WHAT CAN WE DO WITH A MODEL OF THE MULTIPLES ?

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1 Introduction

A very popular and effective way of removing multiples is by creating a multiple model (Verschuur et al., 1992) and using it in order to perform the attenuation of the multiples. From this multiple model, two different strategies are possible. A first strategy might consist in subtracting this model from the data. A second strategy might consist in using this model as an estimate of the noise multivariate spectrum for a pattern-like attenuation of the multiples (Brown and Clapp, 2000). This multivariate spectrum can be reliably estimated with non-stationary prediction-error filters. The first strategy is often referred as adaptive subtraction of the noise. The second is often called pattern-based method.

This paper investigates improvements for both methods when multiples are subtracted from seismic data. For instance, I show how a different norm can be used when weak multiples are present in the data. Then I will show how we can approximate the noise covariance operator with prediction-error filters (Guitton, 2002) to decorrelate the filter estimation step, thus relaxing the assumption that the signal has minimum energy. Finally, I will demonstrate that a pattern-based method with 3-D time domain prediction-error filters greatly improves the attenuation of multiples in complex geology.

2 Adaptive subtraction of predicted multiples

Given a multiple model **m** and the seismic data **d**, the goal of adaptive filtering is to estimate a filter **f** such that

$$g(\mathbf{f}) = \|\mathbf{d} - \mathbf{M}\mathbf{f}\|^2 \tag{1}$$

is minimum where **M** represents the convolution with a multiple model **m**. When weak multiples are surrounded by strong primaries, the filter estimation will preferably match the model of the multiples to the primaries in the data, and not the multiples. This comes from the ℓ^2 norm used in the filter estimation step of equation (1). A better formulation of the filter estimation is

$$g(\mathbf{f}) = |\mathbf{d} - \mathbf{M}\mathbf{f}| \tag{2}$$

where we now use the ℓ^1 norm. Note that in equation (1), we assume that the signal is orthogonal to the noise and has minimum energy. This can lead to a false estimation of the filter. We can improve the formulation of equation (1) by introducing the inverse covariance operator of the signal $\mathbf{C_s^{-1}}$ as follows

$$g(\mathbf{f}) = (\mathbf{d} - \mathbf{M}\mathbf{f})'\mathbf{C_s^{-1}}(\mathbf{d} - \mathbf{M}\mathbf{f}). \tag{3}$$

The role of the covariance operator is to "remove" the signal information present in the residual. Doing so, we do not assume that the signal has minimum energy anymore. One complication arises in the estimation of this covariance operator. In Guitton (2002), I show that the covariance operator can be approximated with prediction-error filters (PEFs). A similar idea has been proposed by Spitz (2000) in the F-X domain. The time domain approach makes the incorporation of weighting function in the signal/noise separation step much easier, however. Therefore we need to estimate the PEFs **S** for the

signal **s** and then estimate the adaptive filters minimizing

$$g(\mathbf{f}) = (\mathbf{d} - \mathbf{M}\mathbf{f})'\mathbf{S}'\mathbf{S}(\mathbf{d} - \mathbf{M}\mathbf{f}), \tag{4}$$

where $\mathbf{S}'\mathbf{S} \approx \mathbf{C}_\mathbf{s}^{-\mathbf{1}}$. The signal being unknown, we can approximate the signal by deconvolving the data PEFs **D** estimated from the data by the noise PEFs **N** estimated from the multiple model (Spitz, 1999). Therefore, for the adaptive subtraction of multiples, we can improve our estimation of the signal by introducing different norms like the ℓ^1 norm when the multiples are weak or by adding the signal covariance operator when the signal has not minimum energy.

3 Multiple attenuation based on multivariate spectra

Instead of subtracting the noise with adaptive filters, we can use the noise model to extract the multivariate spectrum of the multiples. Then, we use this information to discriminate against the multivariate spectrum of the signal. This idea is at the heart of the pattern-based methods. A way of estimating the signal **s** is by solving

$$\hat{\mathbf{s}} = (\mathbf{N}'\mathbf{N} + \epsilon^2\mathbf{S}'\mathbf{S})^{-1}\mathbf{N}'\mathbf{N}\mathbf{d}. \tag{5}$$

Mathematically, equation (5) is a projection of the data components onto the signal (Soubaras, 1994). It has the property of preserving the amplitude of the signal. With a time domain approach, it is very easy to introduce weighting functions in equation (5) to take the amplitudes of the data (primaries and multiples) into account. In addition, with the helical boundary conditions, the PEFs can have any dimension (Mersereau and Dudgeon, 1974; Claerbout, 1998).

4 Examples

Figure 1a shows a shot gather with internal multiples. The internal multiples are shown in Figure 1b. Because the multiples are very weak, the ℓ^1 norm is utilized to perform the multiple attenuation, i.e., equation (2). The estimated primaries are shown in Figure 2a and the multiples in Figure 2b. The internal multiples are correctly removed. Figures 3 and 4 display an example where adaptive filters are estimated with the inverse covariance operator $\mathbf{C}_\mathbf{s}^{-\mathbf{1}}$.

I show in Figure 5 time slices of the time/offset/shot cube in which a pattern-based multiple attenuation technique is performed. As expected, 3-D filters (Figure 5d) attenuate the noise much better than 2-D filters do (Figure 5c). It is quite remarkable that 3-D filters perform so well in areas where the multiple model is known to be inaccurate. In particular, diffracted multiples and off-plane/3-D multiples are better attenuated (between offsets 2000 and 3000 m in Figure 5).

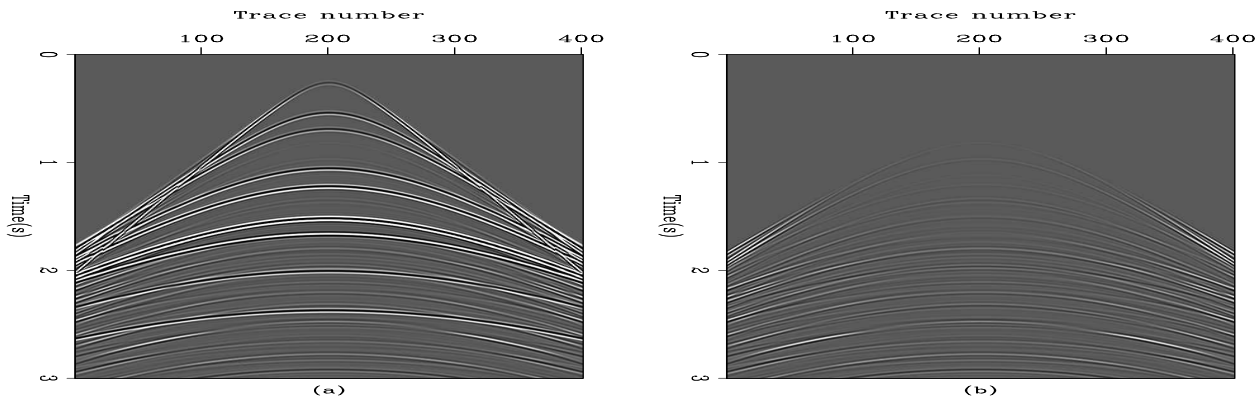

Figure 1: (a) Shot gather infested with internal multiples. (b) Model of the internal multiples. The internal multiples have lower amplitudes than the primaries.

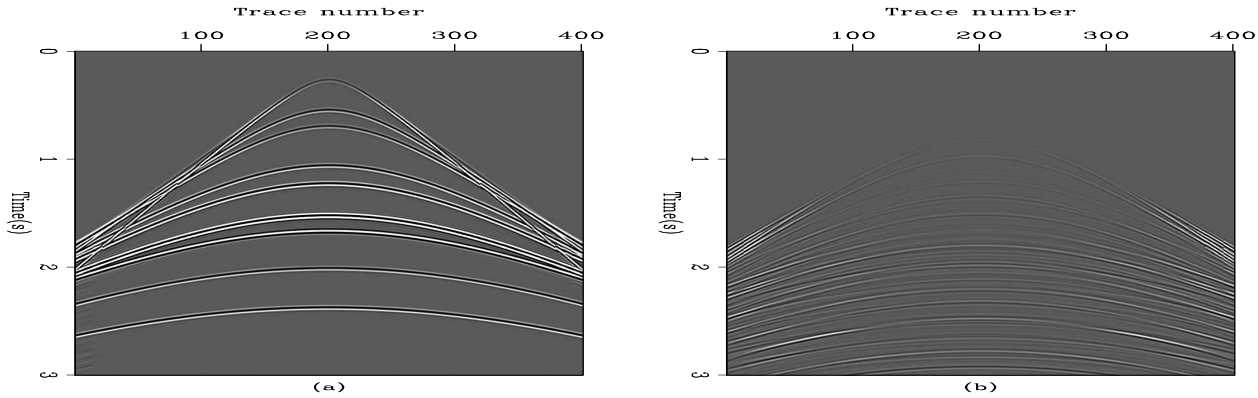

Figure 2: (a) Estimated primaries. (b) Estimated multiples. With the ℓ^1 norm, we can estimate the primaries and the multiples very well.

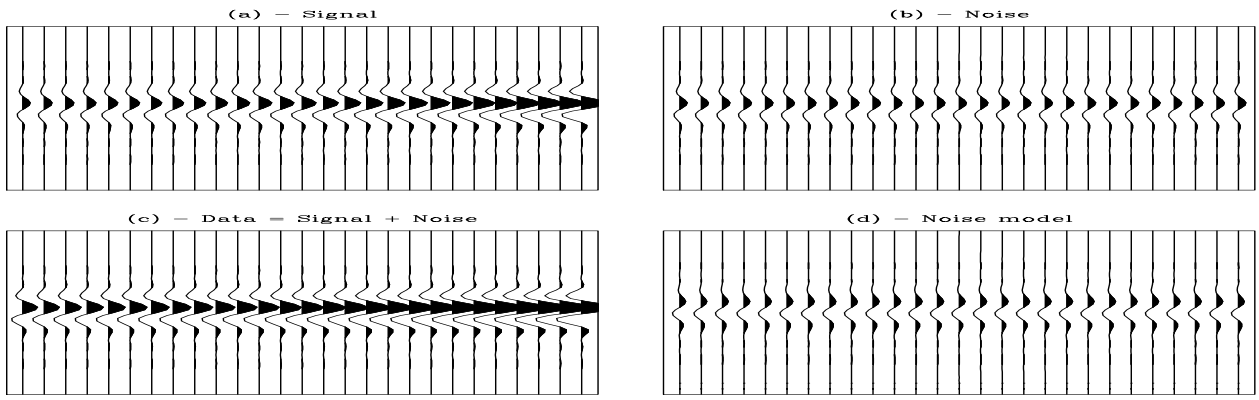

Figure 3: (a) Synthetic signal. (b) Synthetic noise. (c) Synthetic data (a+b). (d) Model of the noise that needs to be adaptively subtracted from (c). The noise and signal are strongly correlated but with a different amplitude pattern. Therefore, equation (4) is utilized for the filter estimation step.

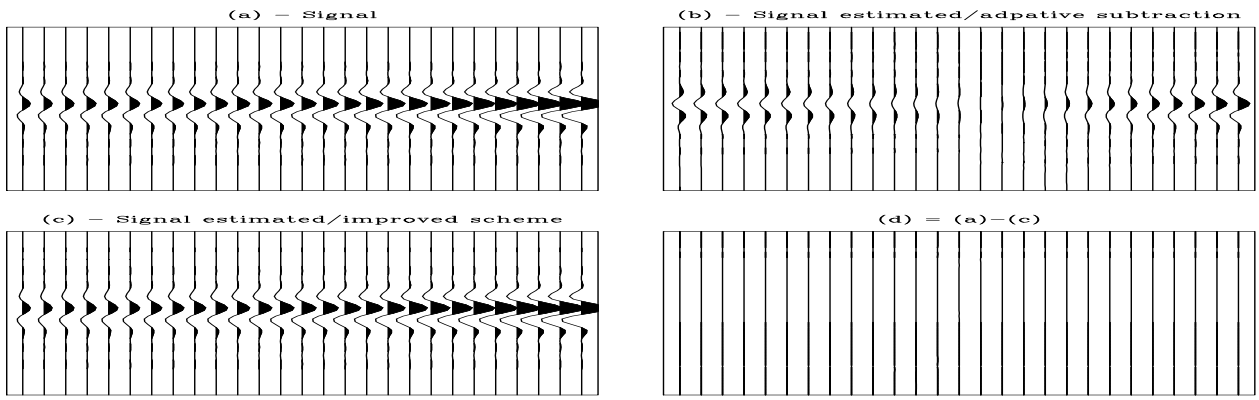

Figure 4: (a) Synthetic signal. (b) Signal estimated with equation (1). The signal is clearly wrong. (c) Signal estimated with an estimation of the signal covariance matrix (equation (4)). The signal is almost identical to the "true" signal in (a). (d) Difference between (a) and (c).

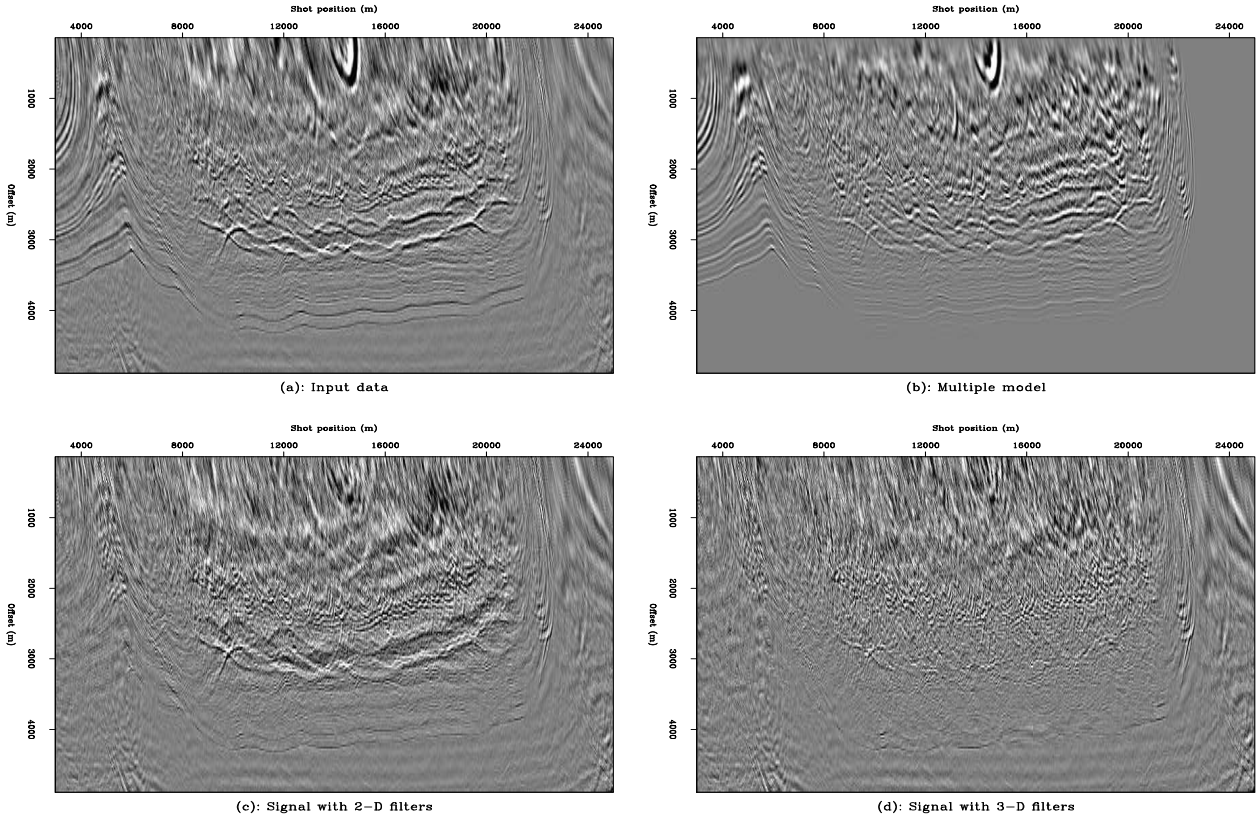

Figure 5: Time slices at 4.5 seconds for a Gulf of Mexico 2-D dataset. (a) Input data. (b) Multiple model. (c) Estimated signal with 2-D filters. (d) Estimated signal with 3-D filters.

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