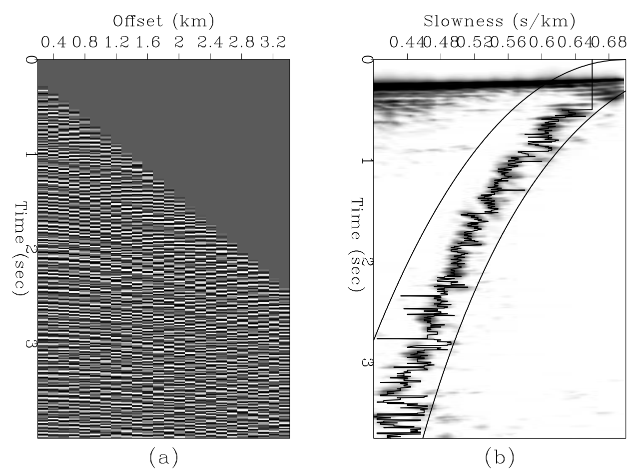

Figure 1: a) CMP gather, b) auto-picked RMS velocity in A.

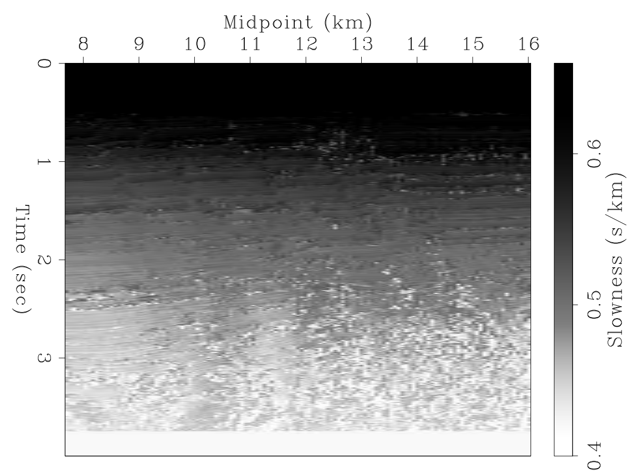

Figure 2: 2-D raw RMS velocity section

We show in figure 4 a graph comparing the interval velocities resulting from solving the inverse problems stated in equations (4),

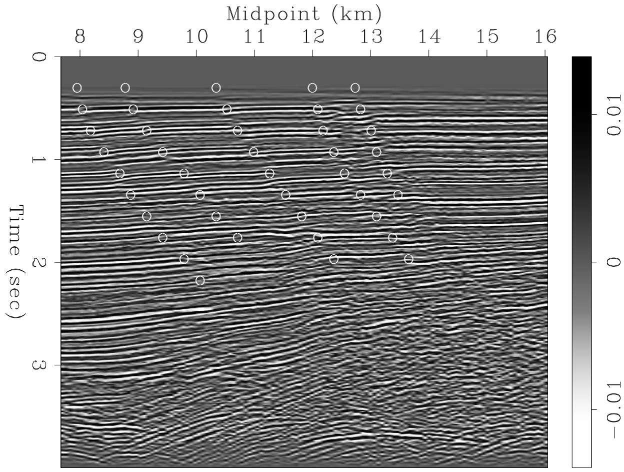

Figure 3: Stacked data section using the raw RMS velocity.

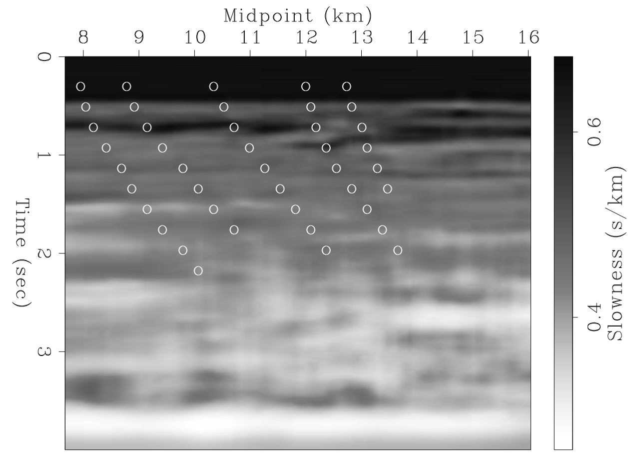

Figure 5: Interval velocity computed by inversion of the RMS velocity (equation (4)).

(7), and (10) respectively and the RMS velocity used as input data at two CMP locations. Figures 5, 6, and 7 show the interval velocities resulting from using the three different methods.

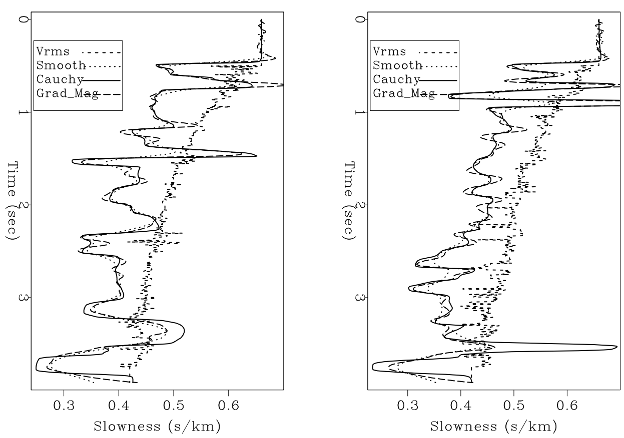

Figure 4: Comparison of the results of solving the inverse problems stated in equations (4) Smooth, (7) Cauchy norm, (10) Gradient Magnitude, and the RMS velocity at the midpoint positions 8.04 and 12.194 km.

In figure 5 the resulting interval velocity is smooth in time and space. Figure 6 shows sharp-edged rectangular shapes all over the image, looking reasonable in the faults but in general geological unappealing. Figure 7 shows sharp objects with more geological meaning.

The preferential shapes can also be seen in the diagonal weight operator. Figures 8 and 9 show ${\bf Q_\tau}^N$ and ${\bf Q_x}^N$, the last nonlinear iteration diagonal weight operator in equation (9). Notice the two preferential directions in what the edges are preserved. Figure 10 shows ${\bf Q}_{|\nabla|}{}^N$, the last nonlinear iteration diagonal weight operator in equation (11). Notice the isotropic behavior of the diagonal weight calculated using the gradient magnitude operator.

CONCLUSIONS

Dix formula can be implemented in a nonlinear least-squares inversion scheme to obtain interval velocities with sharp edges in

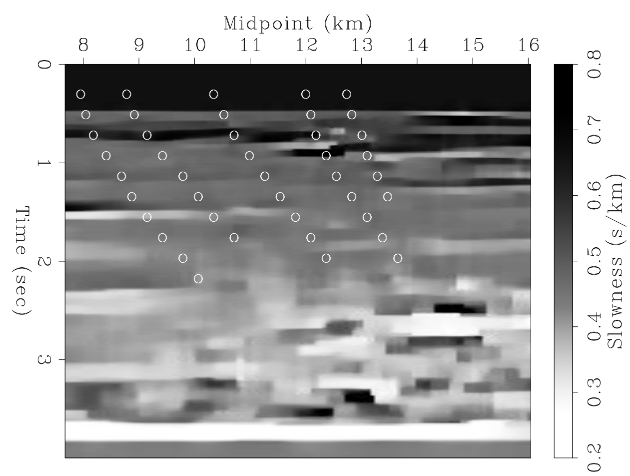

Figure 6: Interval velocity computed by inversion of the RMS velocity using Cauchy norm (equation (7)).

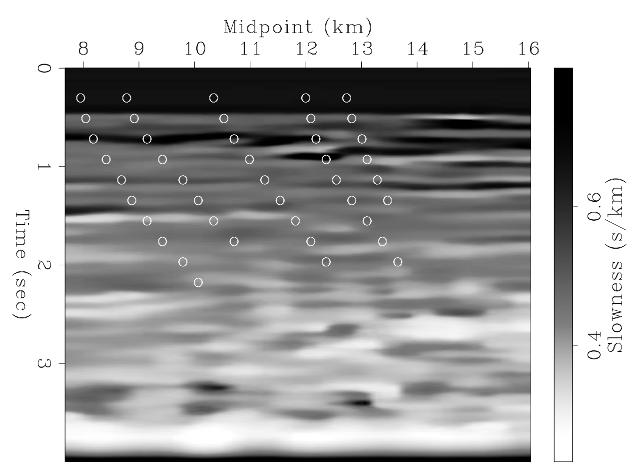

Figure 7: Interval velocity computed by inversion of the RMS velocity using gradient magnitude (equation (10)).

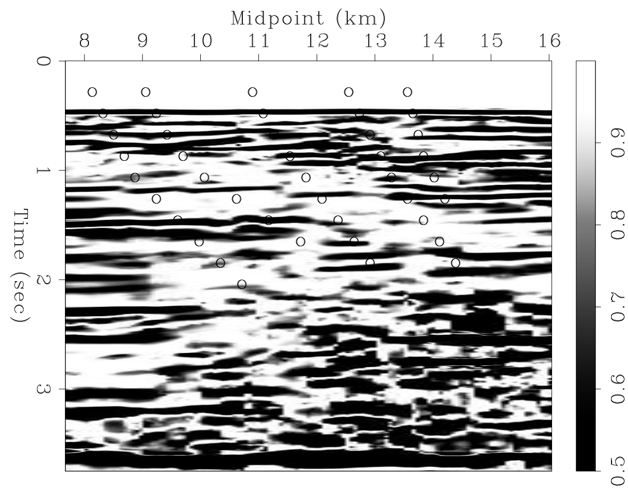

Figure 8: $\mathbf{Q}_{\tau}{}^{N}$ is the last nonlinear iteration diagonal weight operator in equation (8).

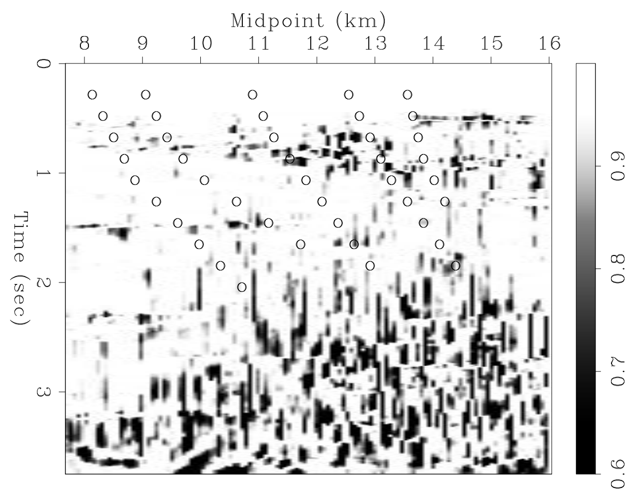

Figure 9: $\mathbf{Q}_{\mathbf{x}}{}^{N}$ is the last nonlinear iteration diagonal weight operator in equation (9).

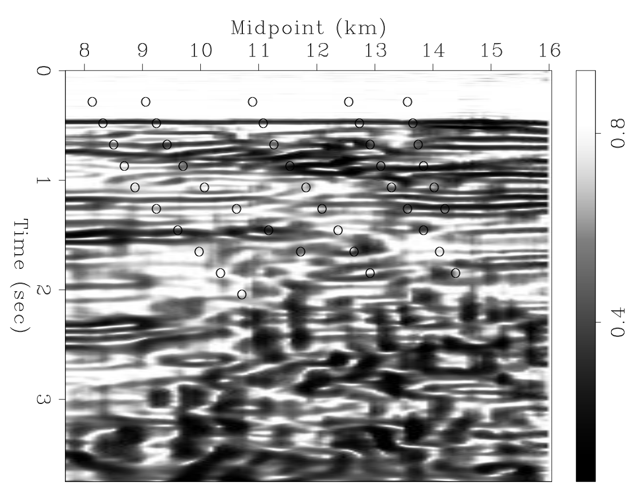

Figure 10: $\mathbf{Q}_{|\nabla|}{}^{N}$ is the last nonlinear iteration diagonal weight operator in equation (11).

the (τ, x) plane. In this paper, we presented two automatic edge-preserving regularization methods to achieve this goal.

Both methods make use iterative reweighted least-squares (IRLS). The first effectively change the norm of the problem to permit a spiky or sparse model residuals, leading to a blocky velocity model. The second uses an isotropic edge detector, the gradient magnitude, to compute the residual weights.

Both methods give the expected results when applied in a 2-D real data set acquired in the Gulf of Mexico. We conclude that the gradient magnitude method shows sharp objects with more geological appeal than the blocky method.

REFERENCES

Biondi, B., and Sava, P., 1999, Wave-equation migration velocity analysis: 69th Ann. Internat. Mtg, 1723–1726.

Claerbout, J. F., and Muir, F., 1973, Robust modeling with erratic data: Geophysics, **38**, no. 5, 826–844.

Claerbout, J., 1999, Everything depends on v(x,y,z): SEP–**100**, 1–10.

Clapp, R. G., Sava, P., and Claerbout, J. F., 1998, Interval velocity estimation with a null-space: SEP–**97**, 147–156.

Clapp, R. G., 2001, Geologically constrained migration velocity analysis: Ph.D. thesis, Stanford University.

Darche, G., 1989, Iterative l_1 deconvolution: SEP–**61**, 281–302.

Dix, C. H., 1952, Seismic prospecting for oil: Harper and Brothers.

Lizarralde, D., and Swift, S., 1999, Smooth inversion of VSP traveltime data: Geophysics, **64**, no. 3, 659–661.

Nichols, D., 1994, Velocity-stack inversion using $\mathbf{L_p}$ norms: SEP–**82**, 1–16.

Youzwishen, C., 2001, Non-linear sparse and blocky constraints for seismic inverse problems: MSc Thesis, Department of Physics, University of Alberta, Canada, http://cm-gw.phys.ualberta.ca/ sacchi/saig/theses.php.