Interpolation of irregularly-sampled data with non-stationary, multi-scale prediction-error filters

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SUMMARY

I develop a multi-scale method to estimate a stationary prediction-error filter on sparse data, which was previously only possible on regularly-sampled data. I then extend this method to estimate non-stationary prediction-error filters on sparse data. I then use these filters to interpolate both stationary and non-stationary test data, with promising results.

INTRODUCTION

Data interpolation can be cast as an inverse problem where the known data remains constant, and the empty bins are regularized to constrain the null space. A two-stage linear approach was developed (Claerbout, 1999) where a prediction-error filter (PEF) is estimated on known data, and is then used to constrain the unknown data by minimizing the output of the model after convolution with the PEF. When the data is not stationary, a non-stationary filter has been used to fill the unknown data (Crawley, 2000). This usually produces better results than a patching approach, where the data is broken up into separate patches that are assumed to be stationary and are treated as independent problems.

Prediction-error filters (PEFs) are estimated by minimizing in a least-squares sense the output of convolution of known data with the unknown PEF, either in the f-x (Spitz, 1991) or t-x domains (Claerbout, 1992).

When interpolating missing data, the fitting equations that depend on missing data must be removed, ie. equations in the convolution where the filter falls upon unknown data cannot be used. As less and less contiguous data is present, the number of possible fitting equations drops. When there is very little contiguous data, a PEF cannot be estimated in this manner, as the number of fitting equations would be too low to determine the coefficients of the PEF. In the case of coarsely sampled data, the filter can be stretched over various scales to fit the data (Crawley, 2000), and create more fitting equations, as shown in case (b) of Figure 1. However, this method fails when the data are not sampled regularly.

I estimate a PEF on sparse data by scaling the data to various different bin sizes and simultaneously estimating a single filter on these data representations in a multi-scale approach. Once the PEF has been estimated, the data can be interpolated as in the second stage of Claerbout's (1999) method. I illustrate this with a 2-D test case.

I take this multi-scale approach for sparse data, and extend it to estimate a non-stationary PEF. Non-stationary prediction-error filters require many parameters, such as micro-patch size, scale choice, regularization, and filter size. I examine how to choose these parameters and how they are related when using this method. I use this approach to interpolate a 3D non-stationary test case with very promising results.

First, I will review how to estimate a prediction-error filter, and then how to estimate a non-stationary prediction error filter. Then I will describe the new multi-scale approach for both stationary and non-stationary PEFs, and then show two examples where this method is used.

PEFS

A PEF is traditionally estimated by minimizing the output of the known data (d_i), convolved with a filter (f_i) that is unknown except for the first coefficient, which is constrained to 1. This can be written as

$$\begin{bmatrix} d_0 & 0 & 0 \\ d_1 & d_0 & 0 \\ d_2 & d_1 & d_0 \\ d_3 & d_2 & d_1 \\ d_4 & d_3 & d_2 \\ d_5 & d_4 & d_3 \\ d_6 & d_5 & d_4 \end{bmatrix} \begin{bmatrix} 1 \\ -f_1 \\ -f_2 \end{bmatrix} = r \approx \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (1)$$

or it can be written more compactly as

$$\mathbf{W}(\mathbf{DKf}+\mathbf{d}) \approx \mathbf{0}, \quad (2)$$

where $\mathbf{K}$ is a mask that constrains the first filter coefficient to 1, $\mathbf{W}$ is a diagonal weighting operator ($\mathbf{W}$) that is equal to 1 only when all filter coefficients lie on known data (and is 0 otherwise), $\mathbf{D}$ represents convolution with the data, and $\mathbf{d}$ is simply a copy of the data.

In equation (2), the left side of the fitting goal is minimized; the standard overdetermined least-squares solution (Tarantola, 1987) is

$$\mathbf{f_{est}} = -(\mathbf{K^T D^T W^T W D K})^{-1}\mathbf{K^T D^T W^T W d}. \quad (3)$$

Equation (2) works well for estimating the PEF if there are contiguous data. However, if the data are inadequately sampled resulting in an insufficient number of fitting equations, a PEF cannot be estimated. For example, if in equation (1) d_0, d_3 and d_5 are missing, a PEF could not be determined. In the case of constant sampling, the PEF can be spaced (Crawley, 2000) so that the filter coefficients fall on known data, creating valid fitting equations. When the data is sparsely sampled, this is not possible, as illustrated in figure 1.

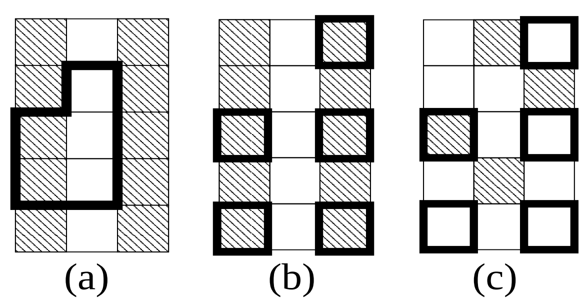

Figure 1: Three cases of a possible fitting equation. The diagonal lines represent gridded data, the white squares are empty bins, and the bold lines represent the 3 x 2 PEF. (a): Original PEF on interlaced data, where an equation is not possible. (b): Spaced PEF on interlaced data, when an equation is possible. (c): Spaced PEF on irregularly spaced data, resulting in no possible equation.

When dealing with seismic data, the covariance of the data varies with position, so the assumption of stationarity is not valid, making stationary PEF-based interpolation less attractive. To overcome this, the data can either be broken up into patches, or non-stationary PEFs can be used.

NON-STATIONARY PEFS

A non-stationary filter varies with position, so instead of only having indices corresponding to the lag of the filter, there are also indices corresponding to the position of the filter. The filter would go from looking like `f(ia)` to `f(ia,id)`, where `ia` is the lag of the filter, and `id` is the position of the filter. Only two indices are used for lag and position, thanks to the helical coordinate system (Claerbout, 1998).

When moving from estimating a stationary filter with `na` filter coefficients to estimating a non-stationary filter with `na*nd` filter coefficients, PEF estimation becomes an under-determined problem instead of an over-determined problem. As a result, we need to incorporate some type of regularization into the estimation in order to adequately constrain the result. Laplacian or radial rougheners of common filter coefficients (constant `ia`) across the spatial axes (`id`) are used to ensure a non-stationary filter that varies smoothly in space (Clapp et al., 1999).

Another method used to better constrain the filter coefficients is called micro-patching (Crawley, 2000). Instead of the filter varying at every data point, micro-patching uses the same filter within a small region of data, reducing the number of filter coefficients that need to be estimated. This has two benefits: the PEF estimation problem becomes less under-determined, and the amount of memory required for the filter goes from `na` times the size of the data (`nd`) to `na` times the number of micro-patches (`np`).

MULTI-SCALE PEFS

In the case of sparse data, more fitting equations for a stationary PEF can be generated by regridding the data at multiple different scales ($\mathbf{d_i}$). All of these different scales of data can then be convolved ($\mathbf{D_i}$) with the unknown PEF, which leads to a better-determined system of equations,

$$\mathbf{W}\left(\begin{bmatrix}\mathbf{D_0}\\ \mathbf{D_1}\\ \mathbf{D_2}\\ \dots\\ \mathbf{D_n}\end{bmatrix}\mathbf{Kf}+\begin{bmatrix}\mathbf{d_0}\\ \mathbf{d_1}\\ \mathbf{d_2}\\ \dots\\ \mathbf{d_n}\end{bmatrix}\right)\approx\mathbf{0}. \tag{4}$$

The weight ($\mathbf{W}$) now includes a weight for missing data in all of the scaled versions of the data. The multiple copies of data ($\mathbf{D_i}$ and $\mathbf{d_i}$) are created by performing linear interpolation on the original data, followed by adjoint linear interpolation. That is: for each bin, create a data point located at the center of that bin, and then use those data points for linear interpolation onto a coarser grid.

Several user-defined parameters must be set during this procedure, including the choice of scales to be used in the estimation as well as the size of the PEF. The only constraint on these parameters is that the aspect ratio of the data remain constant from scale to scale. This means that the ratio of the number of bins in each dimension must remain constant. For example, a 50 x 40 data set should not be regridded to 26 x 21, since the aspect ratio is changed by the round-off from 20.8 to 21. A better choice would be to use 25 x 20 as a scale. The PEF size is only constrained by the size of the coarsest scale of data.

MULTI-SCALE NON-STATIONARY PEFS

The combination of non-stationary filters and PEF estimation on multiple scales of data introduces the new complication that non-stationary filters are linked to the size of the data they operate on. If the dimensions of the data change, the dimensions of the PEF must change to match the data dimensions. This raises issues regarding the consistency of the PEF across scales and limits the choices of possible scales.

In order to maintain a spatially consistent PEF across scales, the filter must be sub-sampled so that the same spatial coordinates of the PEF correspond to the proper locations within the scaled data. Since our non-stationary PEF has micro-patches where the filter coefficients are constant, we can scale the patches to match the scaling of the data. The number of filter coefficients in the non-stationary filter remains constant from scale to scale, but the size of the micro-patches has decreased. The limiting case for this scaling is when a micro-patch reduces to a single point. Beyond that, some micro-patches would be completely unconstrained. An example of this scaling is shown in figure 2.

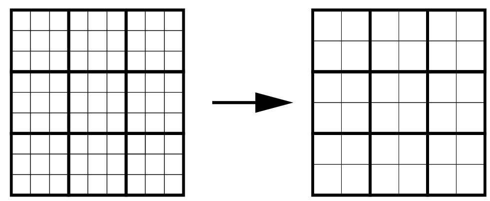

Figure 2: Two scales of the same data (fine grid, $\mathbf{d_i}$) with micro-patches (thick grid) regridded by $\mathbf{P_i}$. In this case a 9 x 9 grid with 3 x 3 micro-patches is regridded to a 6 x 6 grid with 2 x 2 micro-patches. The sizes and locations of the micro-patches remain constant, meaning that the number of filter coefficients does also.

I represent the sub-sampling of the patch table by $\mathbf{P_i}$, which acts upon the non-stationary filter $\mathbf{f}$ in the fitting goal shown below:

$$\mathbf{W}\left(\begin{bmatrix}\mathbf{D_0}\\ \mathbf{D_1}\\ \mathbf{D_2}\\ \dots\\ \mathbf{D_n}\end{bmatrix}\mathbf{K}\begin{bmatrix}\mathbf{P_0}\\ \mathbf{P_1}\\ \mathbf{P_2}\\ \dots\\ \mathbf{P_n}\end{bmatrix}\mathbf{f}+\begin{bmatrix}\mathbf{d_0}\\ \mathbf{d_1}\\ \mathbf{d_2}\\ \dots\\ \mathbf{d_n}\end{bmatrix}\right)\approx\mathbf{0}. \tag{5}$$

Since the above fitting goal is likely under-determined, a set of regularization equations must also be solved, where $\mathbf{A}$ is our regularization operator:

$$\mathbf{Af}\approx\mathbf{0}. \tag{6}$$

Once this non-stationary PEF has been estimated, it is used to interpolate the data in the same manner as a stationary PEF, except that the convolution is now non-stationary.

RESULTS

A stationary, two-dimensional test case has been created as a proof of concept example, shown in figure 3. This data is based upon a simple plane wave model and contains two dipping events of different frequency and a large amount of uniformly distributed noise.

Approximately 85 percent of the traces were randomly removed, leaving the only input to the PEF estimation shown in figure 4.

The results of the interpolation are shown in figure 5. A 5 x 3 PEF was estimated on the 10 different scales of the data, and the system was solved by a conjugate-gradient method (Hestenes and Steifel, 1952).

The PEF appears to capture the dominant dip in the data very easily, but has difficulty capturing the second dip. This is largely due to

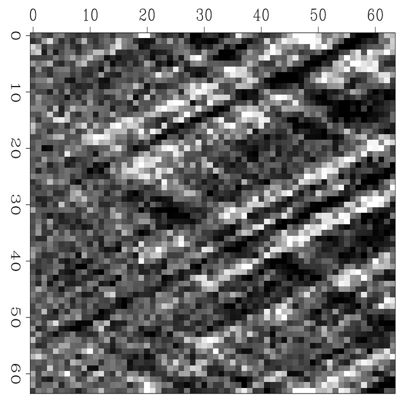

Figure 3: The fully sampled version of the stationary test data.

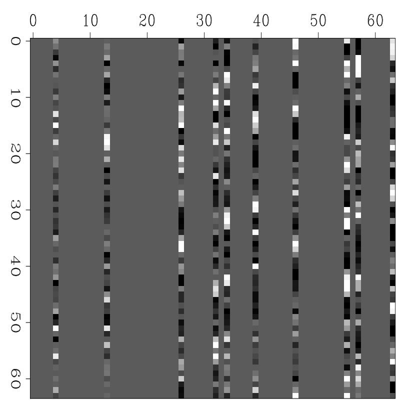

Figure 4: The stationary test data with 85 percent of the data removed.

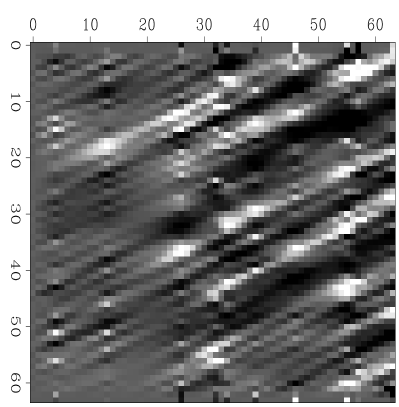

Figure 5: The results of filling the data in Figure 4.

the much lower amplitude of the second event combined with the poor sampling. In this case, the results appear to be more promising than when the data is not fully-sampled in at least one dimension (Curry and Brown, 2001; Curry, 2002).

A more relevant case to interpolating seismic data is the qdome model (Claerbout, 1993), shown in Figure 6. This data has been highly sub-sampled along two of three dimensions, with the vertical axis still fully sampled. The qdome model is a collection of folding layers, flat layers and a fault, which acts as an excellent overall test for this interpolation method. I have randomly removed 88 percent of the traces from the data set, and use the non-stationary multi-scale PEF-based interpolation to attempt to recreate the original model.

The results for the qdome model are very promising. The smoothly varying dips were correctly estimated and interpolated almost everywhere, excluding very steep dips. There are two reasons for this: the size of the PEF might not have been large enough to capture the spatially aliased dips, and the dips were changing rapidly within a small area, which was only covered by a small number of micro-patches (where the PEF is constant).

The results for this 3D case are much more impressive than in the 2D case, even though more of the data were removed. There are several reasons for this. The extra dimension of data allows for more constraints to be applied by the regularization. Also, the greater size of all of the dimensions allows for more fitting equations to be found.

CONCLUSIONS

Previously, it was not possible to estimate a prediction-error filter on sparse data. I have shown a method to estimate both stationary and non-stationary prediction-error filters on sparse data, by simultaneously using multiple re-scaled versions of the data in the estimation.

For the stationary case, a test was shown that was mostly successful. In the presence of a substantial amount of noise, a PEF was estimated, and the interpolated result mostly resembles the original data. By producing multiple equiprobable realizations of the interpolated data (Clapp, 2000), the interpolated traces could more closely resemble the original data.

Estimating a non-stationary prediction-error filter with multiple scales of data appears to be successful. The method interpolates a very heavily decimated 2D test case. Results for a 3D case are more successful, even though the amount of data removed from the case was greater than in the 2D case.

The cost of the non-stationary PEF estimation is naturally substantially higher than in the stationary case, due to the larger number of parameters to be determined. This can be reduced by preconditioning the problem (Claerbout, 1999), where $\mathbf{f} = \mathbf{A}^{-1}\mathbf{s}$ is substituted into equations 5 and 6.

Since this method uses helical coordinates (Claerbout, 1998), extension beyond three dimensions is trivial. In the future, this method can be used on real seismic data in two, three, or even five dimensions, so that prestack 3D data can be interpolated in cases where surface topography, structures, or other obstacles create sparse data.

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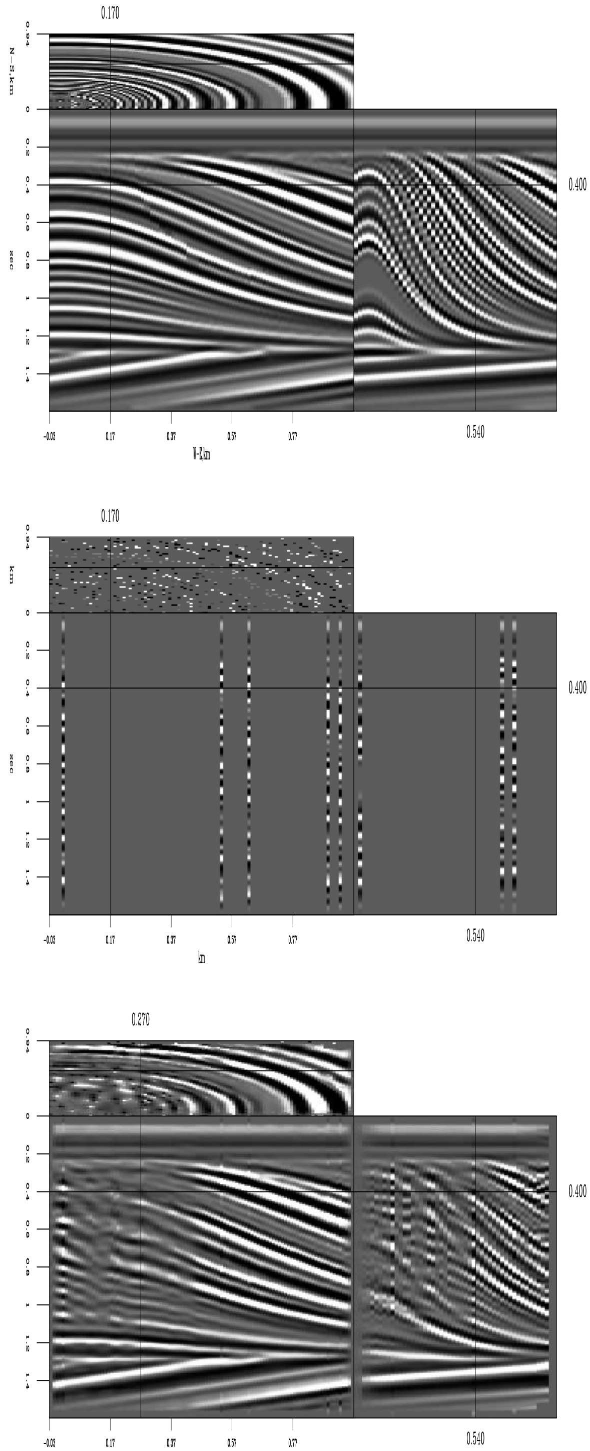

Figure 6: Fully sampled, sub-sampled, and interpolated versions of the qdome model.

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