1 Introduction

To correctly image primary or multiple reflections in a heterogeneous earth, migration reigns supreme, although its computational cost may be nontrivial over complex geology, in 3-D, or when a “true amplitude” image is required. My earlier (Brown, 2002) technique to jointly image primaries and pegleg multiples used normal moveout (NMO), because of NMO’s speed and its amplitude predictability, in spite of its inability to account for nonflat geology. In this paper I present a new time imaging operator, “HEMNO” (Heterogeneous Earth Multiple NMO Operator), which correctly accounts for moderate subsurface structure but retains the speed and amplitude advantages of NMO. The 2-D and 3-D pegleg multiple moveout equations for two dipping planes derived by Levin and Shah (1977) and Ross et al. (1999), respectively, reduce to HEMNO in the small dip angle limit (Brown, 2003b).

When used in conjunction with my earlier joint imaging scheme (Brown, 2002), I show that HEMNO produces improved results on the moderately complex Mississippi Canyon 2-D multiples test dataset, relative to a flat-earth NMO operator. Nevertheless, the real value of HEMNO lies in 3-D. Guitton (2003) demonstrates that the “Delft method” (Verschuur et al., 1992) (plus advanced multiple subtraction technology), almost perfectly separates surface-related multiples from 2-D data with complex 2-D structure. However, in real-world 3-D situations, acquisition and computational cost constraints diminish the method’s applicability. I demonstrate that in a simple, yet realistic, 3-D synthetic example that HEMNO can accurately image pegleg multiples from a seabed that dips in both directions. Coupled with the appropriate joint imaging or subtraction technique, HEMNO should prove a useful advance in 3-D multiple separation.

2 Traveltime Equation for Pegleg Multiples in a Non-Flat Earth

A first-order pegleg multiple consists of two unique arrivals: the event with a multiple bounce over the source, and the event with a bounce over the receiver. In a flat earth, both “legs” of the pegleg arrive simultaneously; when the reflector geometry varies with position, they generally do not. In some cases, pegleg multiples are actually observed to “split”, though humans rarely observe the phenomenon unambiguously in field data. Practical non-observation of split peglegs aside, geologic heterogeneity is a first-order effect on their kinematic and amplitude behavior. Mild variations in reflector depth over a cable length can introduce significant destructive interference between the legs of a pegleg multiple at far offsets – interference impossible to model with a 1-D theory.

Brown (2002) derived an extension to the conventional NMO equation which images pegleg multiples at the zero-offset travelt ime of the target reflector:

\[ t = \sqrt{(\tau + j\tau^*)^2 + x^2/V_{eff}^2}, \]  

where

\[ V_{eff}(\tau) = \frac{(j\tau^*V^*(\tau^*) + \tau V(\tau))}{(j\tau^* + \tau)} \]
$j\tau^*$ is the two-way traveltime of a $j$th-order pegleg in the multiple-generating layer.

Levin and Shah (1977) deduced moveout equations for 2-D peglegs from a two-dipping-layer earth model, and Ross et al. (1999) extended the work to 3-D. Both approaches assume constant velocity and locally planar reflectors; this may be unrealistic in practice. Below, I present “HEMNO” (Heterogeneous Earth Multiple NMO Operator), a simplified moveout equation based upon a more practically realizable conceptual model. It can be shown (Brown, 2003b) that Levin and Shah’s moveout equations reduce to the HEMNO equation in the small dip angle limit. Figure 1 graphically illustrates HEMNO in a constant-velocity earth. The solid line in panel d) is the final result. It has the equation of a hyperbola with zero-offset traveltime $\tau^*(y_0-x_p/2) + \tau(y_0+(x-x_p)/2)$ and the same offset, $x$:

$$
\begin{align*}
    t^2 &= \left[ \tau(y_0-(x-x_p)/2) + \tau^*(y_0-x_p/2) \right]^2 + \frac{x^2}{V_{eff}^2}, \quad \text{where} \\
    x_p &= \left( x_0 \frac{V^2}{(\tau+\tau^*)^2V_{eff}^4+x^2(V_{eff}^2-V^2)} \right)^{1/2}.
\end{align*}
$$

The expression for $x_p$, the width of the pegleg’s primary leg in a 1-D earth, is derived in (Brown, 2003a). The $V_{eff}$ in equation (3) is modified relative to equations (2) and (4); $\tau^*(y_0-x_p/2)$ is substituted for $\tau^*$ and $\tau(y_0-(x-x_p)/2)$ is substituted for $\tau$.

![Figure 1: Raypath for HEMNO. Panel a): True raypath in constant-velocity earth. Panel b): Assumed reflection points under assumed 1-D earth. $x_p$ is defined in equation (4). Panel c): The HEMNO approximation. Stretch legs of raypath vertically to match measured $\tau^*(y_0-x_p/2)$ and $\tau(y_0+(x-x_p)/2)$. Panel d): Final step. The solid line that connects the reassembled raypath is the HEMNO raypath.]

### 2.1 Practical implementation of the HEMNO equation

To implement equation (3) on a computer, we must obtain two quantities. The first, the zero-offset travelttime of the seabed, $\tau^*(y)$, may be obtained by hand- or auto-picking. Unfortunately, the second
quantity, the zero-offset travelt ime to an arbitrary subsea reflector at \( y = y_0 \pm (x - x_p)/2 \), cannot realistically be picked. Panel c) of Figure 1 motivates the problem; starting at \( \tau(y_0) \), the subsea reflector must be followed to \( y = y_0 + (x - x_p)/2 \). My approach to the problem is similar to Lomask and Claerbout’s (2002) algorithm for automatically flattening seismic data. It requires a smooth, unambiguous estimate of reflector dip, and may suffer from some pitfalls, discussed by Brown (2003b). I summarize this approach (in 2-D; extension to 3-D is more involved but conceptually similar) in pseudocode:

Obtain zero offset section, \( d(\tau, y) \), by stacking input data.
Use \( d(\tau, y) \) to compute smooth reflector dip, \( p(\tau, y) \), using technique of Fomel (2002).
Set \( k = 1, y = y_0 \).
do while \( (y + \Delta y \leq y_0 + (x - x_p)/2) \)
\hspace{1em} Set \( y = y_0 + (k - 1)\Delta y \)
\hspace{1em} Set \( \tau(y + \Delta y) = \tau(y + \Delta y) + p(\tau(y), y) \)
\hspace{1em} Set \( k = k + 1 \)
end do

3 Examples

Figure 2: HEMNO versus 1-D NMO comparison in least-squares joint imaging for two midpoint ranges in the Mississippi Canyon dataset. Row 1: Raw data stack. Row 2: Stacked estimated primaries using HEMNO. Row 3: Stacked estimated primaries using 1-D NMO. Row 4: HEMNO difference panel. Row 5: 1-D NMO difference panel. Close inspection shows improved multiple separation with HEMNO, especially where seabed and/or target reflectors dip.

4 Acknowledgement

WesternGeco acquired and released the Mississippi Canyon dataset.
Figure 3: HEMNO demonstration on synthetic 3-D dataset. 8 swaths of 43 shots were acquired over a seabed reflector with 4 degrees of inline dip and 2 degrees of crossline dip. Acquisition was 3 streamers with 300 m crossline separation. Left panel shows the zero offset section after conventional NMO for primaries. Right panel shows zero offset section after HEMNO for multiples. Notice how, over all midpoints, the seabed reflections in each panel are coincident.

5 REFERENCES


