First order lateral interval velocity estimates without picking

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SUMMARY

A new method for estimating interval velocities without picking is proposed. The first step consists in applying a normal move-out correction with a $v(z)$ stacking velocity on common mid-point gathers. In the second step we estimate local stepouts at every offset and time for each gather. We then integrate the local stepouts across offset to obtain local time shifts. The integration is done in the Fourier domain for increased speed. Finally we estimate the interval velocity in the τ space by fitting the time shifts with a tomographic inversion procedure based on a straight rays geometry. This approach is tested on a Gulf of Mexico dataset with flat geology; we demonstrate that lateral velocity variations across faults can be recovered.

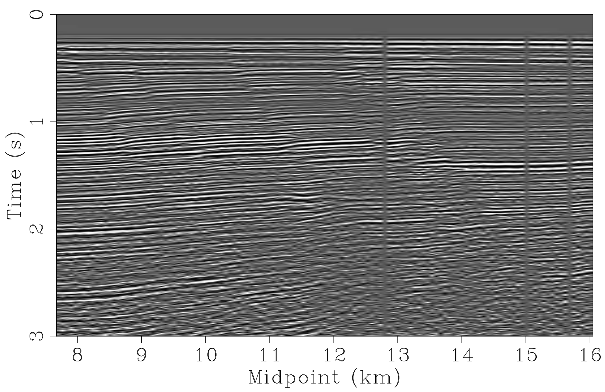

Figure 1: Near-offset section of the 2D field dataset from the Gulf of Mexico. Some normal faults are visible.

INTRODUCTION

Interval velocity estimation requires picking in many circumstances. We might pick parameters indicating how flat is a common depth point (CDP) gather for an initial velocity model (Al-Yahya, 1989; Etgen, 1990) or how well an image focuses after residual migration (Biondi and Sava, 1999). In stereotomography (Billette et al., 2003), slopes and traveltimes are picked for the velocity estimation process. Other imaging techniques such as common-focus-point migration require semblance analysis for updating focusing operators (Berkhout and Verschuur, 2001). In addition for most of these methods, specifically those based on tomography, several reflectors need to be selected (picked) for the velocity inversion (Clapp, 2001). Very few velocity estimation techniques do not require any picking. For instance, Toldi (1989) derives a relationship between interval slowness perturbations and stacking slowness perturbations that is limited to flat geology with constant velocity background. Closest to our approach, Symes and Carazzone (1991) directly invert time shifts between adjacent traces to estimate interval velocities.

It is our belief that picking is inherently flawed and should be replaced by more robust techniques requiring as little human interpretation as possible. Therefore, we propose a fully automated interval velocity estimation technique based on (1) dip estimation, (2) dip integration, and (3), tomographic inversion. Our ultimate goal is to be able to provide a robust technique that can give us a first order estimate of interval velocities.

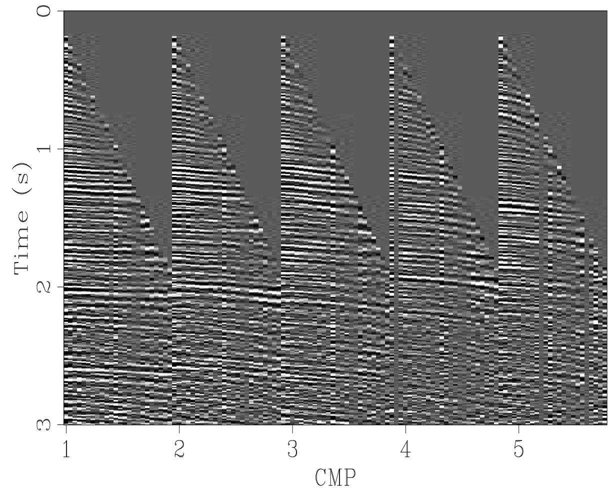

Figure 2: Five CMP gathers every 1.6 km. after NMO correction with the initial 1D RMS velocity derived from the interval slowness. Some residual curvature is apparent throughout the section.

Our initial velocity, or starting guess, is a $v(z)$ model. From this simple model we apply a NMO correction to the CMP gathers. In general, NMO is not able to completely flatten CMP gathers because the velocity is laterally varying. To obtain flat gathers, we estimate local stepouts from the NMO corrected gathers. This estimation is done trace by trace. Once the local stepouts are estimated, they are integrated to form absolute time shifts at every time, offset and midpoint location. Next from the time shifts, we perform a tomographic inversion in the (x, τ) space (Clapp and Biondi, 2000), where x is the mid-point position and τ the zero-offset travel time, by assuming straight rays between the subsurface location and the source/receiver positions.

We check whether the estimated velocity perturbations flatten the gathers or not by applying the forward modeling operator to the estimated velocity perturbations; we then obtain modeled time shifts that we apply to the NMO corrected gathers. From our approach we are able to obtain updated interval velocities and flat CMP gathers. In the next two sections, we describe the time-shifts estimation step and the tomographic inversion.

ESTIMATION OF TIME SHIFTS

Estimating time shifts is a two steps procedure where local stepouts are first estimated and then integrated. The goal of dip estimation is to find the local stepout p_h that destroys the local plane wave such that

$$0 \approx \frac{\partial u}{\partial h} + p_h \frac{\partial u}{\partial \tau}, \quad (1)$$

where u is the wave field at time τ, midpoint x and offset h. For all the gathers, we evaluate the slope p_h with a method based on high-order plane-wave destructor filters (Fomel, 2002). This technique has the advantage of being accurate for steep dips.

Once the dips are estimated, we obtain a vector of local stepouts $\mathbf{p}_h$ that we integrate to obtain time shifts. The dips are integrated for one CMP gather at a time. As mentioned earlier, the dips are

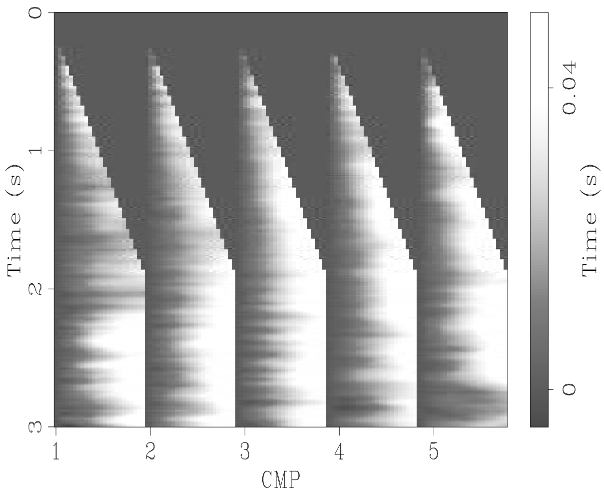

Figure 3: Estimated time shifts for five CMP gathers. The maximum time shift is around 0.05 s. Note that the first trace is set to zero.

smoothed along both spatial directions, but not in time τ. To enforce smoothness in the τ direction we introduce a time component $\mathbf{p}_\tau$ to $\mathbf{p}$. The relationship between the local time shift vector $\ell = \ell(\tau, x, h)$ (x held constant) and the local dip vector $\mathbf{p} = (\mathbf{p}_h, \mathbf{p}_\tau)^T$, $(.)^T$ being the transpose, is defined as follows:

$$\mathbf{p} = \begin{pmatrix} \mathbf{p}_h \\ \mathbf{p}_\tau \end{pmatrix} = \begin{pmatrix} \partial_h \ell \\ \partial_\tau \ell \end{pmatrix}, \qquad (2)$$

where ∂_h is the partial derivative in offset h and ∂_τ is the partial derivative in τ. In practice, we never have to compute $\mathbf{p}_\tau$. $\mathbf{p}_\tau$ can be seen as a regularization term that imposes smoothness in the τ direction for ℓ. To control the amount of smoothness, we introduce a trade-off parameter ϵ as follows:

$$\begin{pmatrix} \partial_h \ell \\ \epsilon \partial_\tau \ell \end{pmatrix} = \begin{pmatrix} \mathbf{p}_h \\ \epsilon \mathbf{p}_\tau \end{pmatrix}. \qquad (3)$$

From equation (3) we wish to minimize the length of the vector $\mathbf{r}_\ell$ that measures the difference between $\mathbf{p}_\epsilon$ and $\nabla_\epsilon \ell$:

$$\mathbf{0} \approx \mathbf{r}_\ell = \nabla_\epsilon \ell - \mathbf{p}_\epsilon, \qquad (4)$$

where $\nabla_\epsilon = (\partial_h, \epsilon \partial_\tau)^T$, and $\mathbf{p}_\epsilon = (\mathbf{p}_h, \epsilon \mathbf{p}_\tau)^T$. We then minimize the following objective function:

$$f(\ell) = \|\mathbf{r}_\ell\|^2, \qquad (5)$$

where $\|.\|$ is the L_2 norm. By setting $\mathbf{p}_\tau = \mathbf{1}$ and by increasing ϵ, the estimated time shifts $\hat{\ell}$ get smoother in time. We solve equation (5) analytically in the Fourier domain (Lomask, 2003), which speeds up the estimation of ℓ:

$$\hat{\ell} \quad \approx \quad \mathrm{FFT_{2D}}^{-1} \left[\frac{\mathrm{FFT_{2D}} \left[\nabla'_\epsilon \mathbf{p}_\epsilon \right]}{-Z_h^{-1} - \epsilon Z_\tau^{-1} + 2 + 2\epsilon - Z_h - \epsilon Z_\tau} \right] \qquad (6)$$

where $Z_h = e^{iw\Delta h}$ and $Z_\tau = e^{iw\Delta\tau}$. The dip integration yields the desired time shifts plus a constant, i.e., a zero frequency component. We remove the zero frequency component by subtracting the near offset panel to the other offsets. We then assume that the time shifts are a measure of the relative moveout errors to the near-offset panel.

TOMOGRAPHY

From the time shifts, we estimate interval velocities in the τ domain. Going from depth to vertical travel time introduces some new variables. As described by Alkhalifah (2003), the transformation from depth coordinates (x, z) into vertical-traveltime coordinates $(\tilde{x}, \tau)$ is governed by the relationships:

$$\tau(x,z) \quad = \quad \int_0^z \frac{2}{v(x,z')} dz', \qquad (7)$$

$$\tilde{x}(x,z) \quad = \quad x. \qquad (8)$$

Therefore, we have the following relationships between the differential quantities (dx, dz) and $(d\tilde{x}, d\tau)$:

$$dz \quad = \quad \frac{v(x,z)}{2} d\tau - \frac{v(x,z)\sigma}{2} d\tilde{x}, \qquad (9)$$

$$dx \quad = \quad d\tilde{x}, \qquad (10)$$

where $v(x,z)$ is the focusing velocity proportional to the mapping velocity (Clapp, 2001) and

$$\sigma = \int_0^z \frac{\partial}{\partial x} \left(\frac{2}{v(x,z')} \right) dz'. \qquad (11)$$

Because we assume that the initial slowness field is horizontally invariant, $\sigma = 0$. We also assume that x and $\tilde{x}$ are equal.

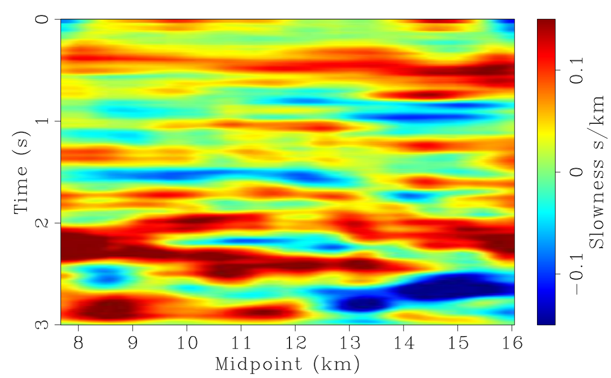

Figure 4: Slowness perturbations estimated from the tomography in the τ space.

The data space for this inverse problem is a cube of time-shifts at every time, offset and midpoint location. This differs from most of the tomographic techniques where few reflectors are usually selected and picked for the inversion. The number of unknowns for the model space (the velocity update) is equal to the number of gathers times the number of time samples; a velocity perturbation is computed for each pixel in the model space. For a CMP location x at time τ and offset h, a total time shift ℓ is estimated. The forward problem relating the velocity perturbations and the time shifts is derived from Fermat's principle:

$$\ell(\tau, x, h) = \int_0^\tau (\Delta s^-(r) + \Delta s^+(r)) dr, \qquad (12)$$

with

$$dr = \frac{d(dt)}{dS}, \qquad (13)$$

where $\Delta s^{-}(r)$ and $\Delta s^{+}(r)$ are the slowness perturbations along the down- and up-going rays respectively (from $x-h/2$ to x and $x+h/2$ to x) and S is the focusing slowness and dt the time increment along the ray (Clapp and Biondi, 2000). To simplify the problem, we assume that the up- and down-going rays are straight lines in the (τ,x) space.

Now we have in equation (12) a linear relationship between the time shifts and the slowness perturbations that we can rewrite

$$\mathbf{d} = \mathbf{L}\mathbf{m}, \tag{14}$$

where $\mathbf{d}$ are the estimated time shifts, $\mathbf{L}$ is the tomographic operator in equation (12) and $\mathbf{m}$ the field of slowness perturbations. Our goal is to find $\mathbf{m}$ such that

$$\mathbf{0} \approx \mathbf{r_d} = \mathbf{L}\mathbf{m} - \mathbf{d}. \tag{15}$$

We also add a regularization operator to enforce smoothness in the horizontal direction as follows:

$$\begin{array}{ccccc} \mathbf{0} & \approx & \mathbf{r_d} & = & \mathbf{L}\mathbf{m}-\mathbf{d}, \\ \mathbf{0} & \approx & \epsilon\mathbf{r_m} & = & \epsilon\nabla_x\mathbf{m}. \end{array} \tag{16}$$

where ∇_x is the horizontal gradient. We estimate $\mathbf{m}$ in a least-squares sense by minimizing the objective function

$$f(\mathbf{m}) = \|\mathbf{r_d}\|^2 + \epsilon^2\|\mathbf{r_m}\|^2. \tag{17}$$

In general, after a first update of the velocity, more iterations are needed to converge towards a satisfying model that will flatten the gathers. It is not our intention to do so. We merely want to get a robust first velocity estimate that, if needed, can be improved further with, for instance, migration velocity analysis (Sava and Biondi, 2003). We now test our method on a 2D Gulf of Mexico dataset.

A GULF OF MEXICO 2D FIELD DATA EXAMPLE

We show in Figure 1 a near-offset section of a 2D dataset. The geology is relatively simple with mostly flat layers and few normal faults. We estimate a first 1D interval slowness model by assuming a $v(z) = v_0 + \alpha z$ function that leads to $v(\tau) = v_0 e^{\alpha\tau/2}$ in the (x,τ) space.

We choose $v_0 = 1.6 km/s$ and $\alpha = 0.5 s^{-1}$. We then transform the velocity into an interval slowness function. In Figure 2 we show a few CMP gathers after NMO correction. The gathers are displayed for every twenty-fifth CMP location. Note that these gathers are not perfectly flat.

From the CMP gathers we estimate local stepouts and time shifts. Figure 3 displays the estimated time shifts for the five selected CMP gathers. It is interesting to notice that the time shifts increase with offset. The fact that the estimated time shifts change with midpoint for a fixed τ and offset prove that lateral velocity variations exist.

Now, we can estimate the velocity perturbations from the time shifts with the τ tomography. Figure 4 shows estimated slowness perturbations and Figure 5 displays the updated slowness field. We used 40 iterations and set $\epsilon = 1$ to obtain this result. Notice that we are effectively able to highlight lateral velocity variations very well. In Figure 5, we pick four fault locations from the seismic. These faults locations seem to be aligned with velocity variations. In particular, it is pleasing to see the change of velocities across the different faults.

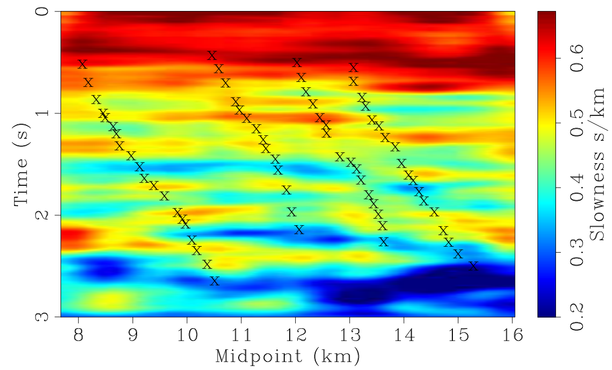

Figure 5: Updated slowness field with four interpreted faults from the seismic. Note the changes of velocity across the faults.

To check whether our method converged, we estimate modeled time shifts from the slowness perturbations in Figure 4 by applying the forward operator in equation (12). The re-modeled time shifts are shown in Figure 6. Comparing Figures 3 and 6, we note that the re-modeled time shifts are smoother. Yet, applying these time shifts to the NMO corrected data in Figure 2 yield flat gathers (Figure 7). The difference between Figures 3 and 6 is that the re-modeled time shifts are constrained by the physics of the tomographic inversion, thus giving well-behaved amplitude variations. In Figure 3, however, the time shifts take any value according to estimated dips. The forward operator of the tomographic inversion can be interpreted as a velocity consistent time shifts estimator.

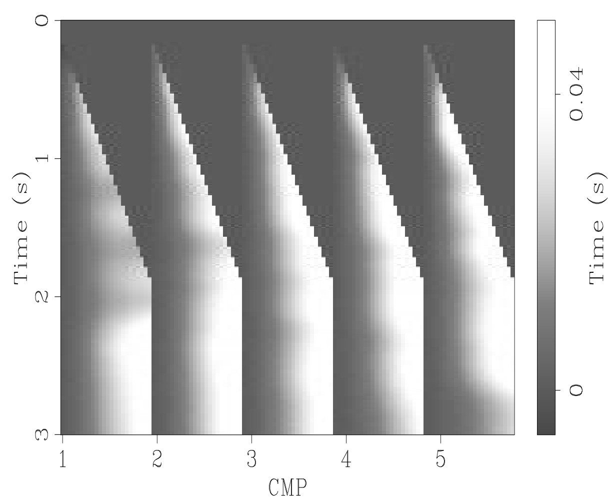

Figure 6: Modeled time shifts from the perturbations in slowness in Figure 4. Note that the remodeled time shifts are smoother than the original ones in Figure 3.

DISCUSSION

We presented a method that estimates interval velocities without picking. In this approach, we estimate time shifts from NMO corrected gathers by first computing local stepouts and then integrating them across offsets. These times shifts are then inverted for with a tomographic inversion in the (x,τ) space.

This technique presents some limitations that require improvements. For instance we assume that the local stepouts are single valued

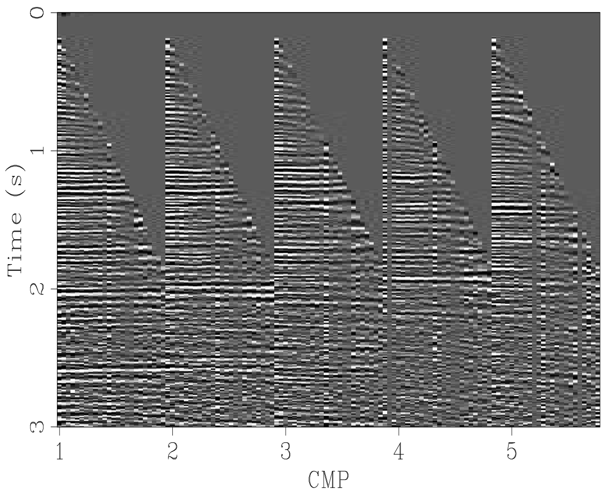

Figure 7: Flattened data with the remodeled time shifts in Figure 6. The gathers are flat showing that the slowness perturbations in Figure 4 form a satisfying model.

with no conflicting dips in the data. We can overcome this limitation by estimating few dips and keeping the ones of interest only. Second, we need an initial 1D velocity model for our starting guess. This approximation, along with the simplified geometry of the rays, prevent us from recovering lateral velocity variations for complex geology, e.g., salt environment.

Having these limitations in mind, we selected a 2D Gulf of Mexico dataset that was compliant with our approximations. We show that our technique is able to recover lateral velocity variations without picking. We also demonstrate that the updated velocity field follows quite closely the geological environment: we can see velocity changes across faults at various locations. We finally show that the estimated velocity perturbations yield a map of time shifts that can be later used to flatten the CMP gathers.

In theory, more iterations of velocity updating should be performed. One problem with more updates is the need for more sophisticated time or depth imaging algorithms. In addition, more updates would mean improving on the tomographic inversion by allowing any type of ray geometry and background slowness field. These changes go beyond the scope of this paper. We believe, however, that all these sources of improvements should be investigated further to provide a robust and picking-free interval velocity estimation tool.

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