

A log spectral approach to bilinear deconvolution: theory only

Jon Claerbout

SEP report 143, page 297
Fallen Leaf Lake, California
June 2011

DEFINITION: Minimum phase == causal inverse

OLD THEOREM:

“The exponential of any causal is minimum phase.”

$$U(Z) = 1 + u_1 Z + u_2 Z^2 + \dots$$

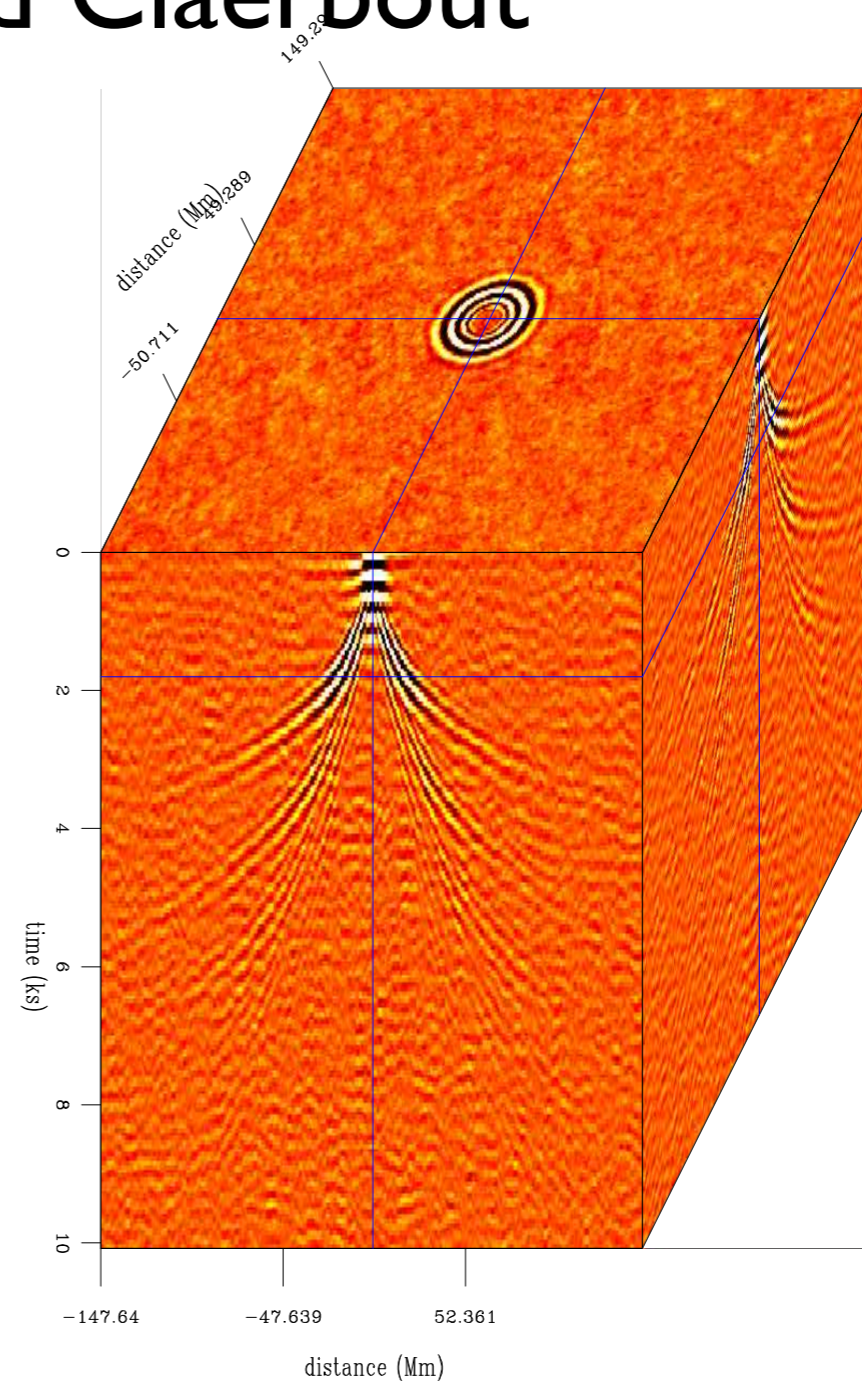
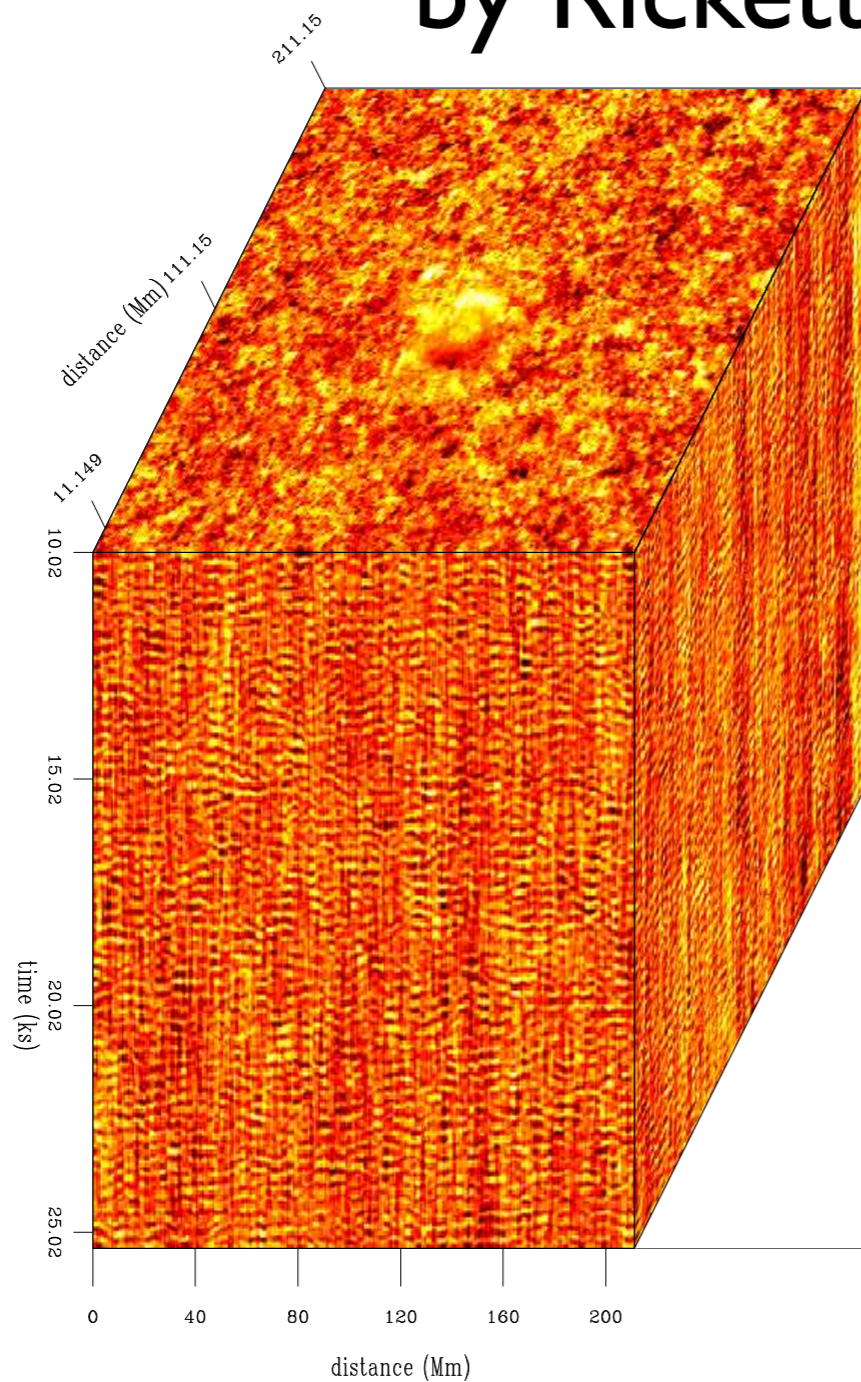
$$e^U = 1 + U + U^2/2! + U^3/3! + \dots$$

$$1/e^U = e^{-U}$$

Both have no negative powers of $Z = e^{i\omega}$. Q.E.D.

TEASER

This theory smells like the seismic impulse response of the sun by Rickett and Claerbout



GOAL to find u or U :

$$U^+ = u_1 Z + u_2 Z^2 + \dots$$

Notice

$$U^- = u_{-1}/Z + u_{-2}/Z^2 + \dots$$

$$u_0 = 0$$

$$e^{U^+} = A = 1 + a_1 Z + a_2 Z^2 + \dots$$

$$e^{U^-} = B = 1 + b_1/Z + b_2/Z^2 + \dots$$

$$U = U^+ + U^-$$

$$e^U = AB$$

Reminder

$$u_0 = 0$$

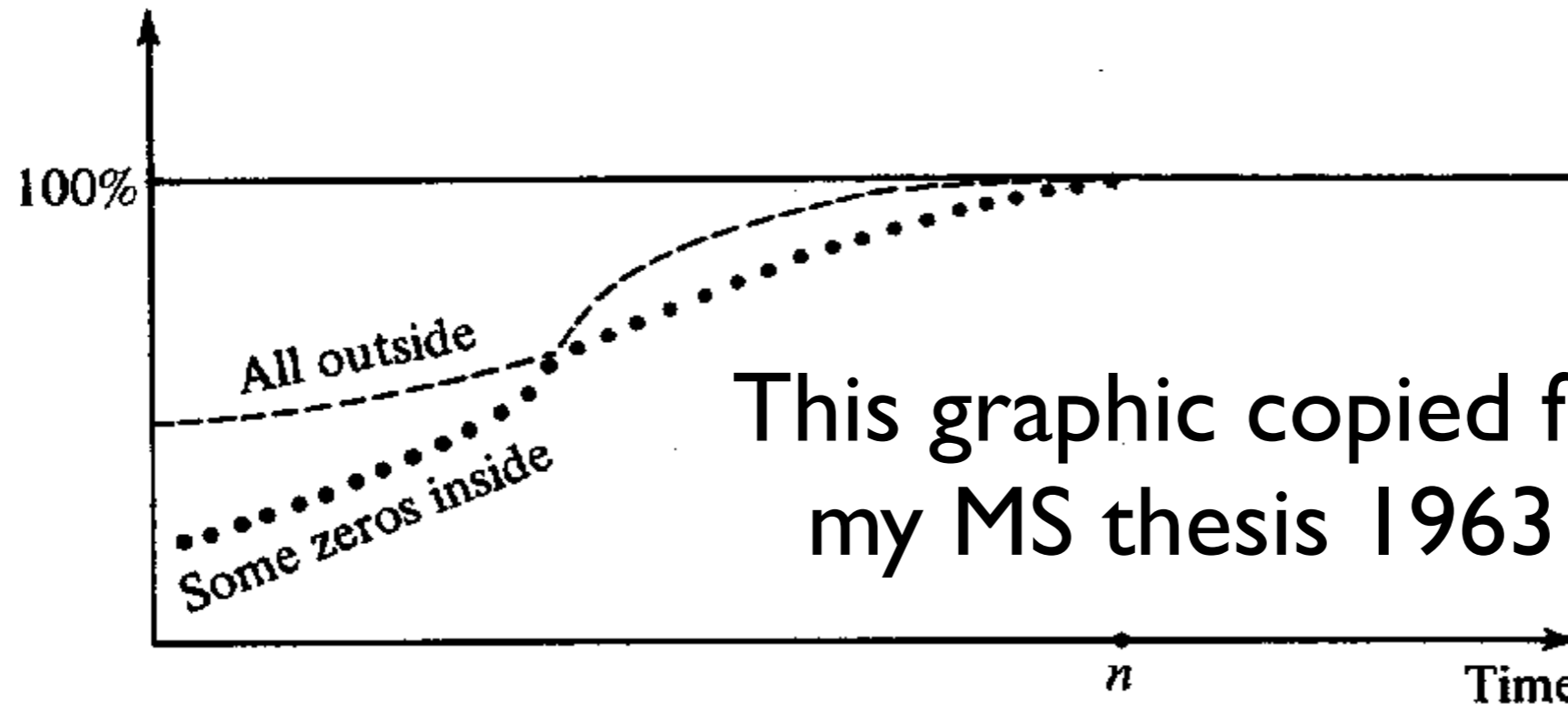
$$\Delta u_0 = 0$$

REMINDER

Robinson's minimum delay theorem

“Comparing all causal wavelets of the same spectrum, being minimum-phase gets energy out the quickest.”

Therefore the bidecon wavelet AB is compact, and has “no” time shift.



This graphic copied from proof in my MS thesis 1963 and FGDP.

Output== residual

Data

Filter

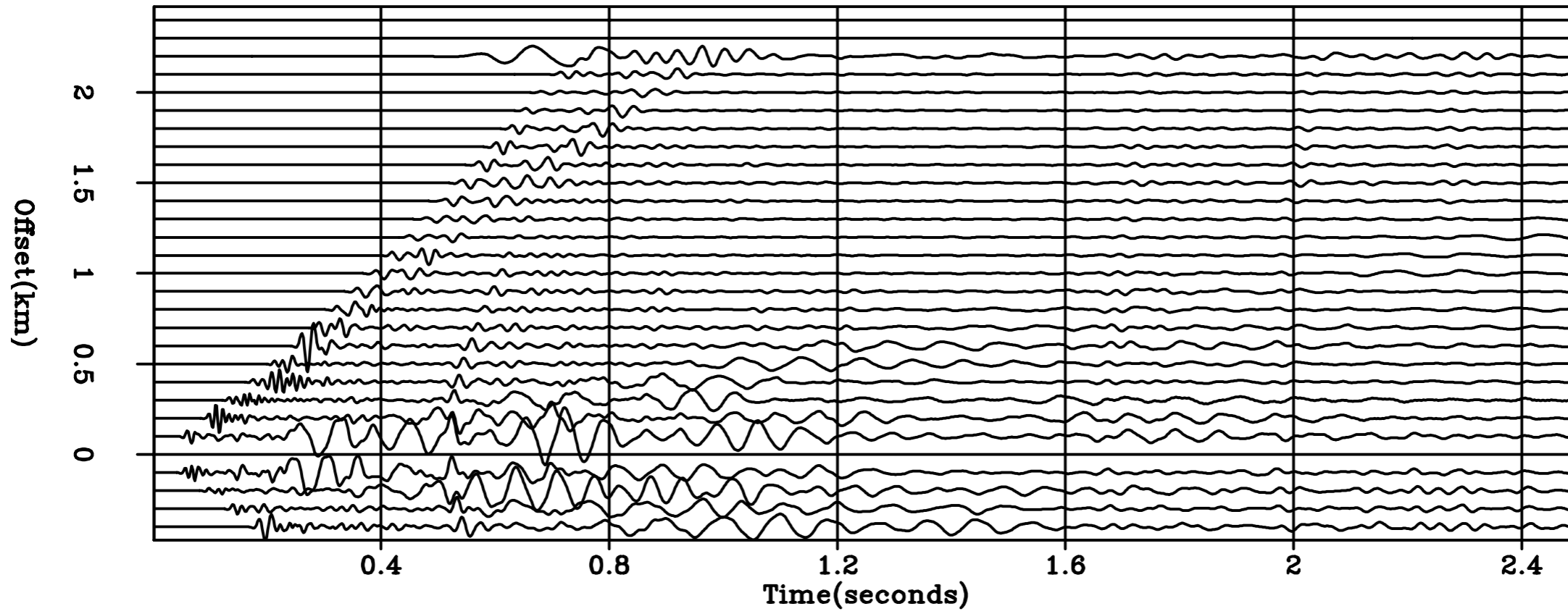
$$\mathbf{r} = (r_t) = \text{FT}^{-1} D(\omega) e^{U(Z(\omega))}$$

$$\text{argmin}_U \left(\text{hyp}(\mathbf{r}) = \sum_t H(r_t) \right)$$

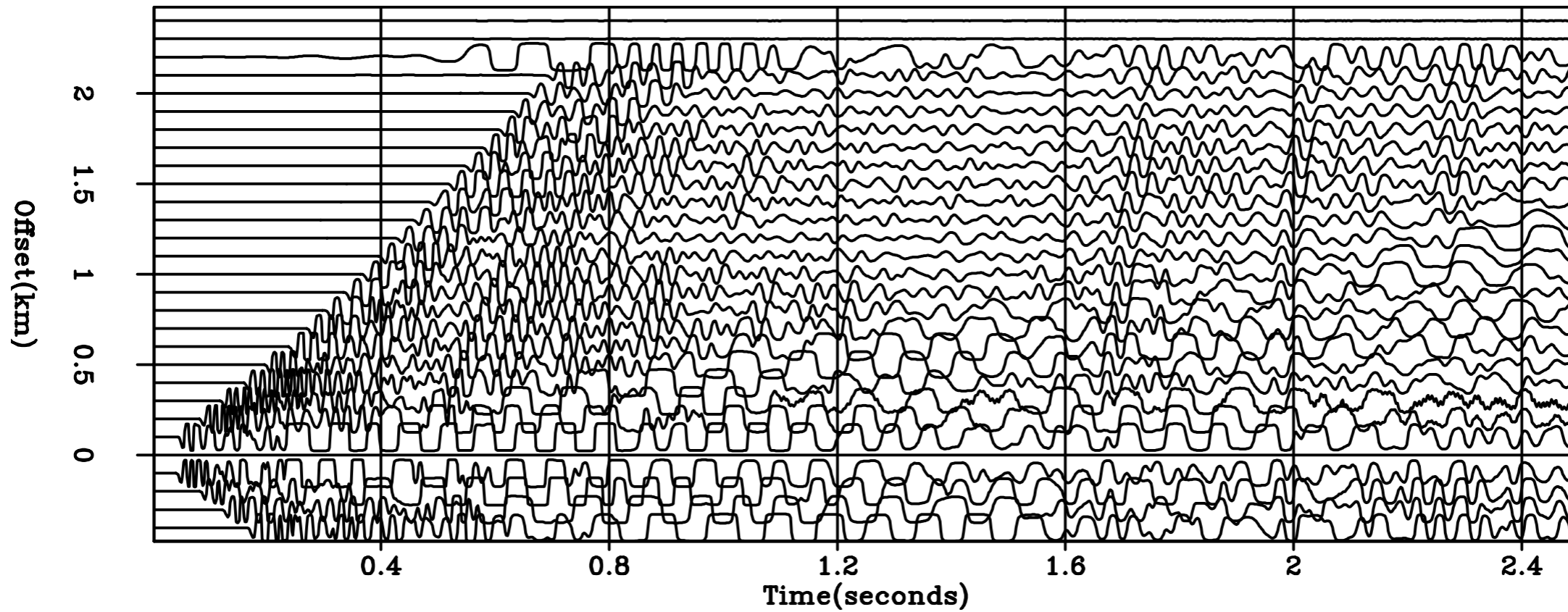
$$H(r_t) = \sqrt{r_t^2 + R_0} - R_0$$

$$H'(r_t) = \frac{\partial H}{\partial r_t} = \text{softclip}(r_t) = \frac{r_t}{\sqrt{1 + r_t^2 / R_0^2}}$$

d=data



softclip(data)



Residual

Data

Filter

$$r_t = \text{FT}^{-1} D(\omega) e^{U(Z(\omega))}$$

$$r_t = \text{FT}^{-1} D(Z) e^{\dots + u_2 Z^2 + u_3 Z^3 + u_4 Z^4 + \dots}$$



$$\frac{\partial r_t}{\partial u_3} \quad \text{Need this.}$$



Play with this.

Residual

Data

Filter

$$r_t = \text{FT}^{-1} D(\omega) e^{U(Z(\omega))}$$

$$r_t = \text{FT}^{-1} D(Z) e^{\dots + u_2 Z^2 + u_3 Z^3 + u_4 Z^4 + \dots}$$

$$0 \approx \frac{\partial \text{hyp}}{\partial u_3} = \sum_t \frac{\partial r_t}{\partial u_3} \frac{\partial H}{\partial r_t}$$

$$0 \approx \sum_t (\text{FT}^{-1} D(\omega) Z^3 e^{U(Z)})_t H'(r_t)$$

$$0 \approx \sum_t r_{t+3} H'(r_t)$$

for $\tau = 0$ $\Delta u_0 = 0$

for $\tau \neq 0$

$$0 \approx \Delta \mathbf{u} = (\Delta u_\tau) = \left(\sum_t r_{t+\tau} H'(r_t) \right)$$

$$\Delta U = \overline{\text{FT}(\mathbf{r})} \text{FT}(\text{softclip}(\mathbf{r}))$$

But must remove the mean from $\Delta U(\omega)$

Amazing generalization of L2 norm

$$0 \approx \Delta \mathbf{u} = (\Delta u_\tau) = \left(\sum_t r_{t+\tau} H'(r_t) \right) \text{ for } \tau \neq 0$$

- ℓ_2 : $H'(r) = r$ (output is white)
- ℓ_1 : $H'(r) = \text{signum}(r)$ (sparseness)
- ℓ_h : $H'(r) = \text{softclip}(r)$ (sparseness)

How does $\Delta \mathbf{u}$ change $\Delta \mathbf{r}$?

First find $\Delta \mathbf{r}$, then α .

Reminder

$$\Delta u_0 = 0$$

$$e^{\alpha \Delta U} = e^{\alpha(\Delta u_1 Z + \Delta u_2 Z^2)}$$

$$e^{\alpha \Delta U} = 1 + \alpha(\Delta u_1 Z + \Delta u_2 Z^2) + \cancel{\alpha^2(\dots)}$$

$$\text{FT}^{-1} e^{\alpha \Delta U} = (1, \alpha \Delta u_1, \alpha \Delta u_2)$$

$$\boxed{\text{FT}^{-1} e^{\alpha \Delta U}} = \boxed{(1, \alpha \Delta \mathbf{u})}$$



Next slide

How does $\Delta \mathbf{u}$ change $\Delta \mathbf{r}$?

First find $\Delta \mathbf{r}$, then α .

$$\begin{aligned} \mathbf{r} + \alpha \Delta \mathbf{r} &= \text{FT}^{-1} (D e^{U + \alpha \Delta U}) \\ &= \text{FT}^{-1} (D e^U e^{\alpha \Delta U}) \\ &= \text{FT}^{-1} (D e^U) * \boxed{\text{FT}^{-1} (e^{\alpha \Delta U})} \\ &= \mathbf{r} * \boxed{(1, \alpha \Delta \mathbf{u})} \\ &= \mathbf{r} + \alpha \mathbf{r} * \Delta \mathbf{u} \\ \Delta \mathbf{r} &= \mathbf{r} * \Delta \mathbf{u} \end{aligned}$$

previous slide

Change in r is proportional to r .

How does $\Delta \mathbf{u}$ change $\Delta \mathbf{r}$?

Now α .

Not this : $\alpha = -(\mathbf{r} \cdot \Delta \mathbf{r}) / (\Delta \mathbf{r} \cdot \Delta \mathbf{r})$

This : $\operatorname{argmin}_{\alpha} H(\mathbf{r} + \alpha \Delta \mathbf{r})$

We'll use Newton's method.

How does $\Delta \mathbf{u}$ change $\Delta \mathbf{r}$?

Now α .

$$0 = \frac{\partial}{\partial \alpha} \sum_t H(r_t) + \alpha \Delta r_t H'(r_t) + (\alpha \Delta r_t)^2 H''(r_t) / 2!$$

$$0 = \sum_t \Delta r_t H'(r_t) + \alpha (\Delta r_t)^2 H''(r_t)$$

$$\alpha = - \sum_t \Delta r_t H'_t / \sum_t (\Delta r_t)^2 H''_t$$

$$\mathbf{r} \leftarrow \mathbf{r} + \alpha \Delta \mathbf{r}$$

$$\mathbf{u} \leftarrow \mathbf{u} + \alpha \Delta \mathbf{u}$$

Newton iteration



Finally $R \leftarrow D e^U$
 $\mathbf{r} \leftarrow \mathbf{F} \mathbf{T}^{-1} R$

Pseudocode

$D(\omega, x) = \text{FT } d(t, x)$

$u=0;$

iteration {

$U = \text{FT}(u)$

 remove mean from $U(\omega)$

$\exp(U(Z))$

$dU = 0$

 for all x

$r(t, x) = \text{IFT}(D \exp(U))$

$\text{softclip}(r)$

$dU += \text{conjg}(\text{FT}(r)) * \text{FT}(\text{softclip})$ # "*" means multiply

 remove mean from $dU(\omega)$

 for all x

$dR = \text{FT}(r) * dU$

"*" means multiply

$dr = \text{IFT}(dR)$

 alpha at minimum of $H(r+\text{alpha}*dr)$

$u = u + \text{alpha } du$

}

CONCLUSIONS

- (1) It's exciting to see a new theory.
- (2) A few test cases were successfully run but we do not know how this method compares to others.
- (3) It's too early to draw conclusions, but maybe....

FUTURE

- (1) Maybe Fourier domain is faster.
- (2) Maybe boundaries are easier.
- (3) Maybe convergence is faster in log domain.
- (4) Helioseismology results suggest to shoot for angle-dependent source waveforms.
- (5) Goal is to identify pitfalls and opportunities.

ACKNOWLEDGEMENT

Qiang Fu and Yi Shen
proofread the theory,
wrote preliminary codes, and
verified the simplest case.

