A pseudo-unitary implementation of the Radial Trace Transform

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SUMMARY

The Radial Trace Transform (RTT) is an attractive tool for wavefield separation because it lowers the apparent temporal frequency of radial events like ground roll, making it possible to remove them from the data by simple bandpass filtering in the RT domain. In Henley's (1999) implementation of the RTT, the RT domain is well-sampled, and thus suitable for post-filtering, but suffers from interpolation errors because it intepolates across offsets. We present an alternate implementation, which is pseudo-unitary in the limit of an infinitely densely sampled RT space, with the side effect that the RT domain has missing data. Using a simple 2-D filter, we estimate the missing data in the RT domain using Claerbout's (1998) optimization methodology without affecting the invertibility of the RTT. Our implementation suppresses radial noise while preserving signal, including static shifts.

INTRODUCTION

The Radial Trace Transform (RTT), is a simple coordinate transform of normal (t,x) domain seismic gathers; a horizontal deformation, accomplished by the following linear mapping

$$\begin{aligned} t &\rightarrow t \\ x &\rightarrow v = \frac{x}{t} \end{aligned} \tag{1}$$

The radial coordinate is termed "v" because the RTT sorts the data by apparent velocity. Idealized ground roll maps to zero temporal frequency in the RT domain. Figure 1 shows 30 radial traces (thick lines) overlain on a rectangular (t,x) mesh. Points along radial traces rarely fall exactly on (t,x) data locations, making the RTT an exercise in interpolation. In matrix notation, denote the RTT of data $\mathbf{d}$ as follows:

$$\mathbf{Rd} = \mathbf{r} \tag{2}$$

$\mathbf{R}$ is generally nonsquare, and even if square, is usually noninvertible. Often, however, such interpolation operators are nearly unitary: $\mathbf{R}^T\mathbf{R} \approx \mathbf{I}$. Define the interpolation error as follows

$$\mathbf{e} = \left[\mathbf{I} - \mathbf{R^T R}\right]\mathbf{d}. \tag{3}$$

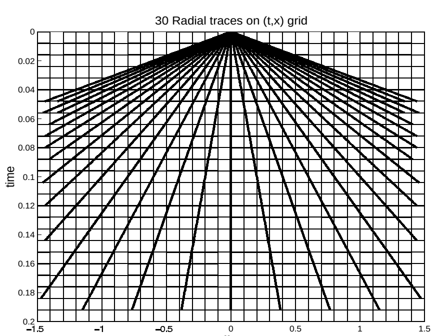

Figure 1: 30 radial traces overlaying a (t,x) grid.

METHODOLOGY

In this abstract, two implementations of the RTT are investigated. Each is illustrated schematically in Figure 2

1. **v-interpolation method:** For fixed (t,x) bins, linearly interpolate between radial traces.

2. **x-interpolation method:** For fixed (t,v) bins, linearly interpolate between offsets.

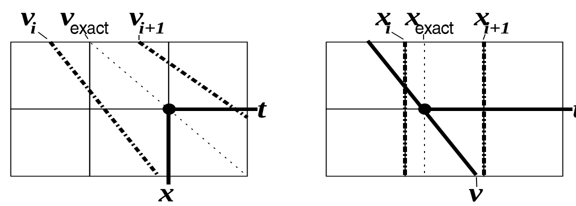

Figure 2: Left: "v-interpolation" implementation. Right: "x-interpolation implementation.

In the v-interpolation approach, $\mathbf{R}$ "pushes" energy — weighted by linear interpolation coefficients — from fixed (t,x) bins into the two radial trace (t,v) bins that bracket them. This implementation will cause interpolation error in regions where many (t,x) bins lie between adjacent radial traces (see Fig. 1). However, the RT domain is nonphysical in the sense that as many radial traces can be used as computer memory permits, so the interpolation error can essentially be driven to zero by sampling densely enough in RT space. Unfortunately, by sampling finely, in many regions of RT space there are pairs of radial traces which bracket no (t,x) bin, so "holes" are introduced into the RT space which inhibit later filtering operations.

In the x-interpolation approach, $\mathbf{R}$ "pulls" energy into fixed (t,v) bins from the two offset (t,x) bins that bracket them. The interpolation error of this implementation depends only on the trace spacing of the data. The net effect of applying the operator is to smooth laterally, making this implementation dangerous if the data has even small static shifts. At typical trace spacing, $\mathbf{R}$ is not pseudo-unitary, but since the RT space is guaranteed to have no "holes", this implementation is appropriate for post-filtering operations. Henley (1999) used the x-interpolation approach.

The two implementations of the RTT discussed above illustrate a fundamental, and oft-ignored duality in the analysis of interpolation operators. Intuition supports the x-interpolation method — any given unknown model point is the weighted average of the two known data points which bracket it. In applications like NMO, where averaging is done along the well-sampled time axis, this intuition is sensible, but it breaks down when the averaging is across offsets. The alternate approach (v-interpolation) is less intuitive, as the interpolation is done across radial traces, in the "virtual" space of the model. Since the model space is not constrained by the parameters of data acquisition, it can be sampled as densely as needed to minimize interpolation error. In fact, in the limit of infinitely dense sampling in model space, simple nearest neighbor binning drives interpolation error to zero.

The idea of this paper is to use the v-interpolation RTT to take advantage of its minimal interpolation error, and then handle the problem of "holes" in RT space by missing data estimation. Ideally, the RT space is composed of nearly-vertical ($h(v)$) and nearly-horizontal ($l(t)$) events, in which case a cascade of derivative operators extinguishes $\mathbf{Rd}$:

$$\frac{\partial}{\partial v}\frac{\partial}{\partial t}\,[h(v)+l(t)] = 0 \tag{4}$$

A finite difference stencil approximating $\partial^2/\partial v \partial t$ is

$$\mathbf{a} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}. \tag{5}$$

Filter **a** is well-suited for the helical least-squares missing data estimation methodology of Claerbout (1998). The missing data problem is driven by two fitting goals: 1) Honor the known data points exactly, and 2) impose any prior knowledge on the unknown model parameters via regularization. Define a known data mask **K**, simply a diagonal operator of the same dimension as **m**; 1 where data is known, 0 otherwise. The prior assumptions about the model are contained in filter **a**. Define **A** as the convolution matrix which applies the filter **a** and combine the two fitting goals into a single regularized optimization problem.

$$\mathbf{Km} = \mathbf{KRd} \tag{6}$$

$$\epsilon \mathbf{Am} \approx 0. \tag{7}$$

Equation (6) forces the model to match the data where the latter is known. Equation (7) minimizes the power of the convolution of **a** with **m**, i.e., optimality is achieved when the unknown model is filled with horizontal and vertical events. ϵ is a Lagrange multiplier.

RESULTS

Figure 3 shows a 2-D shot gather exhibiting dispersive ground roll with an apparent velocity range of 200-500 m/s. A relatively fine trace spacing of 17 m partially mitigates spatial aliasing. Some weak backscattered energy may be present. Additionally, static shifts of $\pm$ one sample were introduced to the data randomly, in order to emphasize the effect of the RTT on data whose lateral coherency may be degraded.

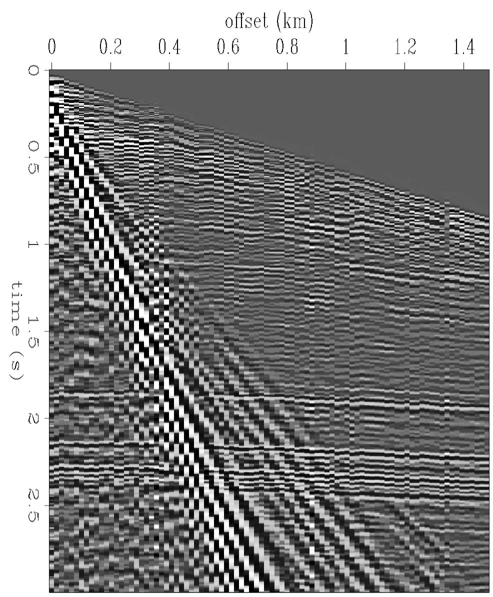

Figure 3: 2-D shot gather.

Figure 4 shows the RTT of the data in Figure 3 for both implementations. Each panel contains 600 radial traces. As expected, the raw v-interpolation panel has many “holes”, while the x-interpolation panel does not. The missing data estimation algorithm described above has plausibly filled the missing data, insofar as visual similarity to the x-interpolation panel is a valid measure.

Figure 5 compares the error arising from both implementations of the RTT: $(\mathbf{I} - \mathbf{R}^T\mathbf{R})\mathbf{d}$. In the RTT panels shown in Figure 4, each containing 600 radial traces, the error in both of the v-interpolation panels is negligible. This figure gives visual proof of the fact that the missing data infill process does not harm the original data, i.e., that the missing data points in RT space are in the nullspace of $\mathbf{R}^T$. As expected, the x-interpolation error is nonzero, particularly for high-wavenumber events like ground roll and likely backscatter. Additionally, and less obviously, the primary events between 1.75 and 2.5 seconds suffer considerable energy loss and a noticeable lateral smoothing as a result of the transform.

Figures 6 and 7 show the result of applying a 5 Hz highpass filter in the RT domain. Figure 6 is $\mathbf{R}^T\mathbf{BRd}$, the “estimated signal,” where **B** is the highpass filter. Notice that the missing data infill has markedly improved the amount of noise suppression obtained by the v-interpolation technique. Still, by visual inspection, we must conclude that the x-interpolation result is the best of the three for noise suppression.

Figure 7 is $(\mathbf{I} - \mathbf{R}^T\mathbf{BR})\mathbf{d}$, the “estimated noise” — simply the data subtracted from the estimated signal. First notice that x-interpolation result contains much more than the radial ground roll that the simple physical model accounts for. From Figure 5, we know that the presence of the primary energy between 1.75 and 2.5 seconds and backscattered noise in the x-interpolation panel is due to interpolation errors, and not to the bandpass filtering. The non-infilled v-interpolation result contains some primary energy around 1 second, while the infilled result does not. From the standpoint of signal preservation, the v-interpolation result with missing data infill is the best of the three. Philosophically, by using a pseudo-unitary RTT operator and thus ensuring that the only thing modifying the original data is the bandpass filter, the v-interpolation implementation honors the physics which drives the problem in the first place.

CONCLUSION

Our implementation of the RTT suppressed radial noise while preserving signal, including static shifts.

REFERENCES

Claerbout, J., 1998, Multidimensional recursive filters via a helix: Geophysics, **63**, no. 5, 1532–1541.

Henley, D., 1999, The radial trace transform: an effective domain for coherent noise attenuation and wavefield separation: 69th Annual Internat. Mtg., Soc. Expl. Geophys., Expanded Abstracts, pages 1204–1207.

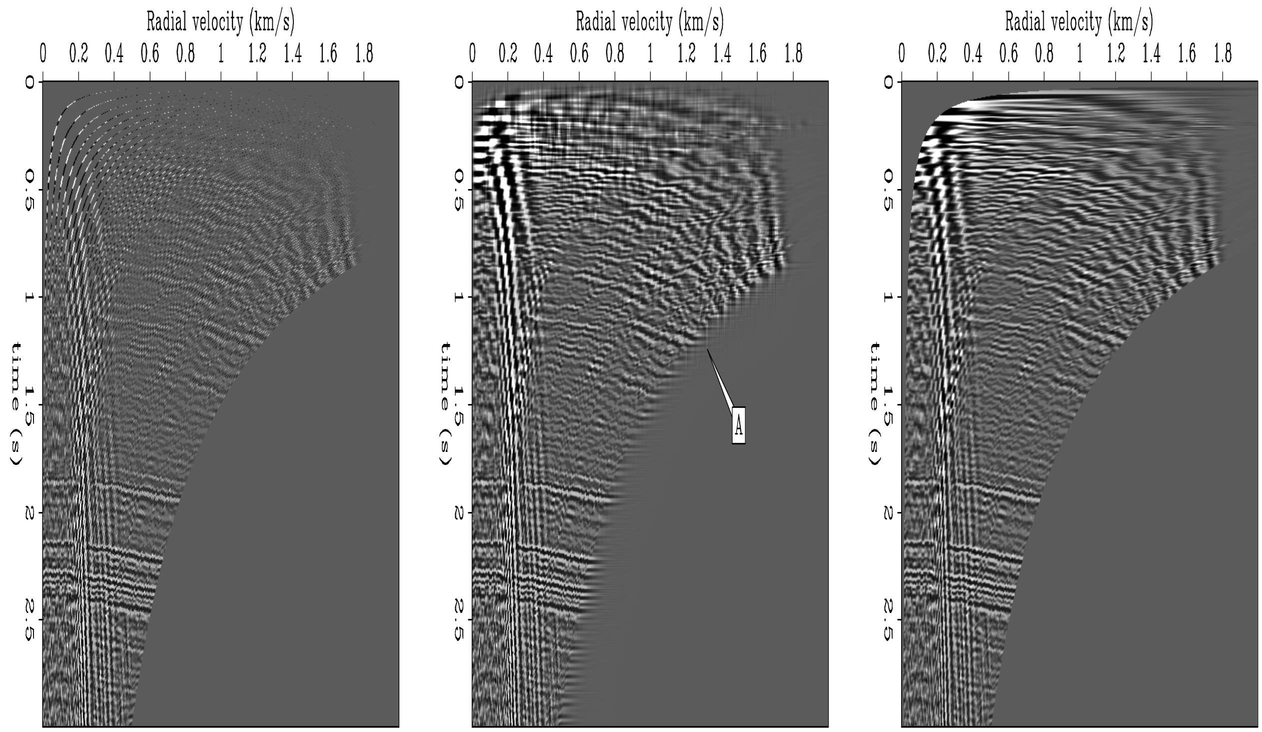

Figure 4: Left: v-interpolation without infill. Center: v-interpolation with infill. “A” points to an example of the “+”-shaped impulse response of the missing data filter of equation (7). Right: x-interpolation.

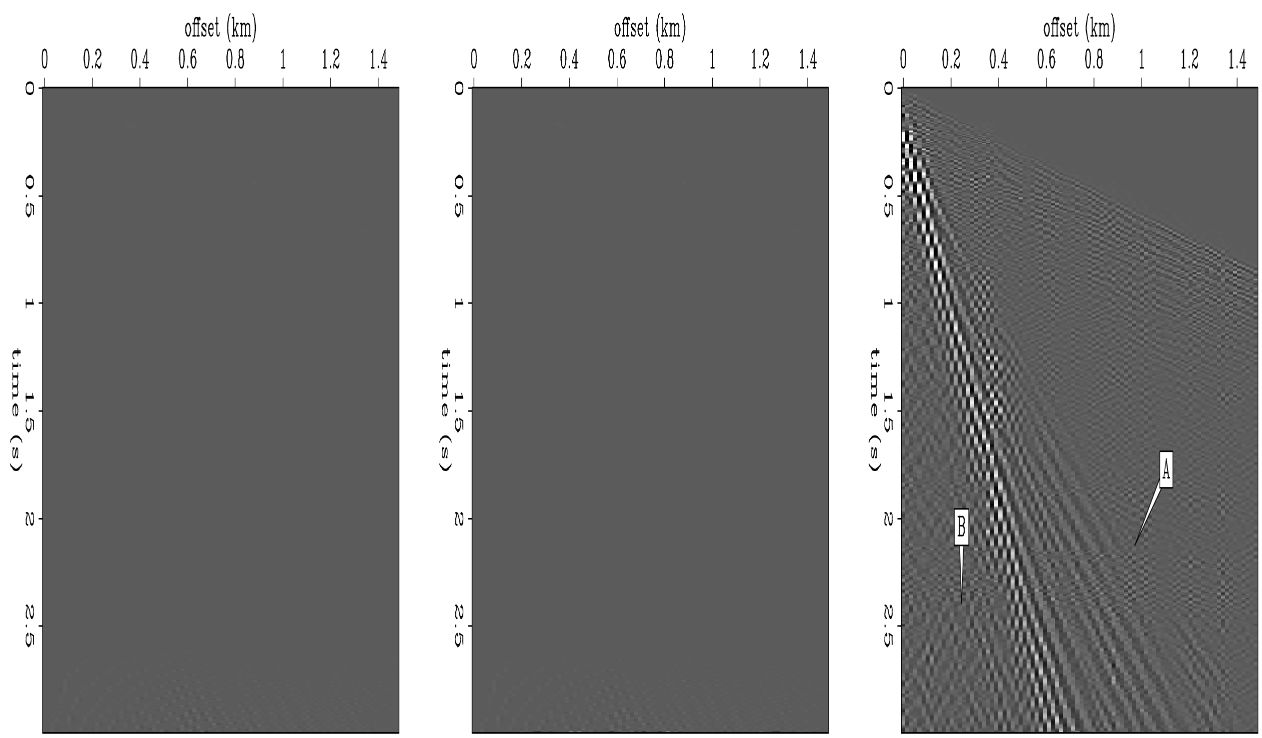

Figure 5: Left: v-interpolation error without infill. Center: v-interpolation error with infill. Right: x-interpolation error. “A” points to lost energy from the primary events around 2 seconds. “B” points to removed backscattered noise.

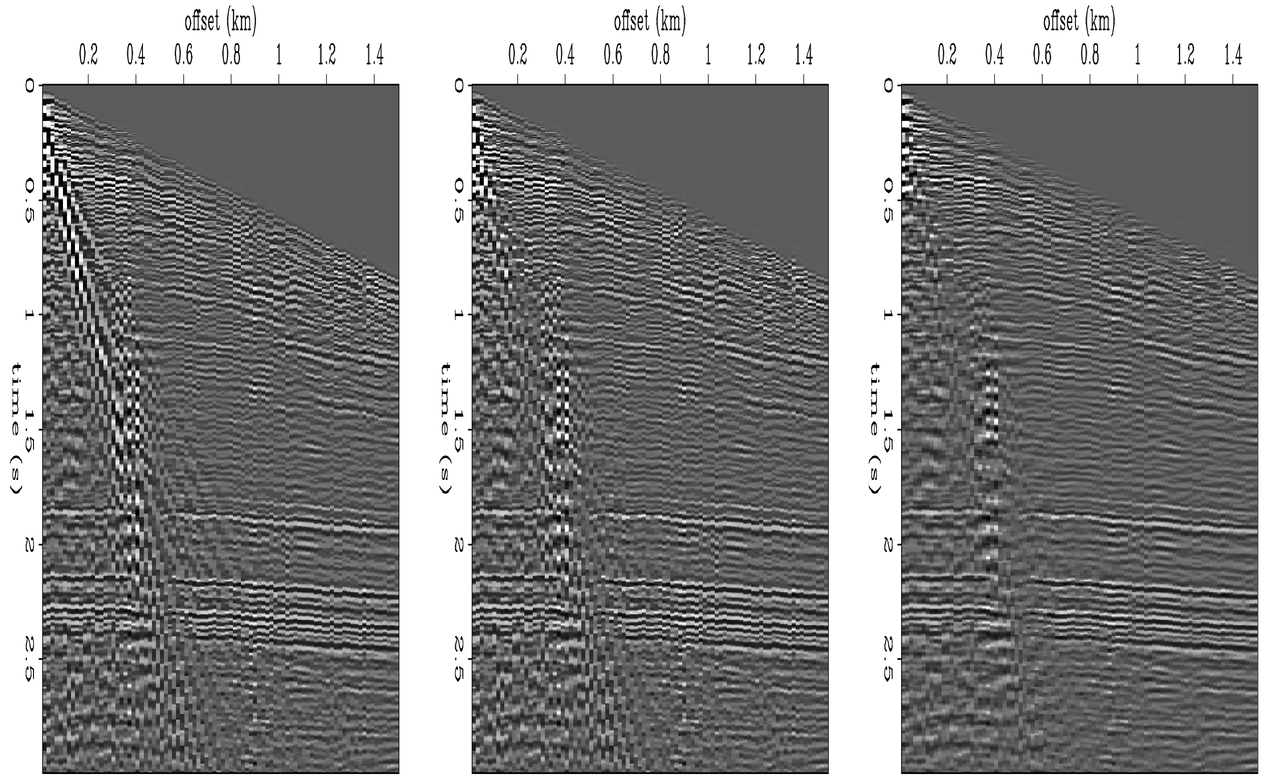

Figure 6: Estimated signal. Left: v-interpolation without infill. Center: v-interpolation with infill. Right: x-interpolation.

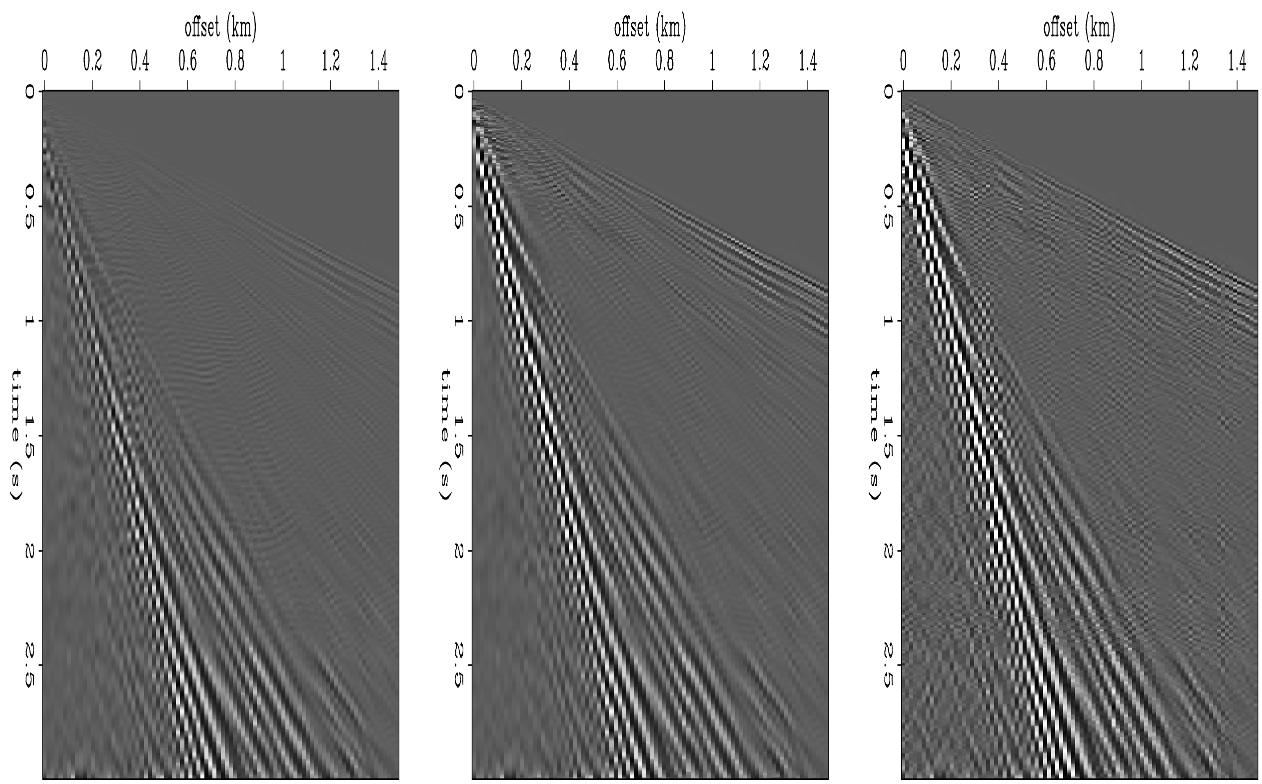

Figure 7: Estimated noise. Panels defined as in Figure 6.