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COHERENT NOISE ATTENUATION USING INVERSE PROBLEMS AND PREDICTION ERROR FILTERS

ANTOINE GUITTON
Department of Geophysics, Stanford University, Stanford, CA 94305, USA

1 SUMMARY

Two iterative methods that remove coherent noise during the inversion of 2-D prestack data are tested. One method approximates the inverse covariance matrices with prediction error filters (PEFs), and the other introduces a coherent noise modeling operator in the objective function. This noise modelling operator is a PEF that has to be estimated either before the inversion from a noise model or directly from the data. These two methods lead to independent, identically distributed (IID) residual variables, thus guaranteeing a stable convergence of the inversion schemes and permitting coherent noise filtering/separation.

2 INTRODUCTION

Seismic processes like migration or deconvolution, can be regarded as the adjoint of forward "modeling" operators (Claerbout, 1992). Unfortunately, most of the time, these operators are not unitary making the preservation of the amplitude a difficult but necessary task. Inverse theory offers an attractive solution to this problem. For example, an approximate inverse can be computed using least-squares inversion . Another possibility is to estimate the unknown vector with an iterative data-fitting approach. However, with both methods, major difficulties arise when the data are contaminated by *outliers*, which are abnormally large or small data components, or *Coherent noise* that the seismic operator is unable to model. The noise will (1) spoil any analysis based on the result of the inversion and (2) affect the amplitude recovery of the input data. From a statistical point of view, the residual components will not be IID. A possible solution to one particular noise problem is to attribute long-tailed PDFs to the residual variables leading to the minimization of the ℓ^1 norm of the data residual. However, this strategy is usually not effective at attenuating coherent noise because it is not generally distinguishable by its histogram, but by its moveout patterns.

In this paper, I focus on the attenuation/separation of the coherent noise only. The first strategy relates to fundamentals in inverse theory as detailed in the general discrete inverse problem (Tarantola, 1987) and approximates the inverse covariance matrices with PEFs. The second strategy proposes to introduce a coherent noise modelling operator in the fitting goal. In the first strategy the coherent noise is filtered. In the second strategy the coherent noise is subtracted from the signal.

3 THE INVERSE PROBLEM

The least-squares criterion comes directly from the hypothesis that the PDF of each observable data and each model parameter is Gaussian. These assumptions lead to the general discrete inverse problem (Tarantola, 1987). Finding a model $\mathbf{m}$ that explains the data $\mathbf{d}$ *via* an operator or matrix $\mathbf{H}$ is then equivalent to minimizing the quadratic objective function

$$f(\mathbf{m}) = (\mathbf{Hm}-\mathbf{d})^{\mathbf{T}}\mathbf{C_n^{-1}}(\mathbf{Hm}-\mathbf{d}) + (\mathbf{m}-\mathbf{m_{prior}})^{\mathbf{T}}\mathbf{C_m^{-1}}(\mathbf{m}-\mathbf{m_{prior}}), \tag{1}$$

where $\mathbf{C_n}$ and $\mathbf{C_m}$ are the *covariance operators*, and $\mathbf{m_{prior}}$ a model given *a priori*. The covariance matrix $\mathbf{C_n}$, often called the *noise covariance* matrix, combines experimental errors and modelling uncertainties. Assuming (1) uniform variance of the model and of the noise, (2) covariance matrices are diagonal , i.e., uncorrelated model an data components, and (3) no *prior* model $\mathbf{m_{prior}}$, the objective function becomes

$$f(\mathbf{m}) = (\mathbf{Hm}-\mathbf{d})^{\mathbf{T}}(\mathbf{Hm}-\mathbf{d}) + \epsilon^{\mathbf{2}}\mathbf{m^Tm}, \tag{2}$$

where ϵ is a function of the noise and model variances. The previous assumptions leading to equation (2) are quite strong when we are dealing with seismic data because the variance of the noise/model may be not uniform and the components of the noise/model are not independent. I will refer to equation (2) as the "simplest" approach. When the assumptions leading to equation (2) are respected, the convergence toward $\mathbf{m}$ is easy to achieve and the components of the residual $\mathbf{r} = \mathbf{Hm}-\mathbf{d}$ become IID.

4 PROPOSED SOLUTIONS TO ATTENUATE COHERENT NOISE

Any dataset may be regarded as the sum of signal and noise $\mathbf{d} = \mathbf{s}+\mathbf{n}$. I assume that the coherent noise $\mathbf{n}$ is made of the inconsistent part (or modeling uncertainties part) of the data $\mathbf{d}$ for any given operator $\mathbf{H}$.

A filtering method: Equation (1) introduces two matrices that are difficult to compute: the noise covariance matrix $\mathbf{C_n}$ and the model covariance matrix $\mathbf{C_m}$. I concentrate my efforts on the noise covariance matrix only. When coherent noise is present in the data the covariance matrices should not be approximated by diagonal operators. IID residual components is equivalent to having a residual with a white spectrum. Thus coherent noise will add "color" to the spectrum of the residual. The goal of the covariance matrices is to absorb this spectrum. Thus experimental residuals (squared) should be weighted inversely by their multivariate spectrum for optimal convergence (Claerbout and Fomel, 1999). Because a PEF whitens data from which it was estimated, it approximates the inverse power spectrum of the data and thus accomplishes the role of the inverse covariance matrices $\mathbf{C_d^{-1}}$ in equation (1). The fitting goal becomes, with $\mathbf{A_r}$ a PEF estimated from the residual and $\mathbf{A_r^TA_r} \approx \mathbf{C_n^{-1}}$,

$$\mathbf{0} \quad \approx \quad \mathbf{A_r}(\mathbf{Hm}-\mathbf{d}), \tag{3}$$

A subtraction method: Instead of removing the noise by filtering, we can remove it by subtraction [e.g Nemeth (1996)]. If an operator is unable to model all the information embedded in the data, then the residual is not IID. The second formulation I propose is based on the idea that if we can model the coherent noise with another operator, then the residual components become IID. Because there should be a different operator for each different coherent noise pattern, this method may become difficult to use. Fortunately, we can use multi-dimensional PEFs to estimate the coherent noise operator. This estimation is possible if we assume that the coherent noise is predictable, i.e., made up of the superposition of local plane wave segments (Claerbout, 1992). The fitting goal becomes

$$\mathbf{0} \approx \mathbf{Hm_s} + \mathbf{A_n^{-1}m_n} - \mathbf{d} \tag{4}$$

where $\mathbf{A_n}$ is the noise PEF, $\mathbf{m_n}$ the noise model and $\mathbf{m_s}$ the signal model. I did not develop any specific algorithm to solve this inverse problem. I assume that we have a strategy that allows us to estimate the operator $\mathbf{A_n}$. We can then minimize the objective function for the fitting goals given in Equation 4 in a least-squares sense, for example.

5 RESULTS

In this section I show some preliminary results from testing the two proposed strategies. The main operator **H** is the hyperbola superposition operator. The adjoint $\mathbf{H}^{\mathbf{T}}$ is the hyperbolic Radon transform (HRT). The model space **m** is called the velocity space. The input data **d** are CMP gathers.

The left panel of Figure 1 displays the input CMP gather for the velocity inversion. These data have been pre-whitened with a 1-D PEF (deconvolution). Notice that this CMP is made up of nearly horizontal events, hyperbolas and a slow velocity, low frequency event crossing the gather. The later is the coherent noise we want to remove. The three other panels in Figure 1 show the remodeled data after inversion for the simplest, filtering and subtraction method. They all look similar and the linear event has been attenuated. This is expected since the range of velocity used in the HRT does not include low velocities. The differences emerge in Figure 2 where the residuals are compared. The residual with the simplest method is not white as opposed to the residual for the filtering and subtraction methods. Thus, if we keep iterating, the simplest method would fit some noise in the data. Figure 2 demonstrates that we obtain IID residual components with the proposed schemes.

6 DISCUSSION

In the filtering method, PEFs are recomputed iteratively from the data residual. For the subtraction method, however, the final result is driven by the orthogonality between the coherent noise operator and the signal operator (Nemeth, 1996). If the two operators can model similar parts of the data, the separation will not be efficient. Nemeth proposes introducing some regularization in equation (4) to mitigate this difficulty. We could perhaps compute a prior coherent noise model from which we estimate the PEFs (Spitz, 1999). In addition, the processing operator **H** should mitigate the crosstalks between the signal and the coherent noise.

Because the data are not time-stationary, the coherent noise operator should be a function of time and space. This difficulty can be overcome using non-stationary filters. In particular, estimating space varying filters with coefficients smoothed along a radial direction might be efficient (Crawley, 1999).

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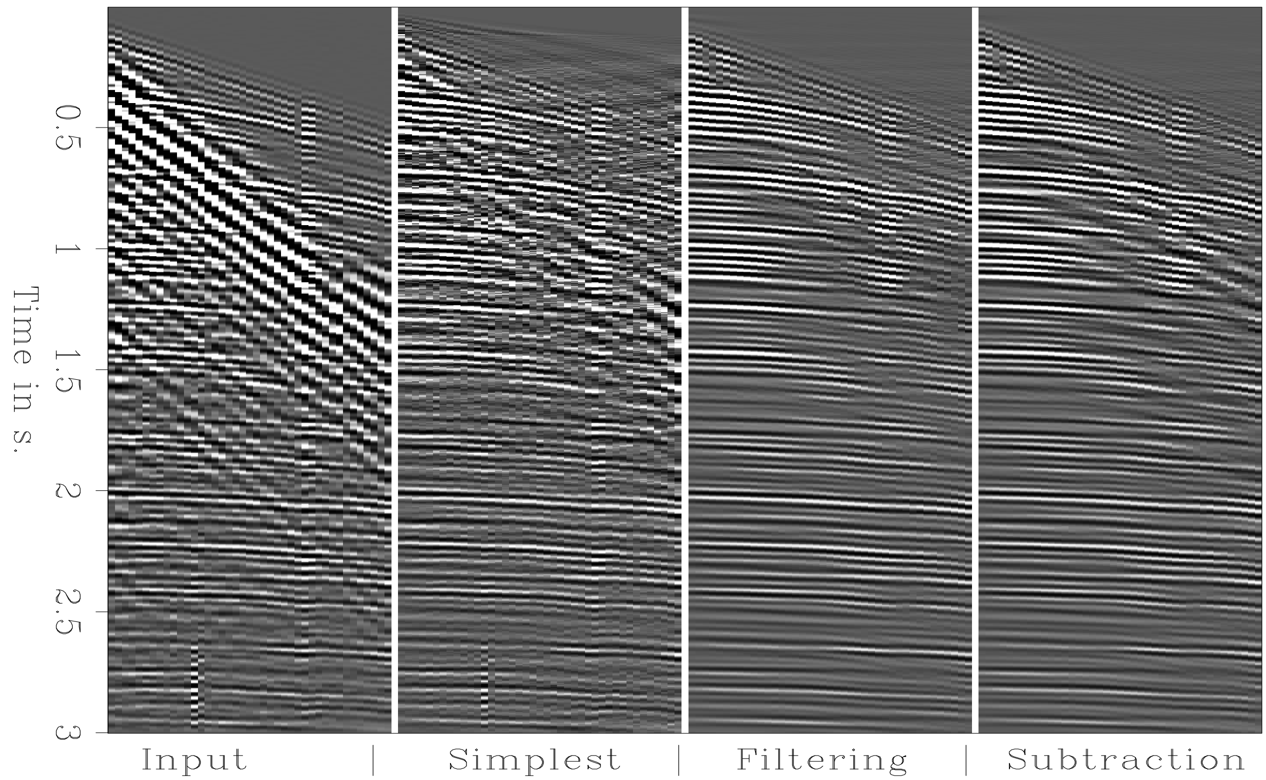

Figure 1: Remodeled data after inversion. Left: input data.

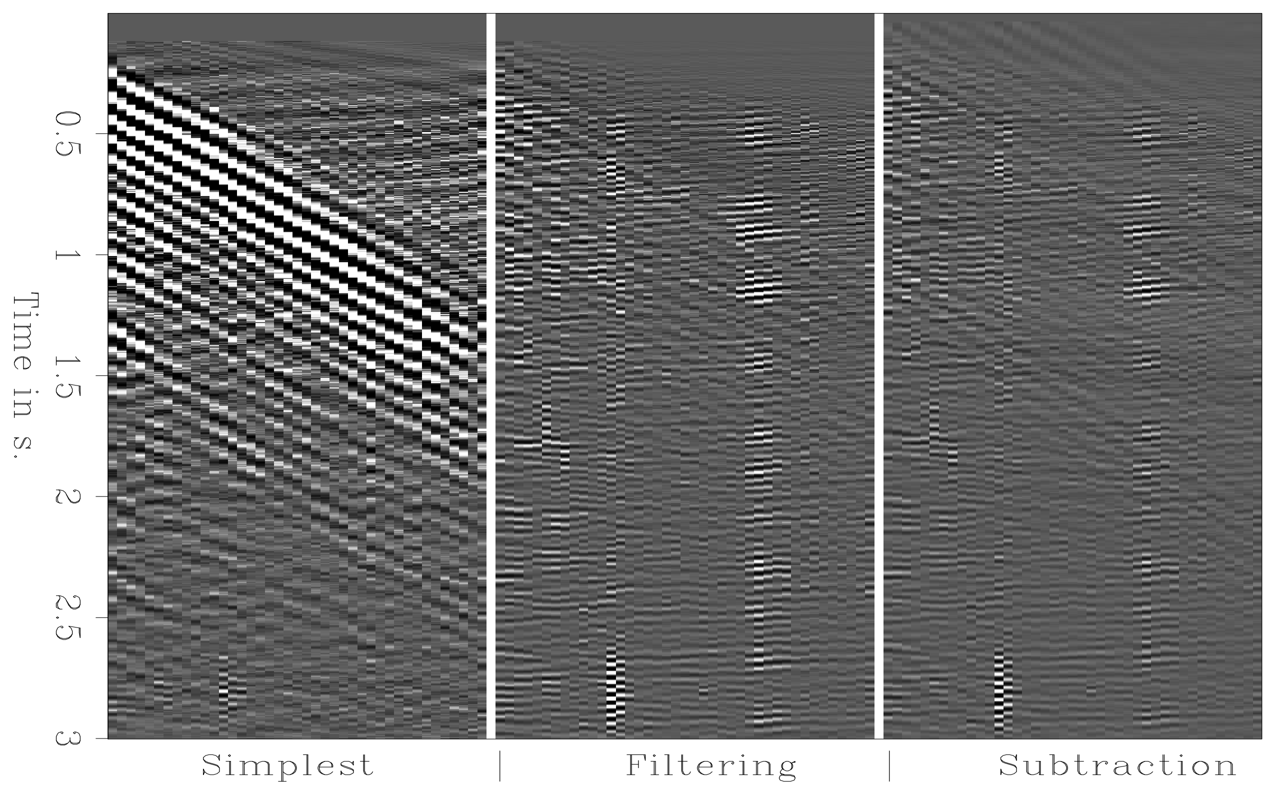

Figure 2: Comparison of the data residuals.