Amplitude and kinematic corrections of migrated images for non-unitary imaging operators

Antoine Guitton, Stanford University

SUMMARY

Obtaining true-amplitude migrated images remains a very challenging problem. One possible solution to address it is inversion. However, inversion is a very expensive process that can be rather difficult and expensive to apply, especially with 3-D data. This paper introduces a method that is applied after the imaging process and costs three migrations in order to estimate two images. The basic idea is then to exploit the relationship between these two images with non-stationary matching filters in order to approximate the Hessian, thus avoiding the need for iterative inversion. Tests on the Marmousi dataset show that this filtering approach gives results similar to iterative least-squares inversion at a lower cost. In addition the filtering method produces an image with fewer artifacts.

INTRODUCTION

It is well known that most of our mathematical operators for seismic processing are not unitary. This means that for any linear transformation $\mathbf{L}$, $\mathbf{L}'\mathbf{L} \neq \mathbf{I}$ where ($'$) stands for the adjoint and $\mathbf{I}$ is the identity matrix. Having non-unitary operators often results from approximations we make when building those operators. For example, we might not take the irregular and finite acquisition geometry of seismic surveys into account.

For migration, different approaches exist to correct for these defects of our operators. Bleistein (1987), based on earlier works from Beylkin (1985), derived an inverse operator for Kirkhhoff migration. A similar path is followed by Thierry et al. (1999) with the addition of non-linear inversion with approximated Hessian. Alternatively, least-squares migration with regularization has proved efficient with incomplete surface data, e.g., Nemeth et al. (1999) and illumination problems due to complex structures, e.g., Prucha et al. (1999); Kuehl and Sacchi (2001). Hu et al. (2001) introduce a deconvolution operator that approximates the inverse Hessian in the least-squares estimate of the migrated image. However, this method assumes a v(z) medium which means that the deconvolution operators are horizontally invariant. Recently, Rickett (2001) proposed estimating weighting functions from reference images to compensate illumination effects for finite-frequency depth migration. This method corrects for amplitude effects only and incorporates some smoothing that can be rather difficult to estimate.

In this paper, I propose a new strategy for approximating the inverse of the Hessian. This approach aims to estimate a bank of non-stationary matching filters (Rickett et al., 2001) between two migrated images that theoretically embed the effects of the Hessian. This approach is implemented after migration and is relatively cheap to apply. It can be applied on the zero-offset migrated images or in the angle domain. I illustrate this method with the Marmousi dataset. I demonstrate that this approach can effectively recover the correct "least-squares" amplitudes of the migrated images with less artifacts than with the least-squares result without regularization at a much reduced cost.

THEORY

In this section, I show how the least-squares estimate of a migrated image can be approximated using non-stationary stationary matching filters. In terms of cost, this approach is comparable to two iterations of conjugate gradient (CG), the first iteration being the migration. The cost of estimating the non-stationary filters is negligible compared to the total cost of migration.

First, given seismic data $\mathbf{d}$ and a migration operator $\mathbf{L}$, we seek a model $\mathbf{m}$ such that

$$\mathbf{Lm} = \mathbf{d}. \tag{1}$$

This goal can be rewritten in the following form

$$\mathbf{0} \approx \mathbf{r_d} = \mathbf{Lm} - \mathbf{d} \tag{2}$$

and is called the fitting goal. For migration, a model styling goal (regularization) is necessary to compensate for irregular geometry artifacts (Prucha et al., 1999). I omit this term in my derivations and focus on the data fitting goal only. The least-squares estimate of the model is given by

$$\hat{\mathbf{m}} = (\mathbf{L}'\mathbf{L})^{-1}\mathbf{L}'\mathbf{d} \tag{3}$$

where $\mathbf{L}'\mathbf{L}$ is the Hessian of the transformation. My goal in this paper is to approximate the effects of the Hessian $\mathbf{L}'\mathbf{L}$ using non-stationary matching filters.

Approximating the Hessian

In equation (3), I define $\mathbf{L}'\mathbf{d}$ as the migrated image $\mathbf{m_1}$ after a first migration such that

$$\hat{\mathbf{m}} = (\mathbf{L}'\mathbf{L})^{-1}\mathbf{m_1}. \tag{4}$$

In equation (4), $\hat{\mathbf{m}}$ and $\mathbf{L}'\mathbf{L}$ are unknown. Since I am looking for an approximate of the Hessian, I need to find two known images that are related by the same expression as in equation (4). This can be easily achieved by remodeling the data from $\mathbf{m_1}$ with $\mathbf{L}$

$$\mathbf{d_1} = \mathbf{Lm_1} \tag{5}$$

and remigrating them with $\mathbf{L}'$ as follows:

$$\mathbf{m_2} = \mathbf{L}'\mathbf{d_1} = \mathbf{L}'\mathbf{Lm_1}. \tag{6}$$

Notice a similarity between equations (4) and (6) except that in equation (6), only $\mathbf{L}'\mathbf{L}$ is unknown. Notice that $\mathbf{m_2}$ has a mathematical significance: it is a vector of the Krylov subspace for the model $\hat{\mathbf{m}}$. Now, I assume that we can write the inverse Hessian as a linear operator $\mathbf{B}$ such that

$$\hat{\mathbf{m}} = \mathbf{Bm_1} \tag{7}$$

and

$$\mathbf{m_1} = \mathbf{Bm_2}. \tag{8}$$

Equation (8) can be approximated as a fitting goal for a matching filter estimation problem where $\mathbf{B}$ is the convolution matrix with a bank of non-stationary filters (Rickett et al., 2001). This choice is rather arbitrary but reflects the general idea that the Hessian is a transform operator between two similar images. My hope is not to "perfectly" represent the Hessian, but to improve the migrated image at a lower cost than least-squares migration. In addition in equations (7) and (8), the deconvolution process becomes a convolution, which makes it much more stable and easy to apply. Hence, I can rewrite equation (8) such that the matrix $\mathbf{B}$ becomes a vector and $\mathbf{m_2}$ becomes a convolution matrix:

$$\mathbf{m_1} = \mathbf{M_2b}. \tag{9}$$

The goal now is to minimize the residual

$$\mathbf{0} \approx \mathbf{r_{m_1}} = \mathbf{m_1} - \mathbf{M_2b} \tag{10}$$

in a least-squares sense. Because we have many unknown filter coefficients in $\mathbf{b}$, I introduce a regularization term that penalizes differences between filters as follows:

$$\begin{array}{ccccc} \mathbf{0} & \approx & \mathbf{r_{m_1}} & = & \mathbf{m_1} - \mathbf{M_2 b} \\ \mathbf{0} & \approx & \mathbf{r_b} & = & \mathbf{Rb} \end{array} \qquad (11)$$

where $\mathbf{R}$ is a regularization operator (a Laplacian). The least-squares inverse of $\mathbf{b}$ is thus given by

$$\hat{\mathbf{b}} = (\mathbf{M_2'M_2} + \epsilon^2\mathbf{R'R})^{-1}\mathbf{M_2'm_1}. \qquad (12)$$

Once $\hat{\mathbf{b}}$ is estimated, the final image is obtained by computing

$$\hat{\mathbf{m}} = \mathbf{m_1} * \hat{\mathbf{b}} \qquad (13)$$

where $(*)$ is the convolution operator.

Therefore, my proposed algorithm is to first compute a migrated image $\mathbf{m_1}$, then to compute a migrated image $\mathbf{m_2}$ (equation (6)), and finally to estimate a bank of non-stationary matching filters $\mathbf{b}$, e.g., equation (11). The final improved image is obtained by applying the matching filters to the first image $\mathbf{m_1}$, e.g., equation (7). In the next section, I illustrate this idea with the Marmousi dataset. I show that an image similar to the least-squares migration image can be effectively obtained.

MIGRATION RESULTS

I illustrate the proposed algorithm with the Marmousi dataset. I use a prestack split-step migration method with one reference velocity. I demonstrate with zero-offset images that the approximation of the Hessian with adaptive filters gives a result comparable to least-squares migration with fewer artifacts.

Figure 1 displays few estimated filters for the Marmousi result. The filters are ten by ten with 40 patches in depth and 80 along the midpoint axes. I show only a fifth of these filters in both axis. It is interesting to notice that these filters have their highest value at zero-lag, meaning that we have a strong amplitude correction with few kinematic changes. The zero-lag values are also larger at the top of the model. Looking more closely at these filters, we see that the coefficients follow the structure of the Marmousi model (upper right corner).

Having estimated the filters $\mathbf{b}$ in equation (11), I apply them to $\mathbf{m_1}$ to obtain an improved image. To validate this approach I show in Figure 3a the result of five conjugate gradient (CG) iterations with the Marmousi data. This results show higher amplitudes at the top but with inversion artifacts. This problem should be addressed with a proper regularization scheme (Prucha et al., 1999). In Figure 3b, I show the corrected image with the approximated Hessian $\mathbf{B}$. The amplitude behavior is very similar to Figure 3a, without the inversion artifacts. Additionally, the cost is much lower.

I show in Figure 2 the ratio of the envelopes of Figure 3b and the migrated image $\mathbf{m_1}$. This Figure illustrates that the effects of the non-stationary filters, i.e, the hessian, are stronger on the top of the model.

DISCUSSION

In this paper, I presented a method that intends to correct migrated images by approximating the Hessian of the imaging operator with non-stationary matching filters. These filters are estimated from two migration results. One migrated image, $\mathbf{m_1}$, corresponds to the first migration result. The second image, $\mathbf{m_2}$, is computed by re-modeling the data from $\mathbf{m_1}$ and then by re-migrating it. It turns out that the relationship between $\mathbf{m_1}$ and $\mathbf{m_2}$ is similar to the relationship that exists between the least-squares inverse $\hat{\mathbf{m}}$ and $\mathbf{m_1}$. In the proposed approach, this relationship is simply "captured" by matching filters.

I demonstrate with the Marmousi dataset that this approach gives a better image than does least-squares without regularization and at a lower cost. In addition this approach can be used in the offset or the angle domain. As opposed to Hu et al. (2001), the correction in the image is completely data driven, does not depend on the velocity, and can be applied with any migration operator. It also works in the poststack or prestack domain without any extra effort. Providing the data and the ability to run at least two migrations to estimate $\mathbf{m_2}$, this method would be easy to apply with 3-D migrated images.

Now, compared to Rickett (2001), this proposed approach does not need reference images. In addition, the multi-dimensional filters offer more degrees of freedom for the correction than does a simple zero-lag weight: in that we might also correct for kinematic changes and move energy locally in the image. In the limit case where we choose one filter per point and only one coefficient (zero-lag) per filter, the matching filter approach would theoretically perform better than Rickett's method because the weights would be optimal in a least-squares sense without ad-hoc smoothing. In the future, it would be valuable to go beyond 2-D filters by extending them to 3-D and to test it with more field data. In addition, these filters could be used as preconditioning operators providing faster convergence for iterative inversion.

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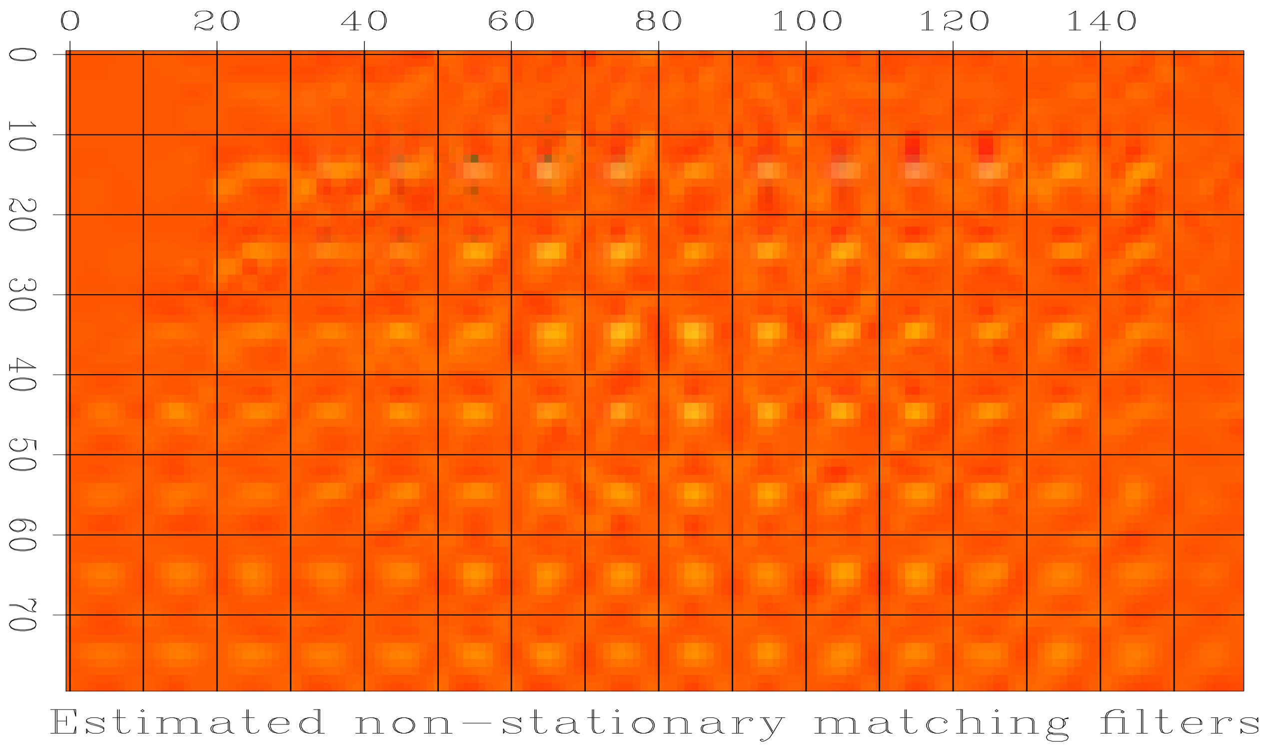


Figure 1: Each cell represents a non-stationary filter with its zero-lag coefficient in the middle. A fifth of the filters are actually shown in both directions. Each filter position corresponds roughly to a similar area in the model space. These filters seem to follow the structure of the Marmousi model. They are also stronger at the top of the model, as expected.

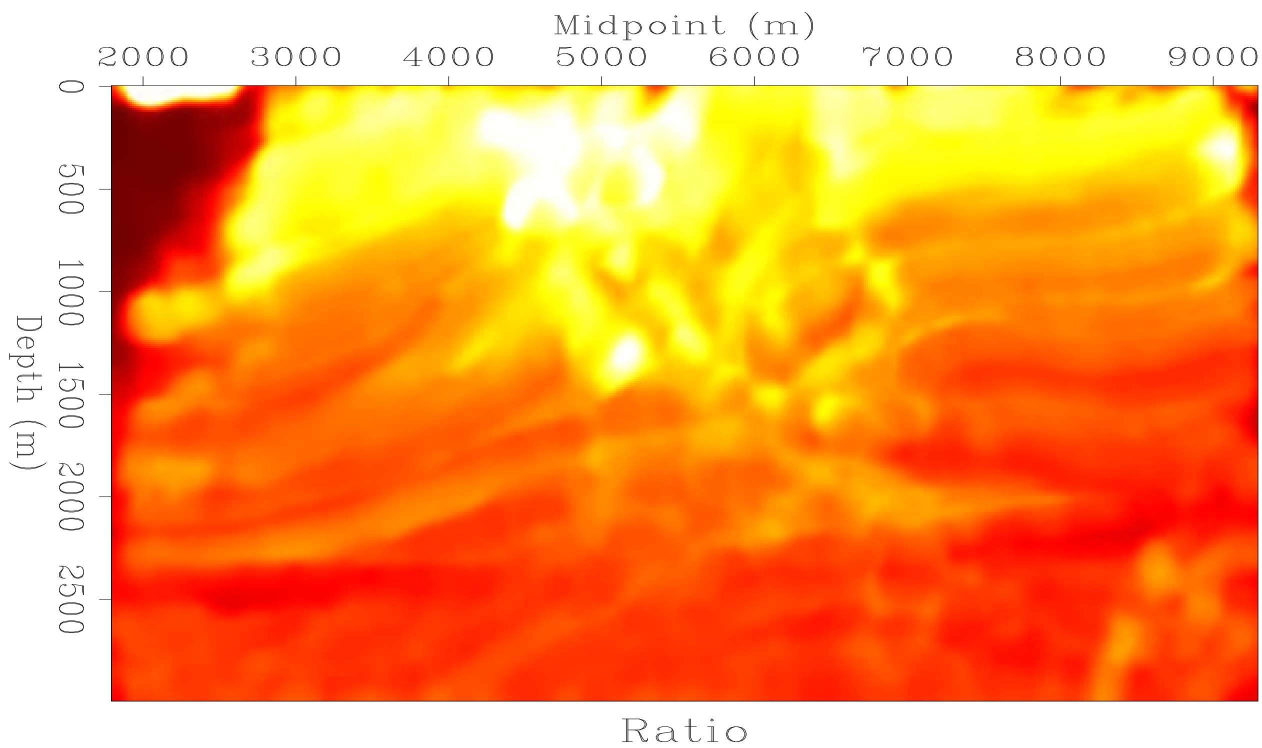
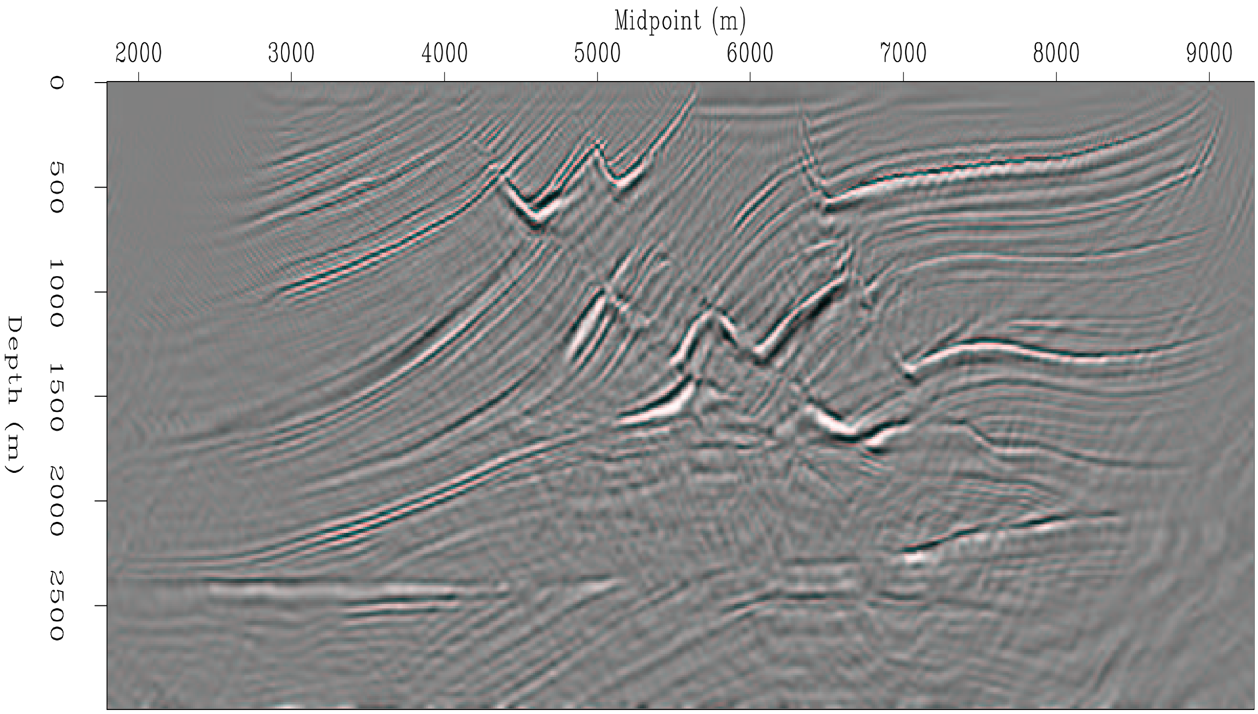
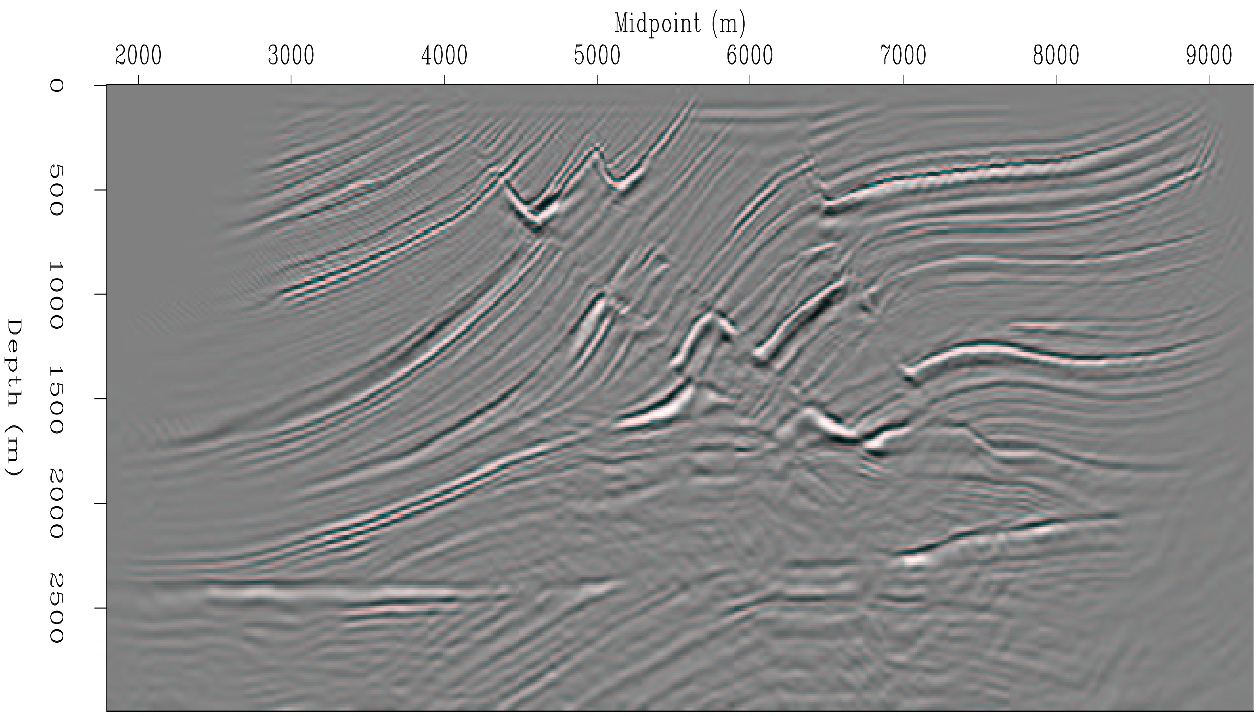


Figure 2: Ratio of the envelopes of $\mathbf{m_1}$ and 3b. Brighter colors correspond to higher values. The main effect of the filters is clearly visible at the top.



(a) Inversion result



(b) Filtering result

Figure 3: (a) Model estimated after five iterations of CG. The model is noisy because no regularization has been applied. (b) Model estimated after applying the adaptive filters to $\mathbf{m_1}$. The amplitude behavior is similar to (a) without the artifacts and with fewer iterations.