Multiple attenuation with multidimensional prediction-error filters

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SUMMARY

Multiple attenuation in complex geology remains a very intensive research area. The proposed technique aims to use the spatial predictability of both the signal (primaries) and noise (multiples), in order to perform the multiple attenuation in the time domain. The spatial predictability is estimated with multidimensional prediction-error filters. These filters are time-variant in order to handle the nonstationarity of the seismic data. Attenuation of surface-related multiples is illustrated with field data from the Gulf of Mexico with 2-D and 3-D filters. The 3-D filters allow the best attenuation result. In particular, 3-D filters seem to cope better with inaccuracies present in the multiple model for short offset and diffracted multiples.

INTRODUCTION

The last decade has seen an exponential growth in the use of 3-D seismic imaging. Contemporaneous with this development, imaging techniques have become more complex in the effort to account for multi-pathing in complex media and to produce true amplitude migrated pictures of the subsurface. Since multiples are not accounted for in the physical model that leads to these migration methods, they can severely affect the final migration result producing erroneous interfaces or amplitude artifacts; consequently, the multiples have to be removed from the data prior to any imaging attempt.

As pointed out by Weglein (1999), the multiple attenuation techniques may be divided into two families: (1) filtering methods which exploit the periodicity and the separability (move-out discrepancies) of the multiples and (2) the wavefield methods, where the multiples are first predicted, for example by autoconvolution of the recorded data, and then subtracted (Verschhur et al., 1992). Traditionally, filtering techniques are the method of choice for multiple processing because of their robustness and cost. However, because these techniques are mainly 1-D methods, they do not extend their multiple attenuation properties very well to higher dimensions, i.e, 2-D or 3-D. Therefore, filtering techniques have some limitations when tackling multiples in complex media.

In this paper I present results of a multiple attenuation technique based on the spatial predictability of both primaries and multiples. The attenuation is based on the assumption that primaries and multiples have different patterns and amplitudes. The pattern is estimated with time-space domain (t-x) multidimensional prediction-error filters (PEFs). In the first section following this introduction, I present the multiple attenuation technique. In the second section I illustrate the proposed method with a Gulf of Mexico field data example. I will show that 3-D PEFs give the best noise attenuation result. More specifically, 3-D PEFs are able to attenuate diffracted multiples better and are also less sensitive to modeling inaccuracies at short offset.

THEORY OF MULTIPLE ATTENUATION AND FILTER ESTIMATION

Recently Spitz (1999) and Brown and Clapp (2000) presented a noise attenuation technique based on patterns. The main idea is simple: the noise and signal have different multidimensional spectra that PEFs can approximate (Claerbout, 1992) in order to perform the separation. In this approach, the multiple attenuation is similar to Wiener filtering (Abma, 1995). One important approximation is that the noise and signal are uncorrelated so that the cross-spectrum between these two components is not needed. In this section, I show how the separation is performed and how the PEFs are estimated.

Multiple attenuation

First, I consider that any seismic data is the sum of signal and noise as follows:

$$\mathbf{d} = \mathbf{s}+\mathbf{n}, \tag{1}$$

where $\mathbf{d}$ are the seismic data, $\mathbf{s}$ the signal we want to preserve and $\mathbf{n}$ the noise we wish to attenuate. In the multiple elimination problem, the noise is the multiples and the signal the primaries.

Now, assuming that we know the multidimensional PEFs $\mathbf{N}$ and $\mathbf{S}$ for the noise and signal components, respectively, we have

$$\begin{aligned} \mathbf{Nn} &\approx \mathbf{0} \\ \mathbf{Ss} &\approx \mathbf{0} \end{aligned} \tag{2}$$

by definition of the PEFs. Equations (1) and (2) can be combined to solve a constrained problem to separate signal from spatially uncorrelated noise as follows:

$$\begin{array}{rcccl} \mathbf{0} & \approx & \mathbf{r_n} & = & \mathbf{Nn} \\ \mathbf{0} & \approx & \mathbf{r_s} & = & \mathbf{Ss} \\ \text{subject to} & \leftrightarrow & \mathbf{d} & = & \mathbf{s}+\mathbf{n} \end{array} \tag{3}$$

We can easily eliminate $\mathbf{n}$ in the last equation of the fitting goal (3) by convolving with $\mathbf{N}$. For some field data, it might be usefull to add a masking operator on the data and signal residual $\mathbf{r_d}$ and $\mathbf{r_s}$ to perform the noise attenuation. It happens for example when the noise appears after a certain time or offset. I call $\mathbf{M}$ this masking operator and I weight the fitting goals in equation (3) as follows:

$$\begin{array}{rcccl} \mathbf{0} & \approx & \mathbf{r_d} & = & \mathbf{M}(\mathbf{Ns}-\mathbf{Nd}) \\ \mathbf{0} & \approx & \mathbf{r_s} & = & \mathbf{MSs} \end{array} \tag{4}$$

The least-squares inverse for $\mathbf{s}$ becomes

$$\hat{\mathbf{s}} = (\mathbf{N'MN}+\epsilon^2\mathbf{S'MS})^{-1}\mathbf{N'MNd}, \tag{5}$$

where ($'$) stands for the adjoint and ϵ is a constant to be chosen a-priori. Note that since $\mathbf{M}$ is a diagonal operator of zeros and ones, we have $\mathbf{M'M} = \mathbf{M^2} = \mathbf{M}$. It is interesting to note that $\mathbf{N'MN}$ is the inverse spectrum of the noise and $\mathbf{S'MS}$ is the inverse spectrum of the signal where we perform the attenuation. Soubaras (1994) use a very similar approach for random noise and more recently for coherent noise attenuation (Soubaras, 2001) with F-X PEFs. Because the size of the data space can be quite large, we estimate $\mathbf{s}$ iteratively with a conjugate-gradient method. In the next section, I describe the PEFs estimation method I use to compute $\mathbf{N}$ and $\mathbf{S}$ needed in equation (5).

Filter estimation

The PEFs I estimate are time domain non-stationary filters to cope with the variability of seismic data with time and offset. The basic relationships for the filter estimation are

$$\begin{array}{rcccl} \mathbf{0} & \approx & \mathbf{r_y} & = & \mathbf{YKa}+\mathbf{y} \\ \mathbf{0} & \approx & \mathbf{r_a} & = & \mathbf{Ra}, \end{array} \tag{6}$$

where $\mathbf{Y}$ is a matrix for non-stationary convolution with the data, $\mathbf{K}$ is a masking operator, $\mathbf{a}$ a vector of the unknown PEFs coefficients where the first coefficient is held at one, $\mathbf{y}$ the data vector from which we want to estimate the PEFs and $\mathbf{R}$ a regularization operator. The first equation states that the PEFs applied to the data give a white residual. The second equation states that the difference between coefficients for neighboring PEFs should be small.

Often with seismic data, the amplitude varies accross offset and time. These amplitude variations can be troublesome when we want to use least-squares inversion because they tend to bias the final result (Claerbout, 1992). Therefore, it is important to make sure that the amplitude variation does not affect our processing. One solution is to apply a weight to the data like Amplitude Gain Control (AGC) or a geometrical spreading correction. However, a better way is to incorporate the weight inside our inversion by weighting the residual. Introducing a weighting function $\mathbf{W}$ in the PEFs estimation, we have the following fitting goals:

$$\begin{array}{rcccl} \mathbf{0} & \approx & \mathbf{r_y} & = & \mathbf{W}(\mathbf{YKa}+\mathbf{y}) \\ \mathbf{0} & \approx & \mathbf{r_a} & = & \mathbf{Ra}. \end{array} \quad (7)$$

This weighting improves the signal/noise separation results. The choice of the weighting function will be discuss later for the multiple attenuation example. This weight can also be different for the noise and signal PEFs. The least-squares estimate of $\mathbf{a}$ becomes

$$\hat{\mathbf{a}} = -(\mathbf{K}'\mathbf{Y}'\mathbf{W}^2\mathbf{YK}+\epsilon^2\mathbf{R}'\mathbf{R})^{-1}\mathbf{K}'\mathbf{Y}'\mathbf{W}^2\mathbf{y}. \quad (8)$$

Because we have many filter coefficients to estimate, $\mathbf{a}$ is estimated iteratively with a conjugate-gradient method.

Now, prior to the signal estimation, $\mathbf{S}$ and $\mathbf{N}$ need to be computed from a signal and noise model, respectively. The multiple model is derived by autoconvolving the recorded data (Verschhur et al., 1992). I then obtain a prestack model of the multiples that I use to estimate the bank of nonstationnary PEFs $\mathbf{N}$. The signal PEFs are more difficult to estimate since the signal is usually unknown. To estimate the signal PEFs $\mathbf{S}$, I apply the noise PEFs $\mathbf{N}$ to the data vector $\mathbf{d}$ and estimate the signal PEFs from the convolution result. This will give me an approximation of $\mathbf{S}$ that I later use for the noise attenuation (Spitz, 2001, personnal communication). Thanks to the Helix (Mersereau and Dudgeon, 1974; Claerbout, 1998), the PEFs can have any dimension. In this paper, I use 2-D and 3-D filters and demonstrate that 3-D filters lead to the best noise attenuation result. In the next section, I show a prestack multiple attenuation example with field data from the Gulf of Mexico.

A 2-D FIELD DATA EXAMPLE

This section demonstrates that our multiple attenuation method yields an efficient attenuation of surface-related multiples for complex subsurface. The modeling of the multiples is made in 2-D and does not incorporate off-plane events. This dataset has been extensively used in a special edition of The Leading Edge (January 1999) on multiple attenuation.

First, I estimate a weighting function $\mathbf{W}$ for the filter estimation and a masking operator $\mathbf{M}$ for the noise attenuation. For the noise and signal PEFs weighting operators, I first applied an AGC of 0.8 second (200 samples) on the shot gather for the data and the multiple model. From these gathers, I estimated a weighting function by dividing the gathers with AGC by the gathers without AGC, making sure that no division by zero would occur. Note that the weight for the signal PEFs is estimated from the data.

Having computed the masking and weighting operators, we can proceed to the multiple attenuation. The first step is to estimate the noise and signal PEFs, i.e., equation (8). Then, I estimate the signal, i.e. equation (5). I show the result of the multiple attenuation for one shot location in Figure 1. In Figure 1b, we notice that the multiple model has weak amplitudes at short offset. This comes from the acquisition geometry at short offset.

In Figures 1, I compare the estimated signal when 2-D and 3-D filters are utilized for the signal/noise separation. It is quite clear that the 3-D filters yield a better noise attenuation result. It is interesting to see that the 3-D filters can handle modelling uncertainties and diffracted multiples very well. I show in Figure 2 time slices of the time/offset/shot cube in which the multiple attenuation is performed. As expected, 3-D filters (Figure 2d) attenuate the noise much better than 2-D filters (Figure 2c). It is quite remarkable that 3-D filters perform so well in areas where the multiple model is known to be inaccurate. In particular, diffracted multiples and off-plane/3-D multiples are better attenuated (between offsets 2000 and 3000 m in Figure 2).

DISCUSSION-CONCLUSION

I have presented a multiple-attenuation technique that aims to model and separate the noise and signal with time-variant, multidimensional prediction-error filters. These filters can be 2-D or 3-D thanks to the helical boundary conditions. The estimation of these filters incorporate weighting functions to cope with amplitude variations in the data and in the noise model. In addition, a masking operator is introduced in the signal/noise separation method in order to preserve areas where no multiples are present. Tests with a field data example from the Gulf of Mexico prove that the multiple attenuation works much better when 3-D filters are utilized, as opposed to 2-D filters. These 3-D filters allow a better signal/separation in areas where the multiple model is known to be inaccurate, e.g., short offset traces, diffracted multiples and off-plane/3-D events.

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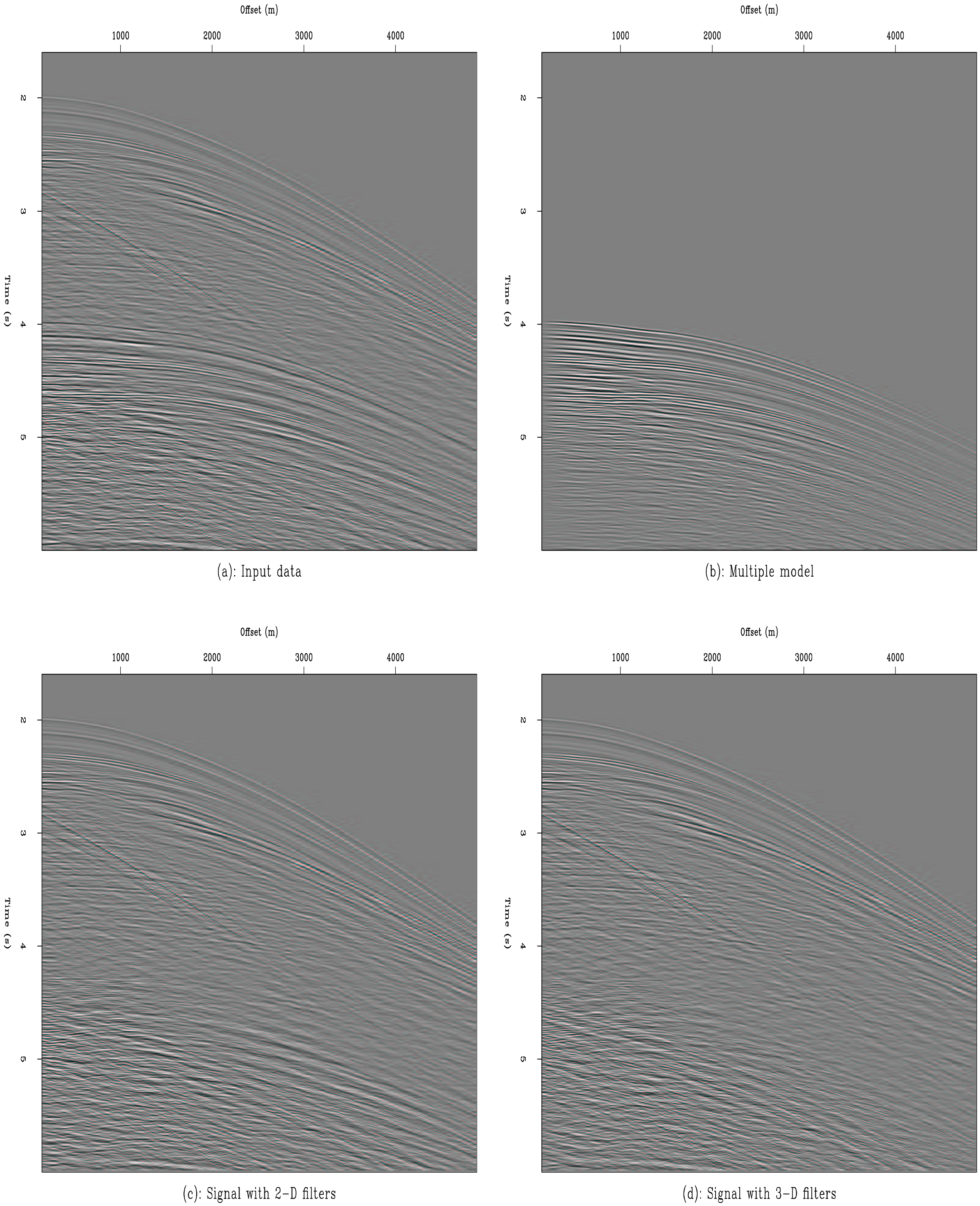

Figure 1: Multiple attenuation result at location 4500 m. (a) Input data. (b) Multiple model. (c) Estimated signal with 2-D filters. (d) Estimated signal with 3-D filters.

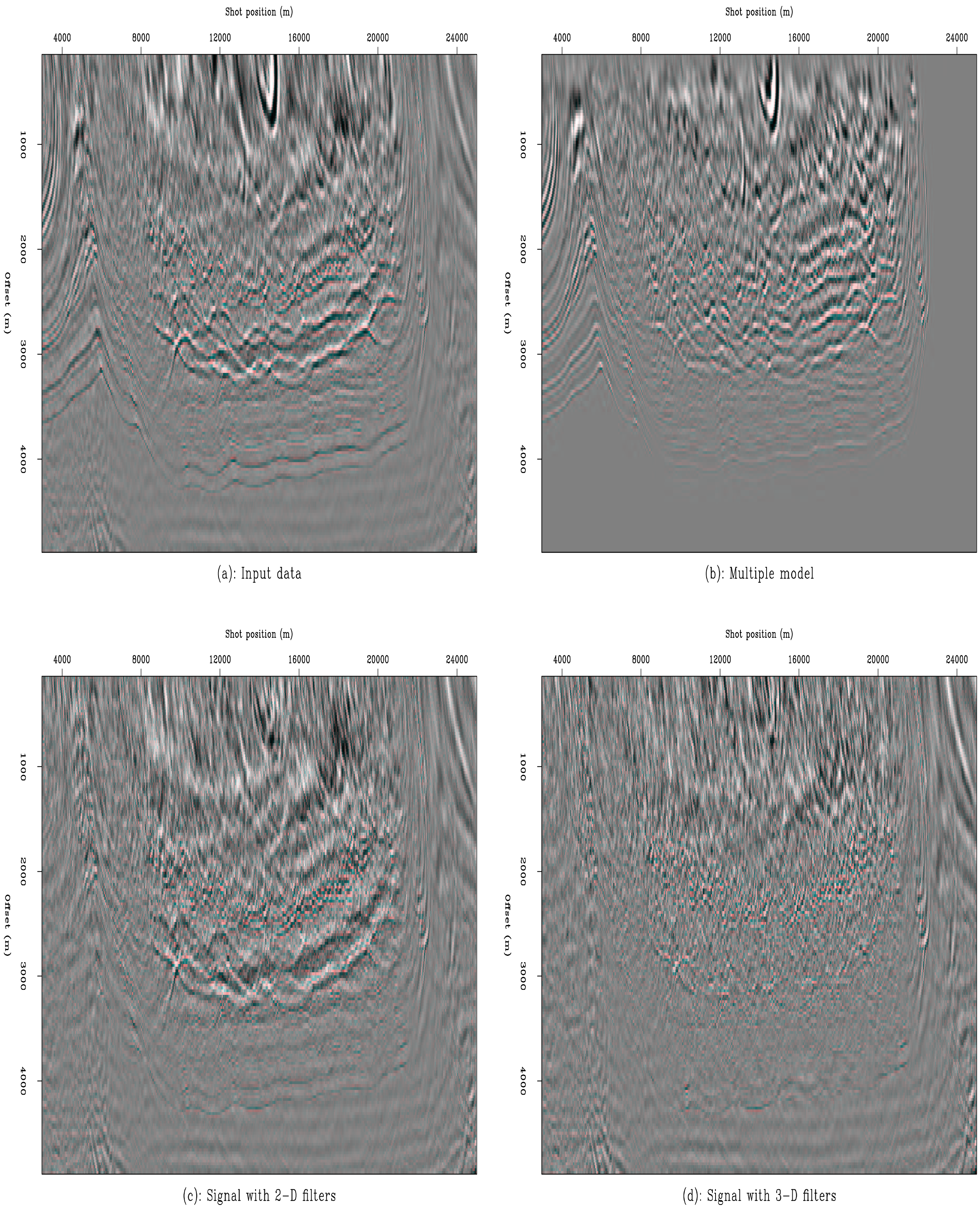

Figure 2: Time slices at 4.5 seconds. (a) Input data. (b) Multiple model. (c) Estimated signal with 2-D filters. (d) Estimated signal with 3-D filters.