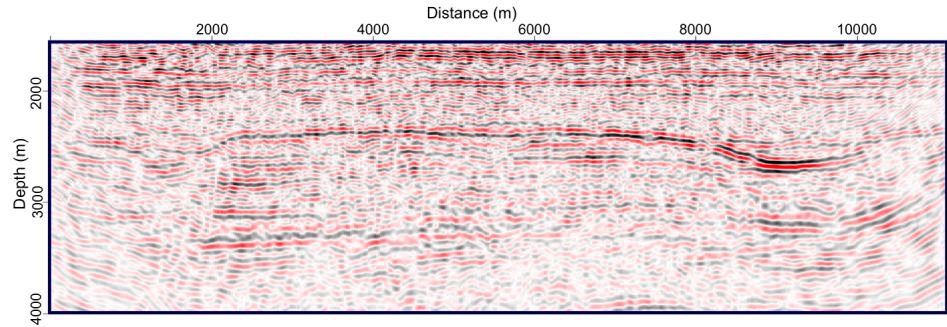
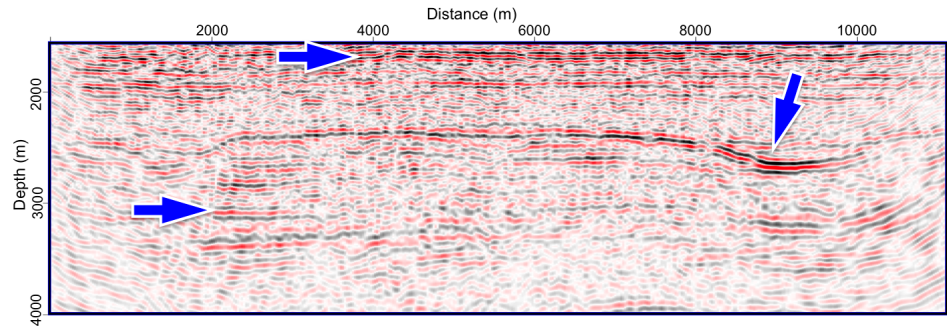


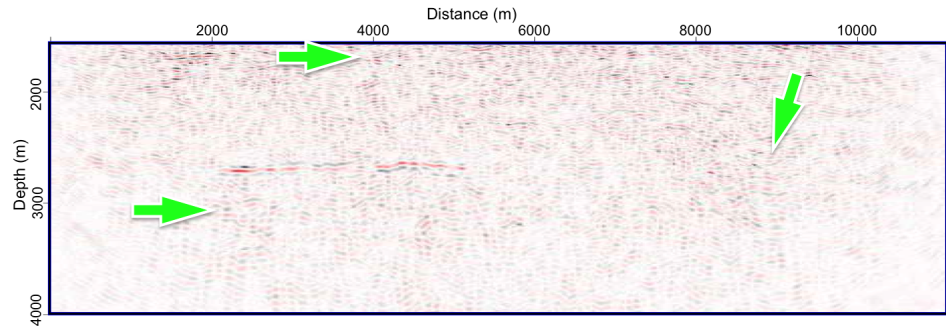
Time-lapse image



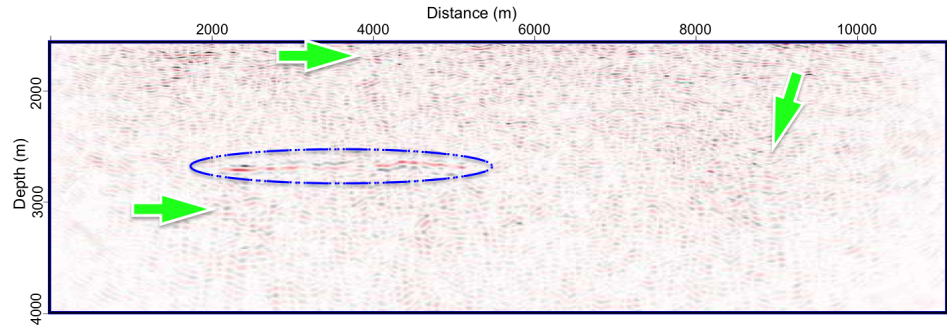
Time-lapse image: raw



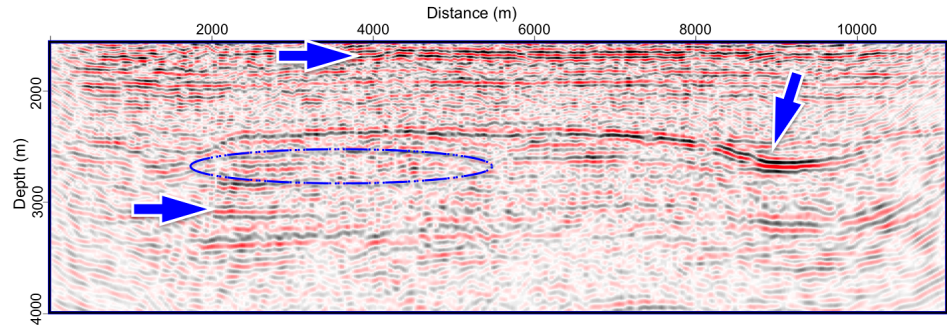
Time-lapse image: inverted



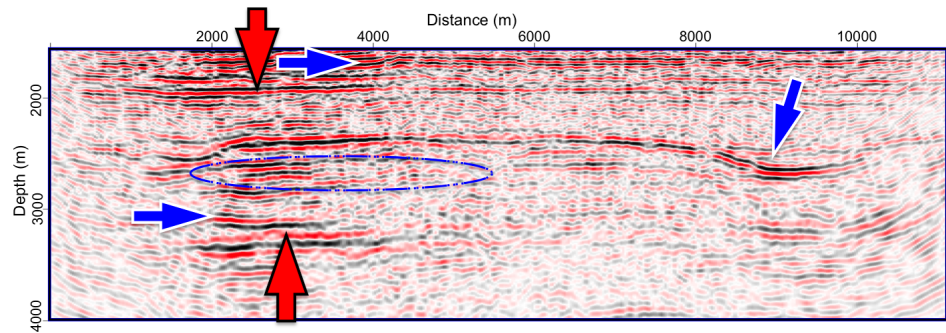
Time-lapse image: inverted



Time-lapse image: raw



Time-lapse image: raw+obstruction



Wave-equation inversion of time-lapse seismic data sets

Gboyega Ayeni & Biondi Biondi

SEP 143: Pgs 119-136

SEP Sponsors' Meeting

June, 2011

Wave-equation inversion of time-lapse seismic data sets

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SEP Sponsors' Meeting

June, 2011

1 Introduction

- Time-lapse imaging
- Key assumptions
- Time-lapse processing workflow
- Time-lapse inversion workflow

2 Method

- Data-space inversion
- Regularized inversion
- Image-space inversion

3 Examples

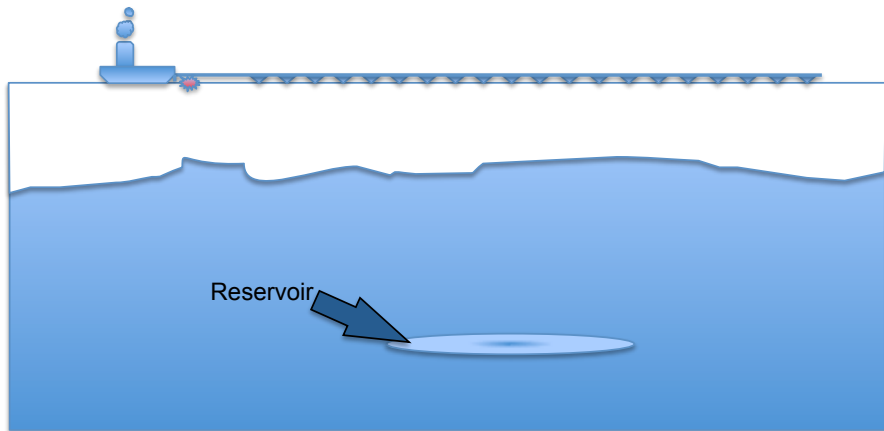
- Field data set
- Complete data example
- Gapped data example

4 Conclusions

Time-lapse imaging – Acquisition

Baseline

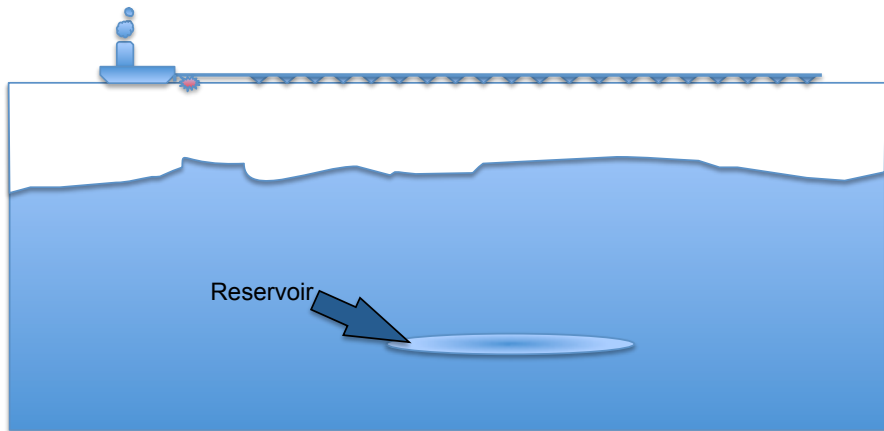
$$\mathbf{L}_0 \mathbf{m}_0 = \mathbf{d}_0$$



Time-lapse imaging – Acquisition

Monitor

$$L_1 m_1 = d_1$$



Time-lapse imaging

$$\mathbf{L}_0 \mathbf{m}_0 = \mathbf{d}_0$$

$$\mathbf{L}_1 \mathbf{m}_1 = \mathbf{d}_1$$

$$\Delta \mathbf{m} = \mathbf{m}_1 - \mathbf{m}_0$$

\mathbf{L}_i : modeling (acquisition) operator

\mathbf{d}_i : seismic data

\mathbf{m}_i : earth reflectivity model

$\Delta \mathbf{m}$: time-lapse image

$i=0$: Baseline

$i=1$: Monitor

Time-lapse imaging

$$\mathbf{L}_0 \mathbf{m}_0 = \mathbf{d}_0$$

$$\mathbf{L}_1 \mathbf{m}_1 = \mathbf{d}_1$$

$$\Delta \mathbf{m} = \mathbf{m}_1 - \mathbf{m}_0$$

$$\Delta \tilde{\mathbf{m}} = \tilde{\mathbf{m}}_1 - \tilde{\mathbf{m}}_0$$

\mathbf{L}_i : modeling (acquisition) operator

\mathbf{d}_i : seismic data

\mathbf{m}_i : earth reflectivity model

$\Delta \mathbf{m}$: time-lapse image

$i=0$: Baseline

$i=1$: Monitor

$\tilde{\mathbf{m}}_i$: migrated image ($\mathbf{L}_i^T \mathbf{d}_i$)

Key assumptions

- Production- (or injection-) related changes in acoustic properties are measurable
- The baseline migration velocity model is accurate

Key assumptions

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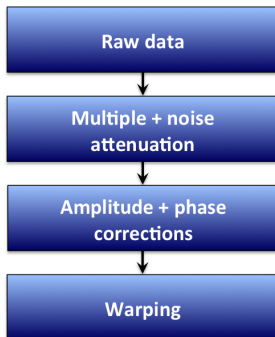
Key assumptions

- Production- (or injection-) related changes in acoustic properties are measurable
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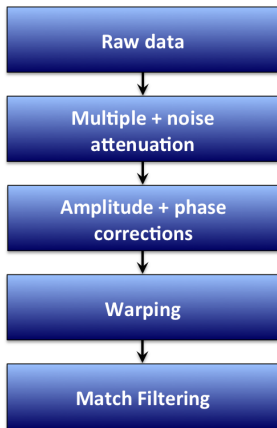
Key assumptions

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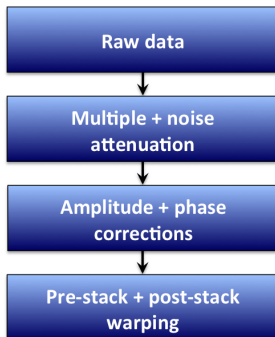
Workflow: 4D processing



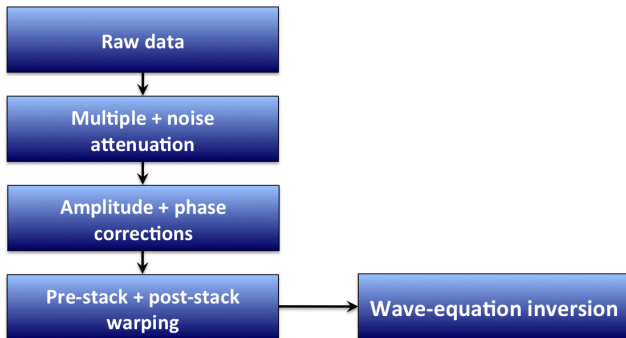
Workflow: 4D processing



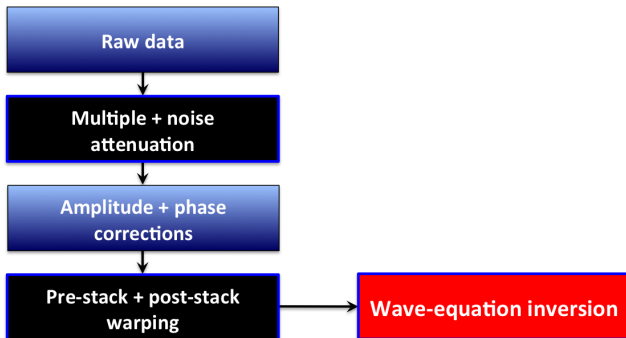
Workflow: 4D processing–updated



Workflow: 4D processing+inversion



Workflow: 4D processing+inversion



Data-space inversion

$$\mathbf{L}_0 \mathbf{m}_0 = \mathbf{d}_0$$

$$\mathbf{L}_1 \mathbf{m}_1 = \mathbf{d}_1$$

\mathbf{d}_i – data

\mathbf{m}_i – reflectivity image

\mathbf{L}_i – modeling operator

$\Delta \mathbf{m}$ – time-lapse image

Data-space inversion

$$\left\| \begin{bmatrix} \mathbf{L}_0 & \mathbf{0} \\ \mathbf{L}_1 \mathbf{S}^{m^-} & \mathbf{L}_1 \end{bmatrix} \begin{bmatrix} \mathbf{m}_0 \\ \Delta \mathbf{m}^m \end{bmatrix} - \begin{bmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \end{bmatrix} \right\| \approx 0$$

\mathbf{S}^{m^-} – forward (baseline to monitor) warping
 $\Delta \mathbf{m}^m$ – time-lapse image at monitor position

Data-space inversion

$$\left\| \begin{bmatrix} \mathbf{L}_0 & \mathbf{0} \\ \mathbf{L}_1 \mathbf{S}^{m^-} & \mathbf{L}_1 \mathbf{S}^{m^-} \end{bmatrix} \begin{bmatrix} \mathbf{m}_0 \\ \Delta \mathbf{m}^b \end{bmatrix} - \begin{bmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \end{bmatrix} \right\| \approx 0$$

\mathbf{S}^{m^-} – forward (baseline to monitor) warping
 $\Delta \mathbf{m}^b$ – time-lapse image at baseline position

Data-space inversion

$$\left\| \begin{bmatrix} \mathbf{L}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{U}^{m^-} \mathbf{L}_1^b \end{bmatrix} \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1^b \end{bmatrix} - \begin{bmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \end{bmatrix} \right\| \approx 0$$

\mathbf{U}^{m^-} – mapping operator

$$\mathbf{U}^{m^-} \approx \mathbf{L}_1 \mathbf{S}^{m^-} \left[[(\mathbf{L}_1^b)^T \mathbf{L}_1^b]^{-1} (\mathbf{L}_1^b)^T \right]$$

Regularized-inversion

$$\left\| \left[\begin{array}{cc} \mathbf{L}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{U}^{m-b} \mathbf{L}_1 \\ \hline \epsilon_0 \mathbf{R}_0 & \mathbf{0} \\ \mathbf{0} & \epsilon_1 \mathbf{R}_1 \\ \hline -\zeta_0 \mathbf{\Lambda}_0 & \zeta_1 \mathbf{\Lambda}_1 \end{array} \right] \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1^b \end{bmatrix} - \begin{bmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \right\| \approx 0$$

Regularized-inversion

$$\left\| \left\| \begin{bmatrix} \mathbf{L}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{U}^{m-b} \mathbf{L}_1 \\ \hline \epsilon_0 \mathbf{R}_0 & \mathbf{0} \\ \mathbf{0} & \epsilon_1 \mathbf{R}_1 \\ \hline -\zeta_0 \mathbf{\Lambda}_0 & \zeta_1 \mathbf{\Lambda}_1 \end{bmatrix} \begin{bmatrix} \mathbf{m}_0 \\ \mathbf{m}_1^b \end{bmatrix} - \begin{bmatrix} \mathbf{d}_0 \\ \mathbf{d}_1 \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \right\| \right\| \approx 0$$

\mathbf{R}_i – spatial regularization

$\mathbf{\Lambda}_i$ – temporal regularization

ϵ_i – spatial regularization parameter

ζ_i – coupling parameter

Image-space inversion

$$\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1^b \\ \hline \mathbf{R}_{00} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{11}^b \\ \hline \mathbf{\Lambda}_{00} & -\mathbf{\Lambda}_{01}^b \\ -\mathbf{\Lambda}_{10}^b & \mathbf{\Lambda}_{11}^b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{m}}_0 \\ \hat{\mathbf{m}}_1^b \end{bmatrix} \approx \begin{bmatrix} \tilde{\mathbf{m}}_0 \\ \tilde{\mathbf{m}}_1^b \\ \hline \mathbf{0} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$\mathbf{H}_i = \mathbf{L}_i^T \mathbf{L}_i$ – Hessian

$\tilde{\mathbf{m}}_i = \mathbf{L}_i^T d_i$ – migrated image

$\hat{\mathbf{m}}_i$ – inverted image

\mathbf{R}_{ij} – spatial regularization

$\mathbf{\Lambda}_{ij}$ – temporal regularization

Image-space inversion

$$\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1^b \\ \hline \mathbf{R}_{00} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{11}^b \\ \hline \mathbf{\Lambda}_{00} & -\mathbf{\Lambda}_{01}^b \\ -\mathbf{\Lambda}_{10}^b & \mathbf{\Lambda}_{11}^b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{m}}_0 \\ \hat{\mathbf{m}}_1^b \end{bmatrix} \approx \begin{bmatrix} \tilde{\mathbf{m}}_0 \\ \tilde{\mathbf{m}}_1^b \\ \hline \mathbf{0} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

H – target-oriented approximation to the full Hessian

Image-space inversion

$$\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1^b \\ \hline \mathbf{R}_{00} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{11}^b \\ \hline \mathbf{\Lambda}_{00} & -\mathbf{\Lambda}_{01}^b \\ -\mathbf{\Lambda}_{10}^b & \mathbf{\Lambda}_{11}^b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{m}}_0 \\ \hat{\mathbf{m}}_1^b \end{bmatrix} \approx \begin{bmatrix} \tilde{\mathbf{m}}_0 \\ \tilde{\mathbf{m}}_1^b \\ \hline \mathbf{0} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

\mathbf{H}_i^b – Hessian with monitor geometry but baseline velocity

$\tilde{\mathbf{m}}_1^b$ – migrated monitor warped to the baseline

$\hat{\mathbf{m}}_1^b$ – inverted monitor at the baseline position

Image-space inversion

$$\begin{bmatrix}
 \mathbf{H}_0 & \mathbf{0} \\
 \mathbf{0} & \mathbf{H}_1^b \\
 \hline
 \mathbf{R}_{00} & \mathbf{0} \\
 \mathbf{0} & \mathbf{R}_{11}^b \\
 \hline
 \mathbf{\Lambda}_{00} & -\mathbf{\Lambda}_{01}^b \\
 -\mathbf{\Lambda}_{10}^b & \mathbf{\Lambda}_{11}^b
 \end{bmatrix}
 \begin{bmatrix}
 \hat{\mathbf{m}}_0 \\
 \hat{\mathbf{m}}_1^b
 \end{bmatrix}
 \approx
 \begin{bmatrix}
 \tilde{\mathbf{m}}_0 \\
 \tilde{\mathbf{m}}_1^b \\
 \hline
 \mathbf{0} \\
 \mathbf{0} \\
 \hline
 \mathbf{0} \\
 \mathbf{0}
 \end{bmatrix}$$

Data-fitting

Image-space inversion

$$\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1^b \\ \hline \mathbf{R}_{00} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{11}^b \\ \hline \Lambda_{00} & -\Lambda_{01}^b \\ -\Lambda_{10}^b & \Lambda_{11}^b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{m}}_0 \\ \hat{\mathbf{m}}_1^b \end{bmatrix} \approx \begin{bmatrix} \tilde{\mathbf{m}}_0 \\ \tilde{\mathbf{m}}_1^b \\ \hline \mathbf{0} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

Spatial regularization – non-stationary dip-filtering

Temporal regularization – difference operator

Image-space inversion

$$\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1^b \\ \hline \mathbf{R}_{00} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{11}^b \\ \hline \mathbf{\Lambda}_{00} & -\mathbf{\Lambda}_{01}^b \\ -\mathbf{\Lambda}_{10}^b & \mathbf{\Lambda}_{11}^b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{m}}_0 \\ \hat{\mathbf{m}}_1^b \end{bmatrix} \approx \begin{bmatrix} \tilde{\mathbf{m}}_0 \\ \tilde{\mathbf{m}}_1^b \\ \hline \mathbf{0} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

- Can be easily extended to multiple surveys

Image-space inversion

$$\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1^b \\ \hline \mathbf{R}_{00} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{11}^b \\ \hline \mathbf{\Lambda}_{00} & -\mathbf{\Lambda}_{01}^b \\ -\mathbf{\Lambda}_{10}^b & \mathbf{\Lambda}_{11}^b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{m}}_0 \\ \hat{\mathbf{m}}_1^b \end{bmatrix} \approx \begin{bmatrix} \tilde{\mathbf{m}}_0 \\ \tilde{\mathbf{m}}_1^b \\ \hline \mathbf{0} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

- Can be easily extended to multiple surveys
- Can be solved in a target-oriented way (e.g., around the reservoir)

Image-space inversion

$$\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1^b \\ \hline \mathbf{R}_{00} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{11}^b \\ \hline \mathbf{\Lambda}_{00} & -\mathbf{\Lambda}_{01}^b \\ -\mathbf{\Lambda}_{10}^b & \mathbf{\Lambda}_{11}^b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{m}}_0 \\ \hat{\mathbf{m}}_1^b \end{bmatrix} \approx \begin{bmatrix} \tilde{\mathbf{m}}_0 \\ \tilde{\mathbf{m}}_1^b \\ \hline \mathbf{0} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

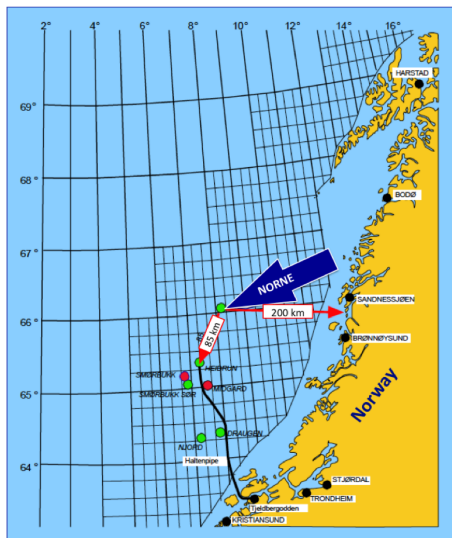
- Can be easily extended to multiple surveys
- Can be solved in a target-oriented way (e.g., around the reservoir)
- Can be efficiently repeated with different regularization parameters

Image-space inversion

$$\begin{bmatrix} \mathbf{H}_0 & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_1^b \\ \hline \mathbf{R}_{00} & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_{11}^b \\ \hline \mathbf{\Lambda}_{00} & -\mathbf{\Lambda}_{01}^b \\ -\mathbf{\Lambda}_{10}^b & \mathbf{\Lambda}_{11}^b \end{bmatrix} \begin{bmatrix} \hat{\mathbf{m}}_0 \\ \hat{\mathbf{m}}_1^b \end{bmatrix} \approx \begin{bmatrix} \tilde{\mathbf{m}}_0 \\ \tilde{\mathbf{m}}_1^b \\ \hline \mathbf{0} \\ \mathbf{0} \\ \hline \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

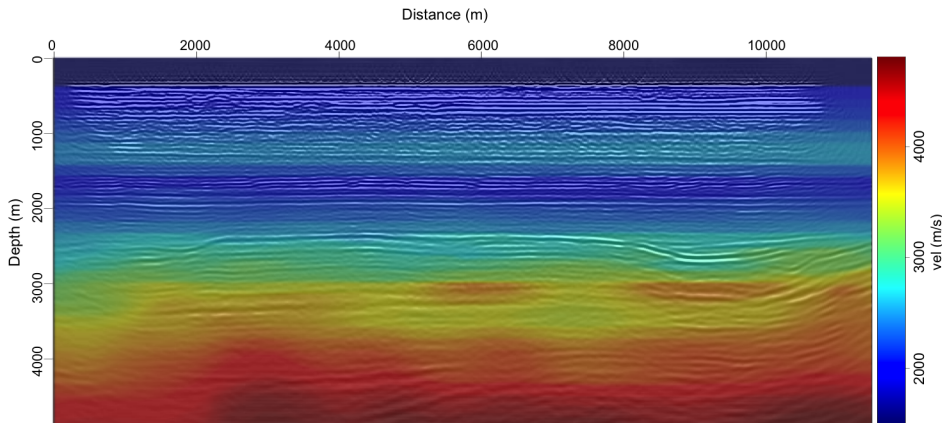
- Can be easily extended to multiple surveys
- Can be solved in a target-oriented way (e.g., around the reservoir)
- Can be efficiently repeated with different regularization parameters
- Can be easily integrated with *conventional* inversion framework

Location: Norwegian North Sea

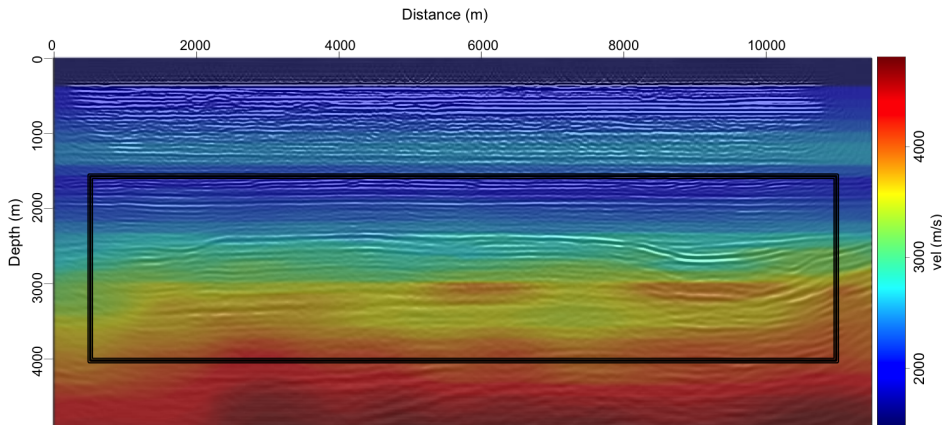


Modified from: STATOIL Annual Report (2006)

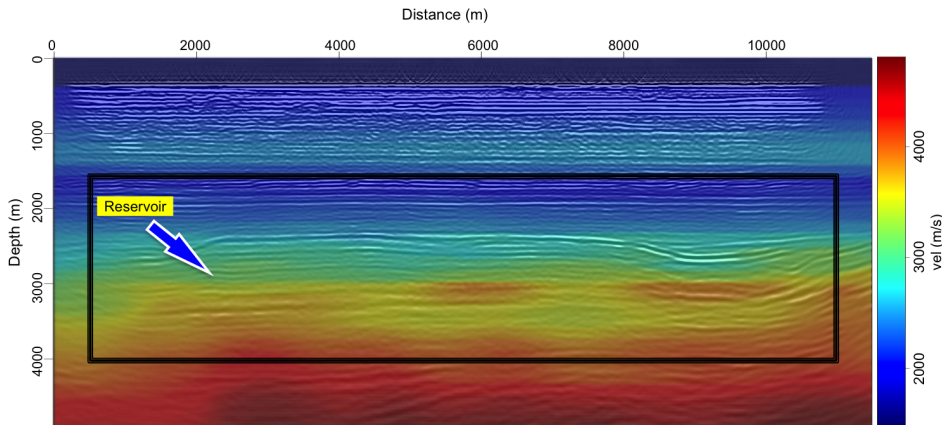
Baseline: stacked image/velocity model



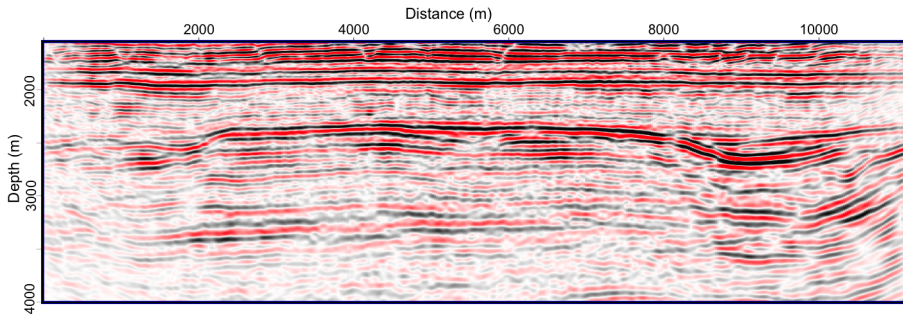
Baseline – Target area



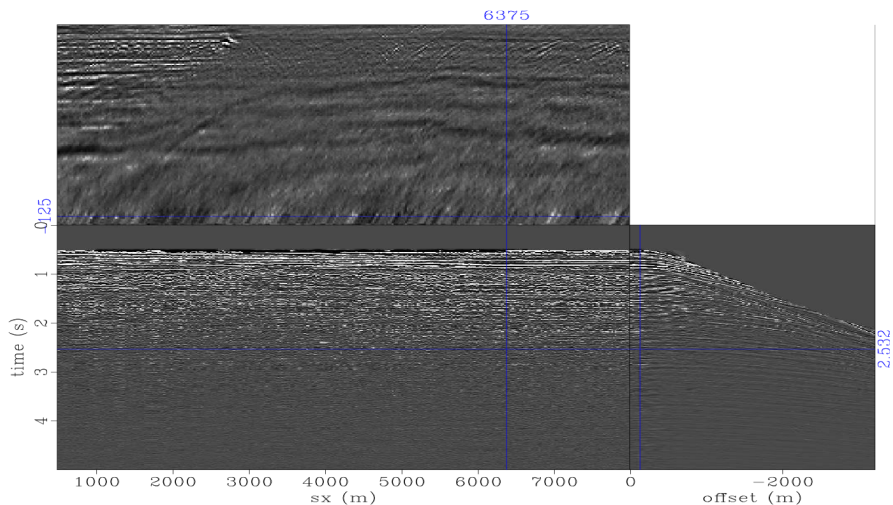
Baseline – Target area



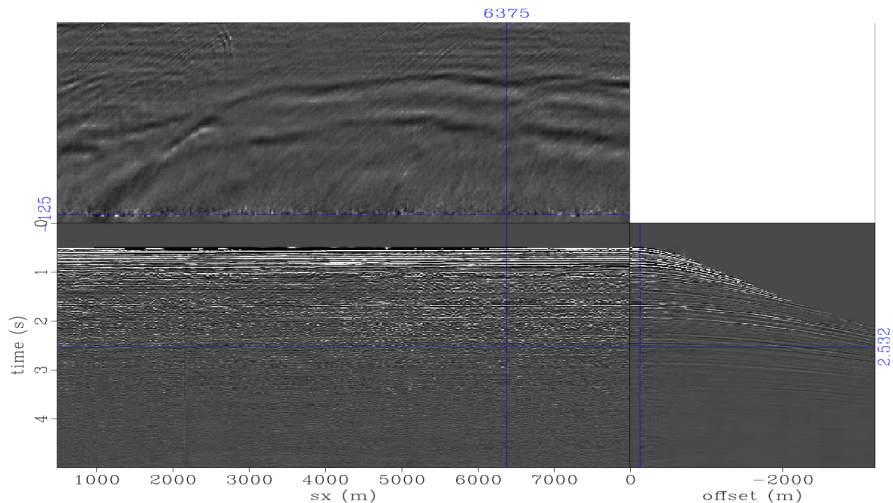
Baseline – Target area



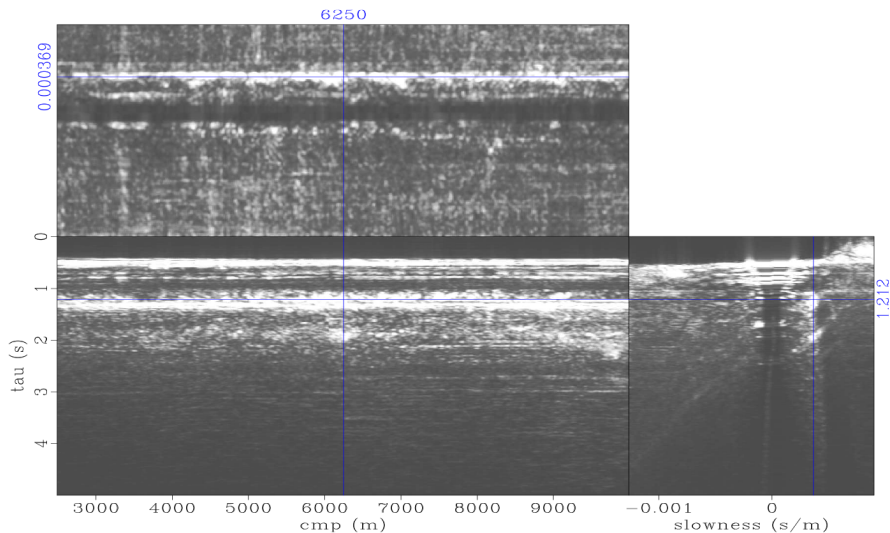
Baseline: Raw



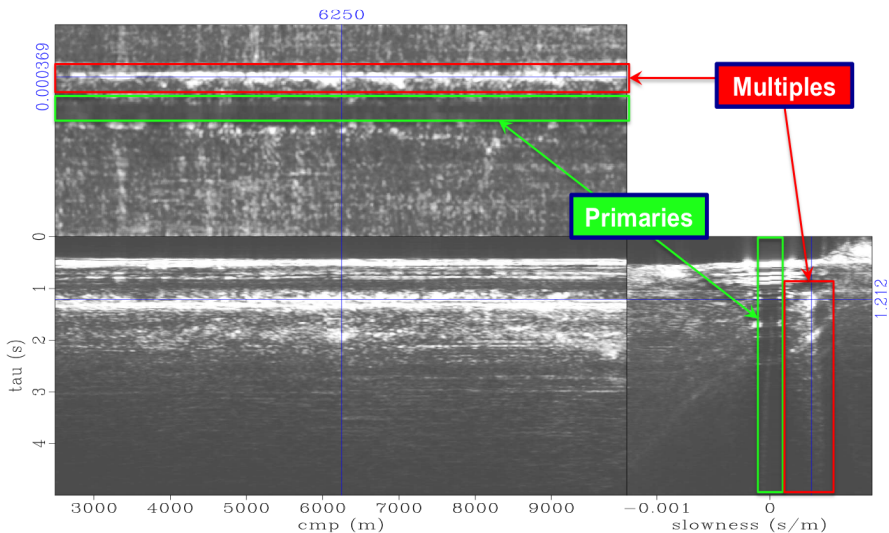
Baseline: After demultiple



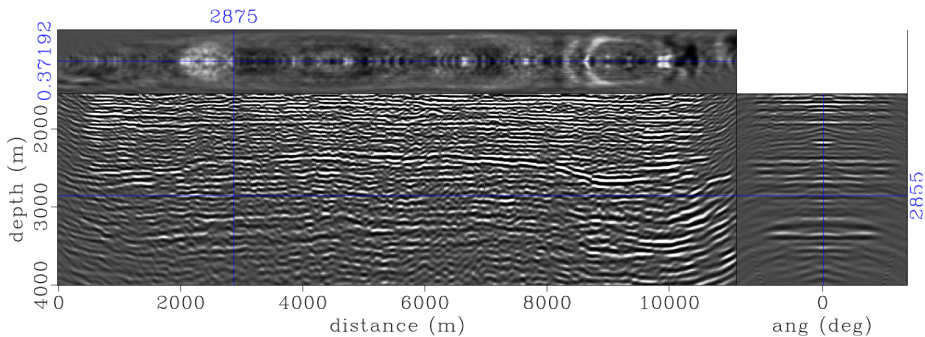
Baseline: Radon model



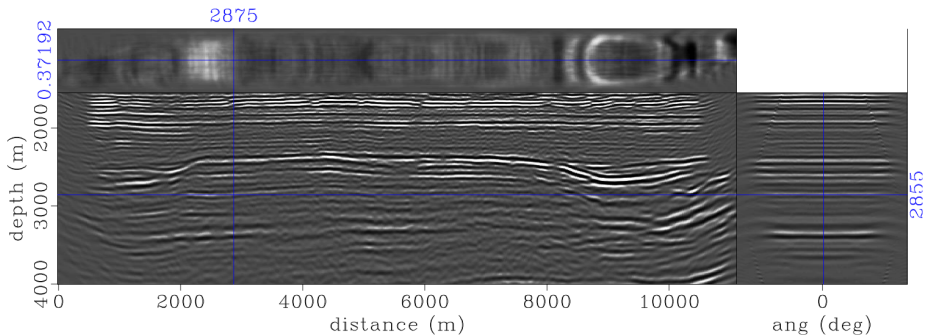
Baseline: Radon model



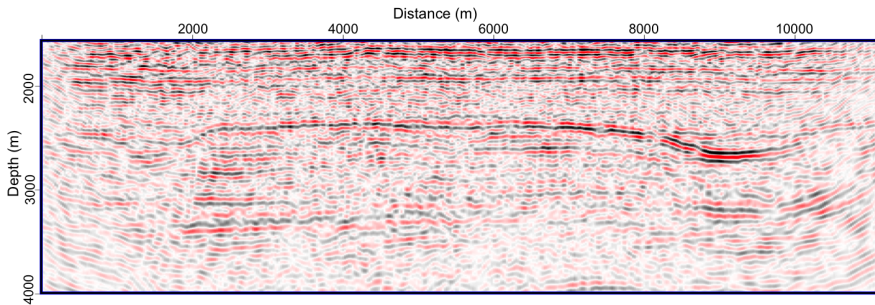
Baseline: Prestack image before processing



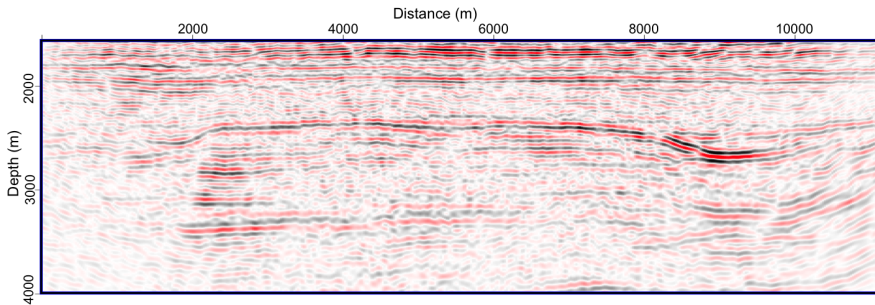
Baseline: Prestack image after processing



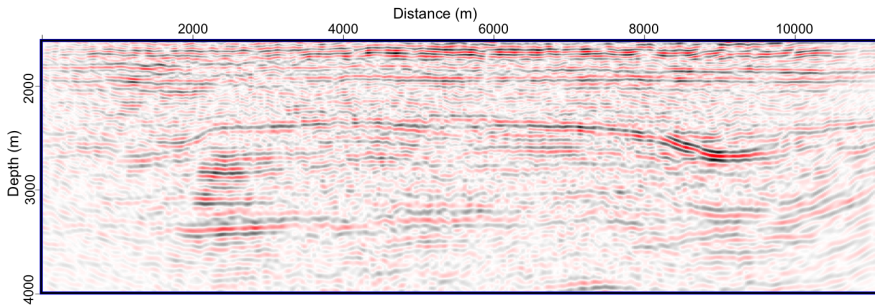
Time-lapse image: raw



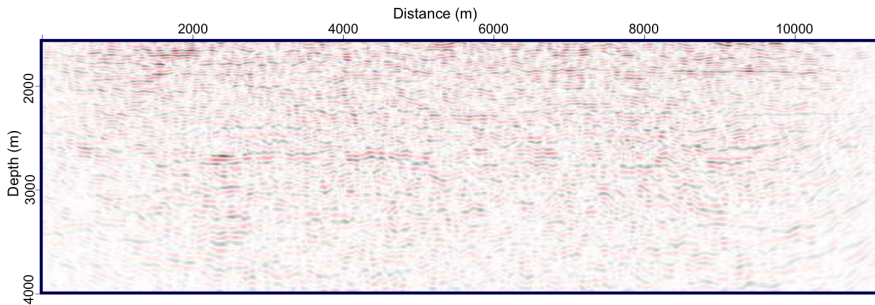
Time-lapse image: demultiple



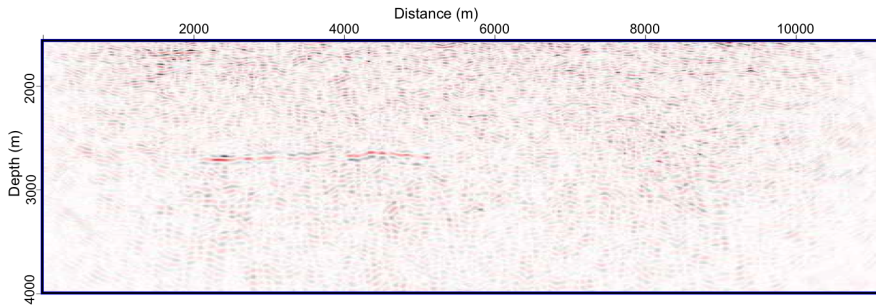
Time-lapse image: amplitude correction



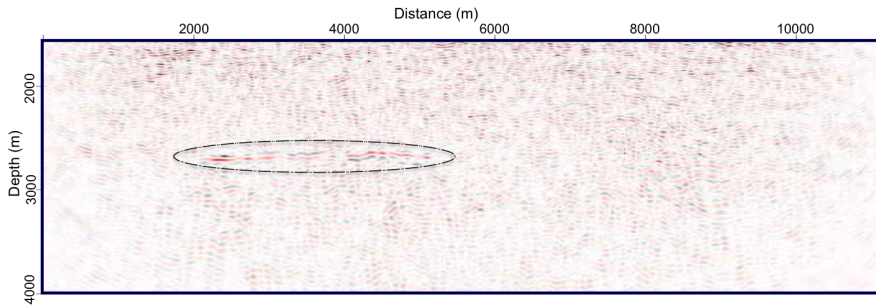
Time-lapse image: warping (prestack+ poststack)



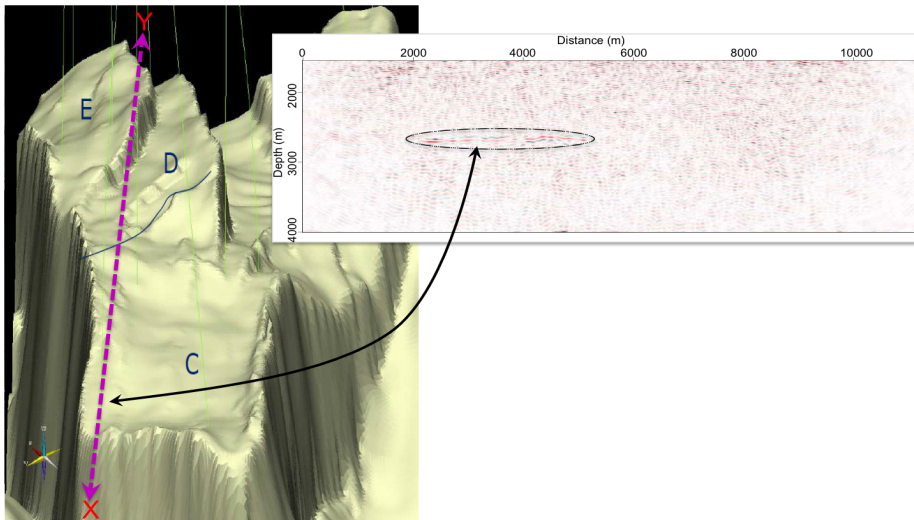
Time-lapse image: inversion



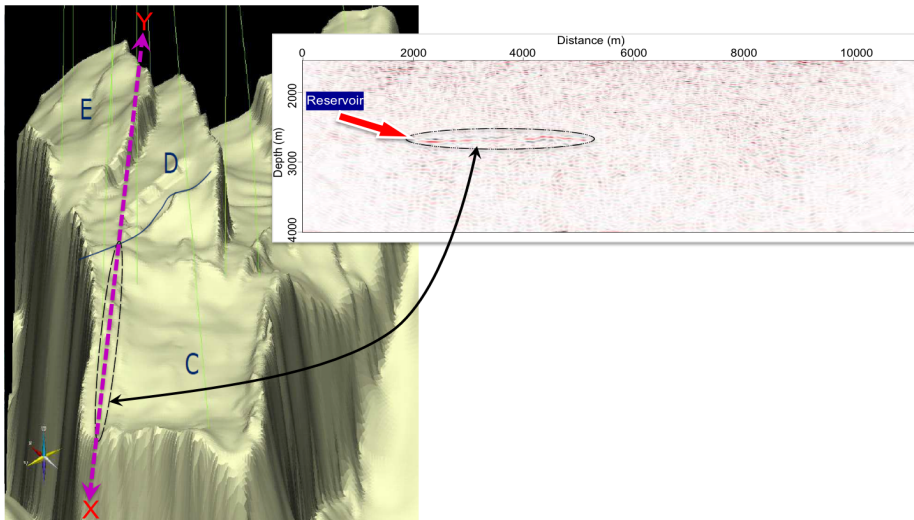
Time-lapse image: inversion



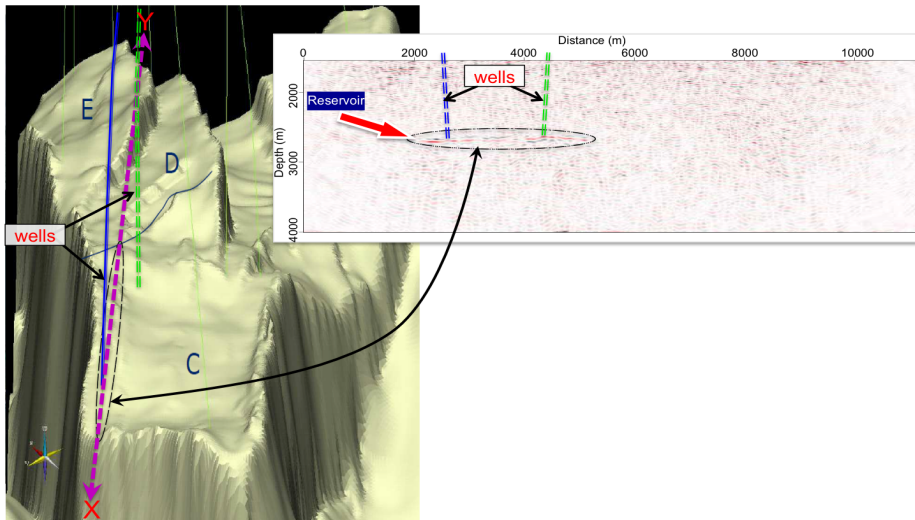
Time-lapse image: inversion



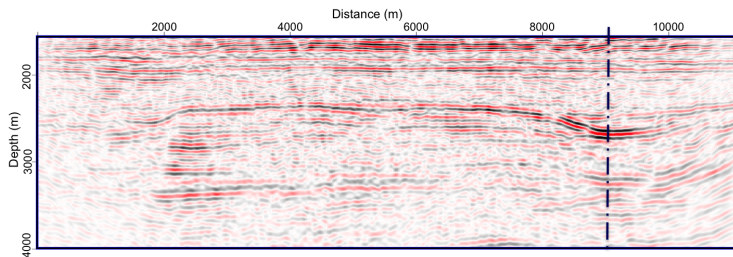
Time-lapse image: inversion



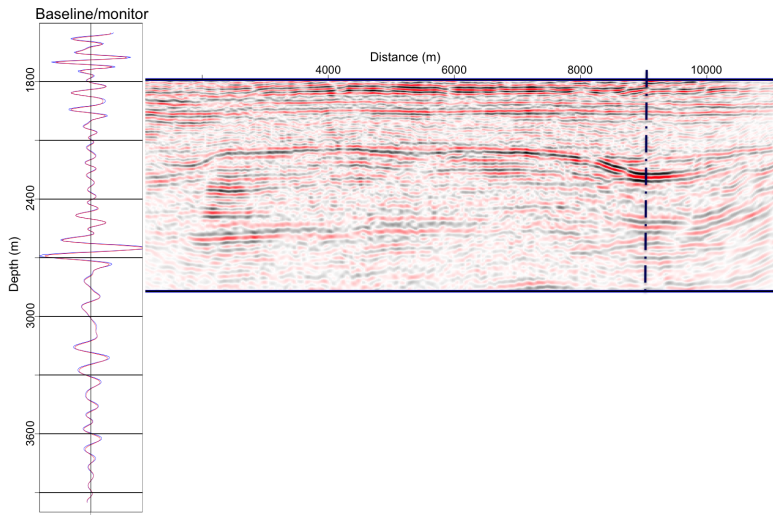
Time-lapse image: inversion



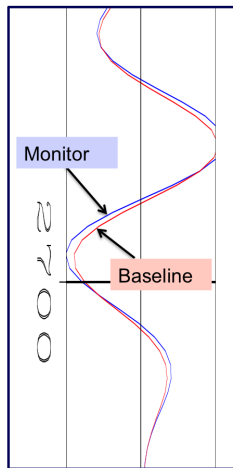
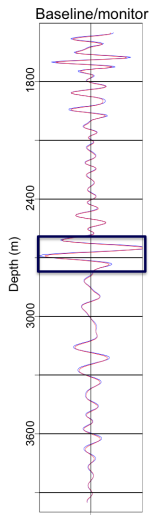
Time-lapse requires careful processing/interpretation!



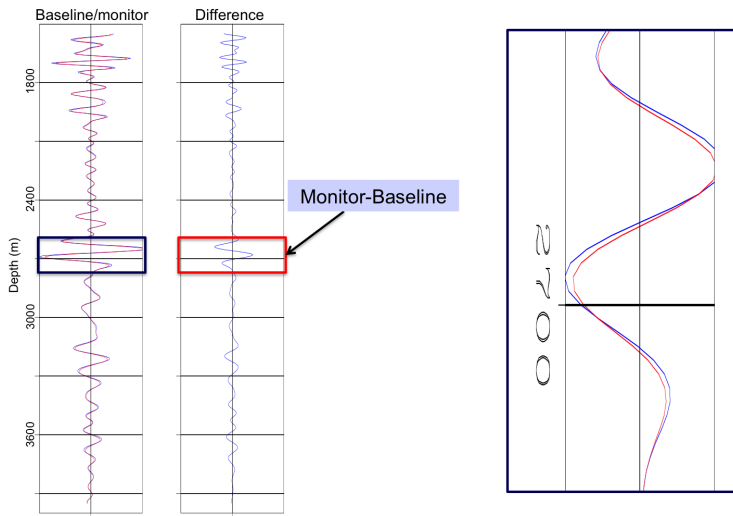
Raw time-lapse – Artifacts



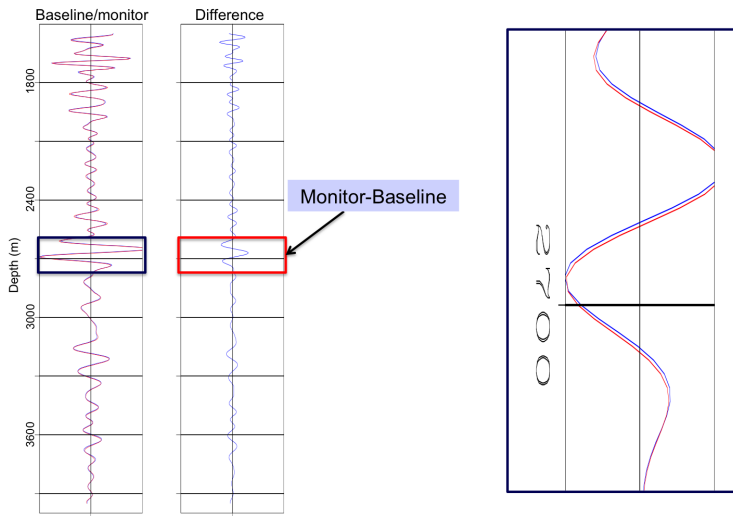
Raw time-lapse – Artifacts



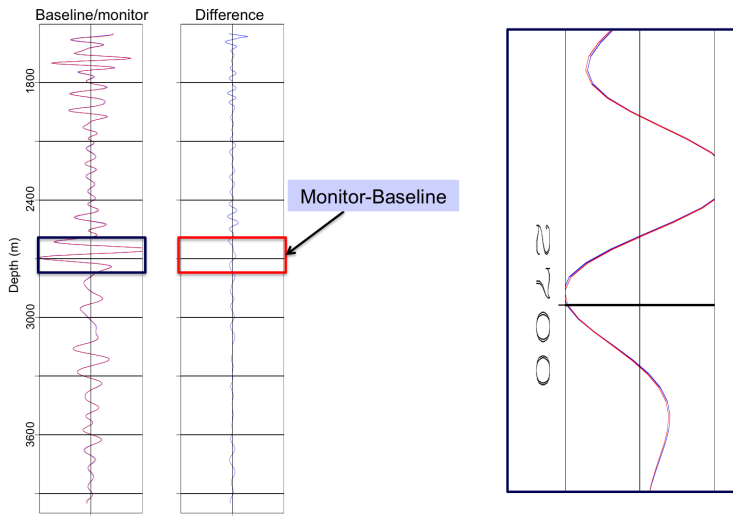
After multiple attenuation



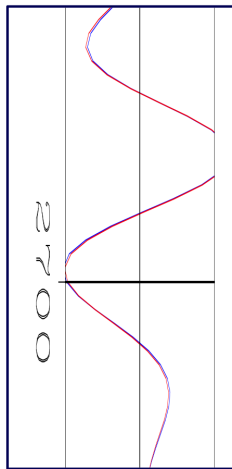
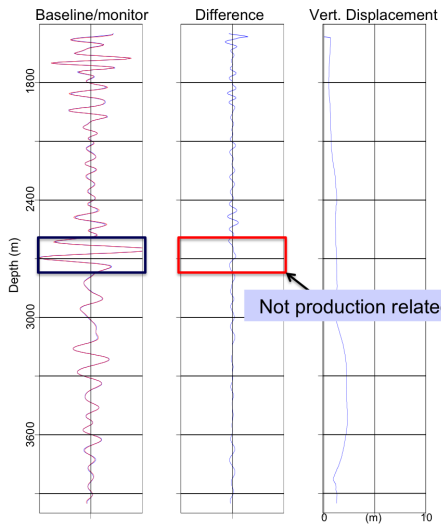
After amplitude balancing



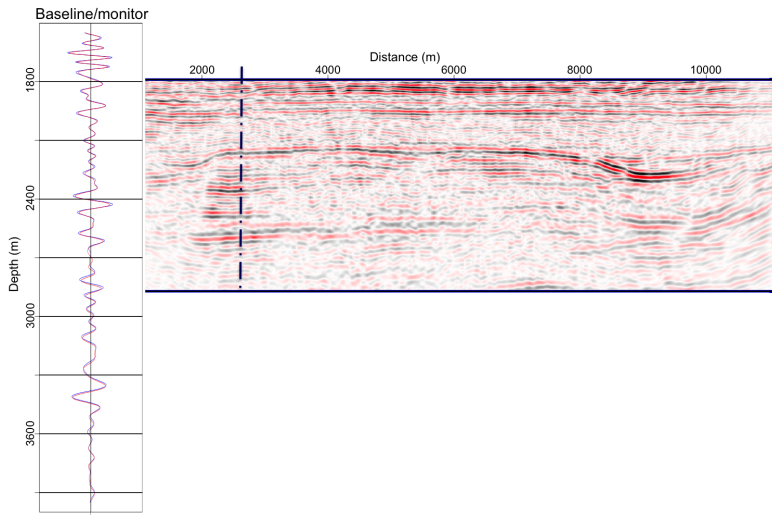
After warping



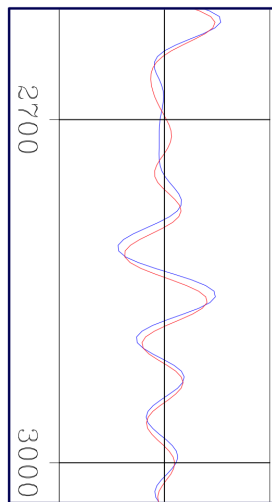
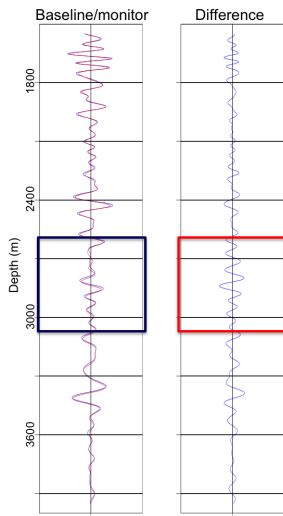
After warping



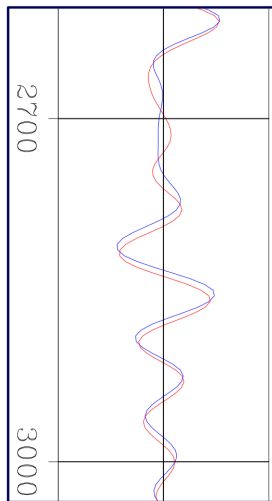
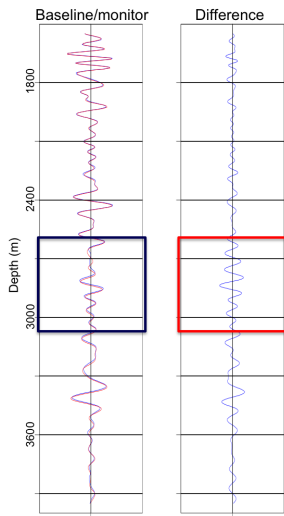
Raw time-lapse – Reservoir



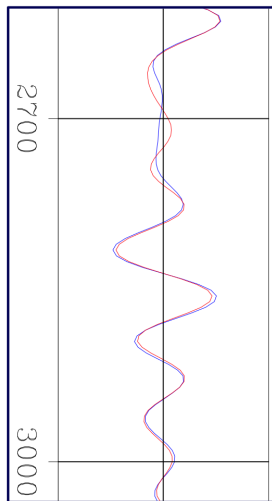
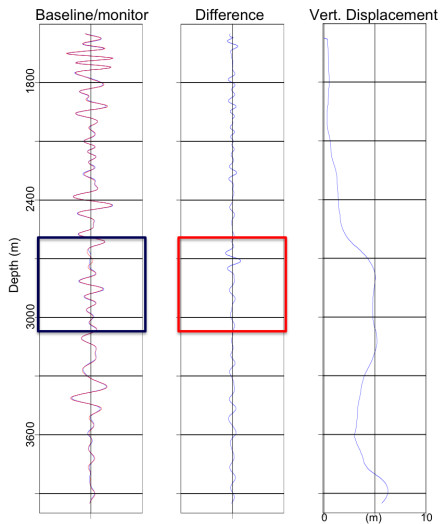
After multiple attenuation



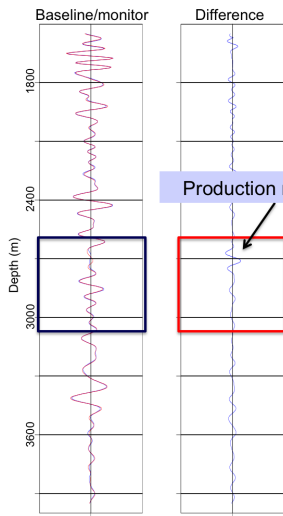
After amplitude balancing



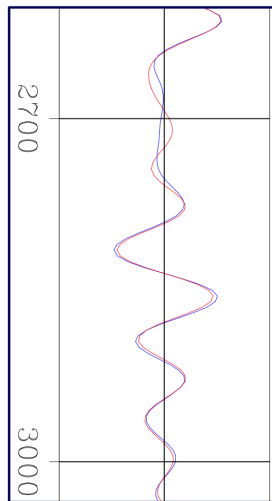
After warping



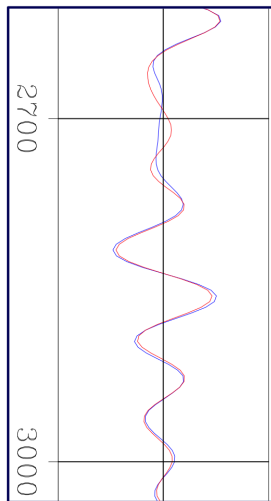
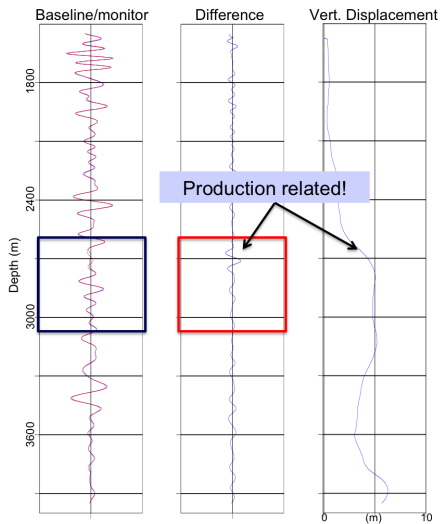
After warping



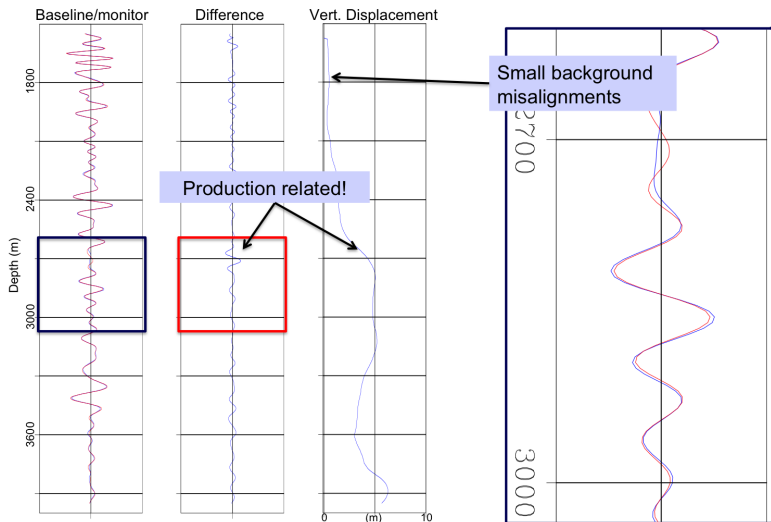
Production related!



After warping

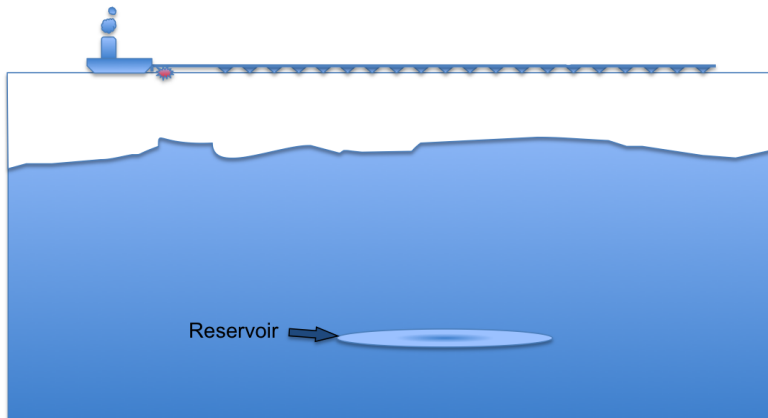


After warping



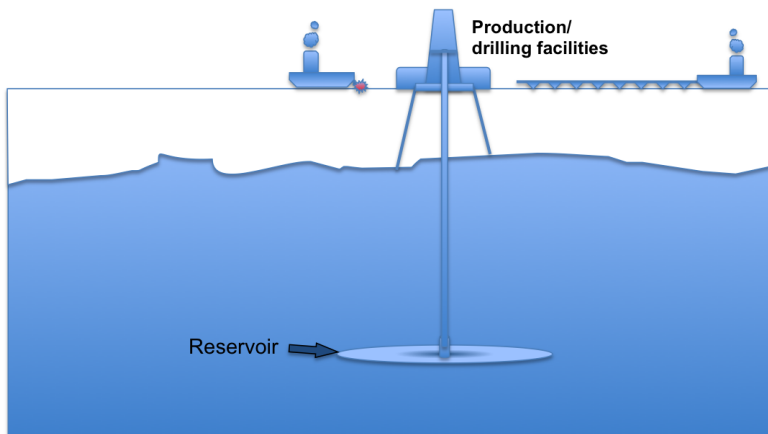
Complete baseline

Baseline

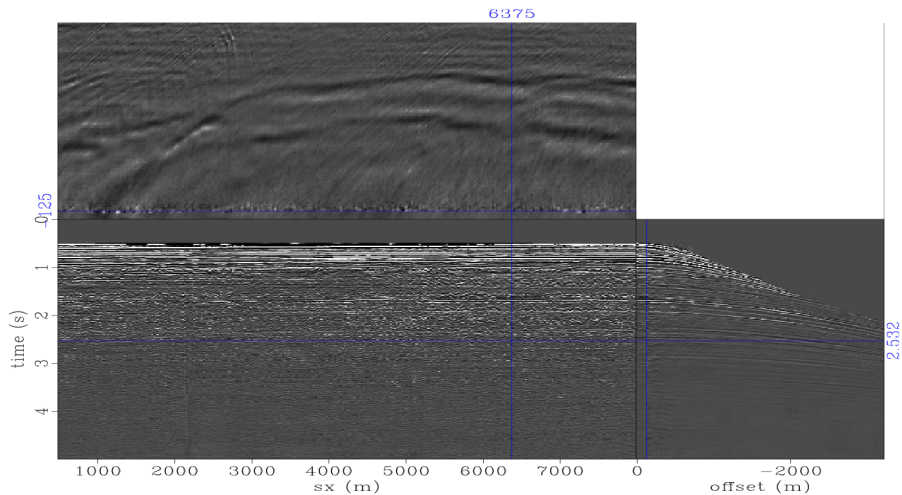


Gapped monitor

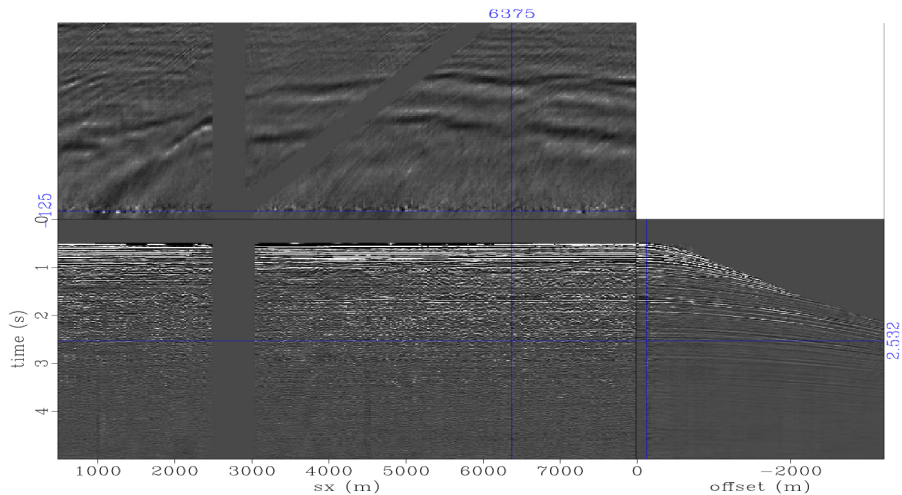
Monitor



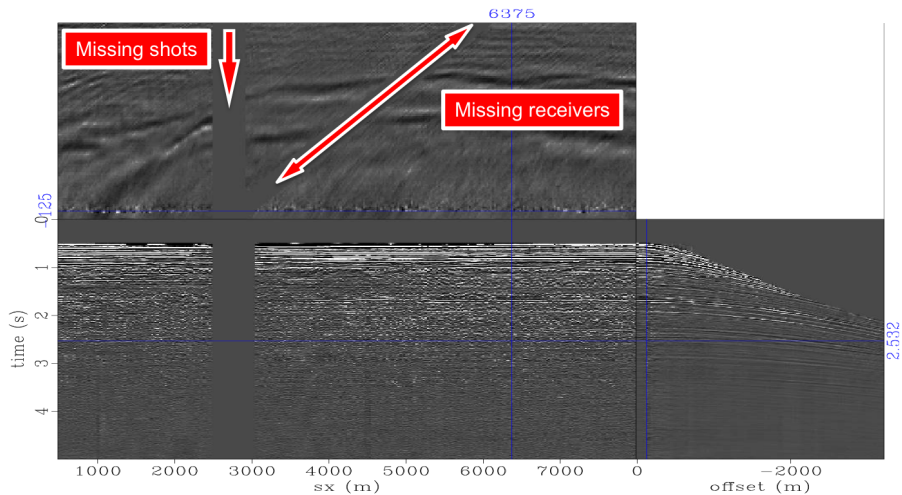
Data: Complete baseline



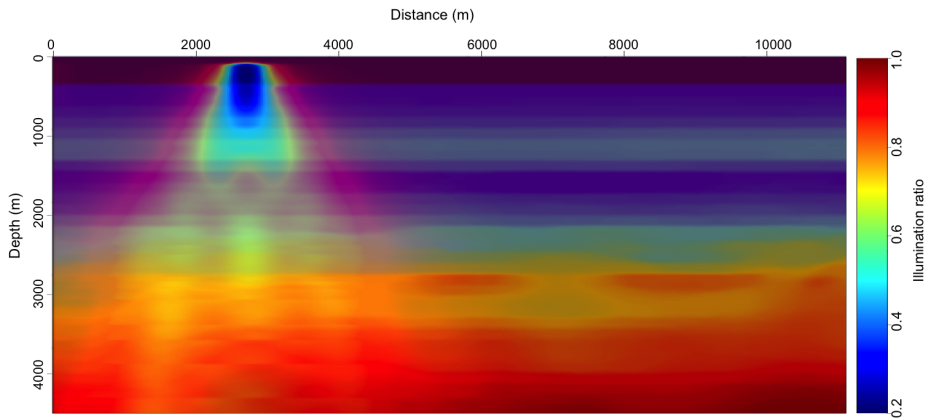
Data: Gapped monitor



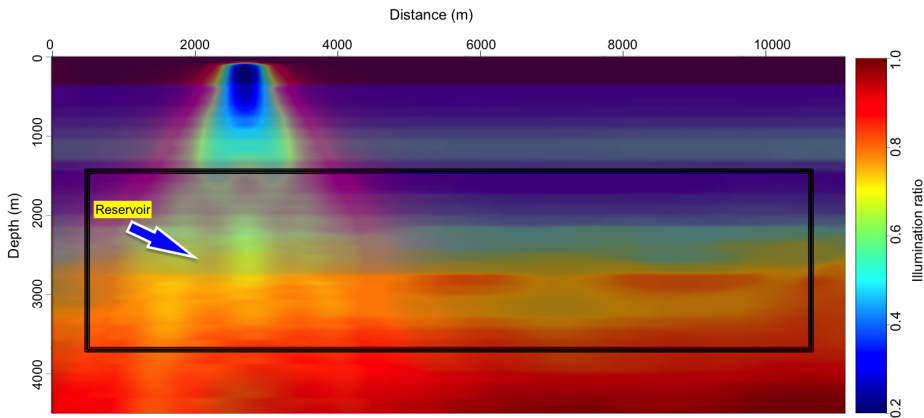
Data: Gapped monitor



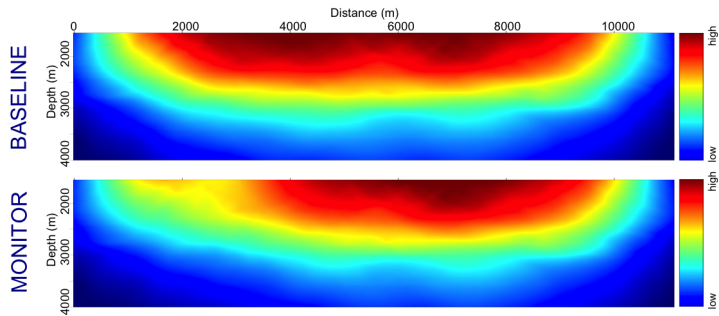
Illumination ratio: monitor/base



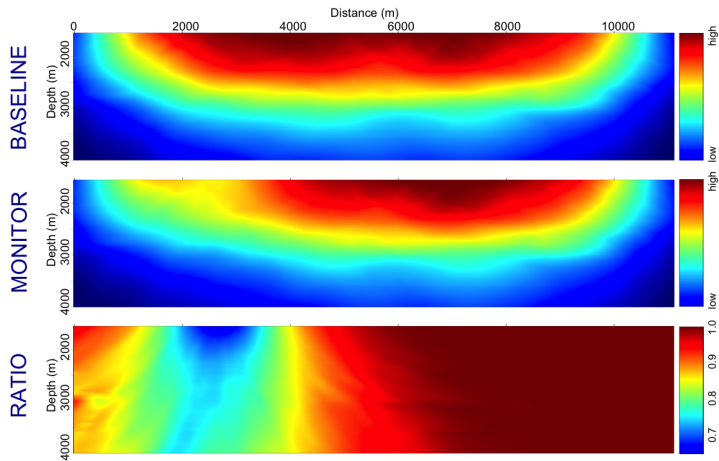
Illumination ratio: monitor/base



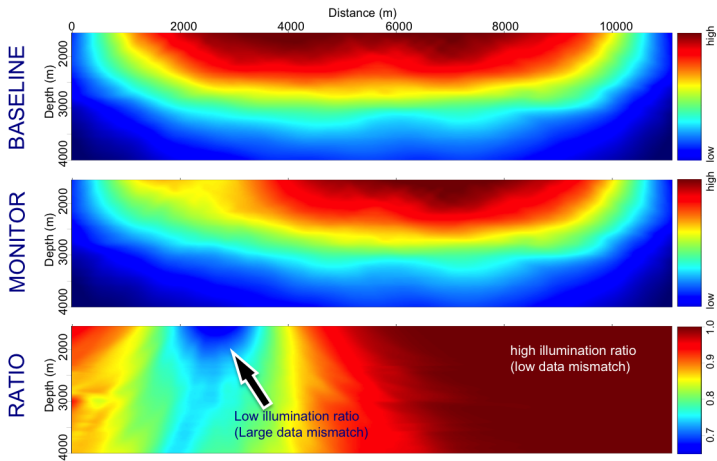
Illumination ratio: monitor/base



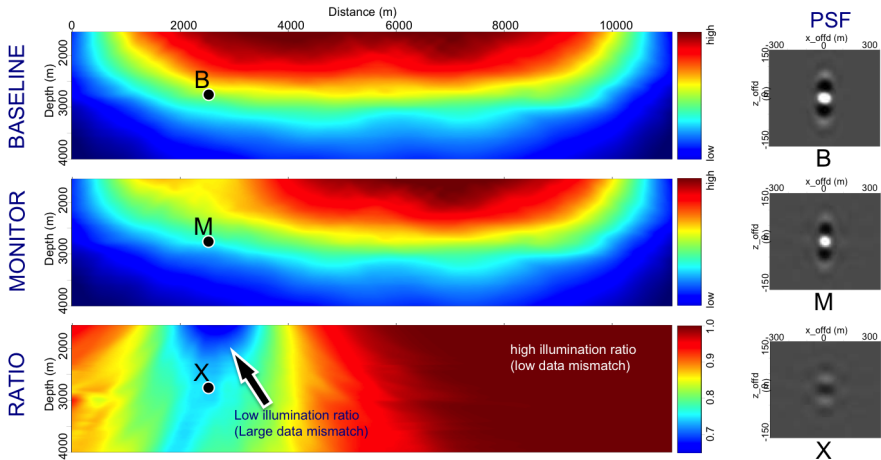
Illumination ratio: monitor/base



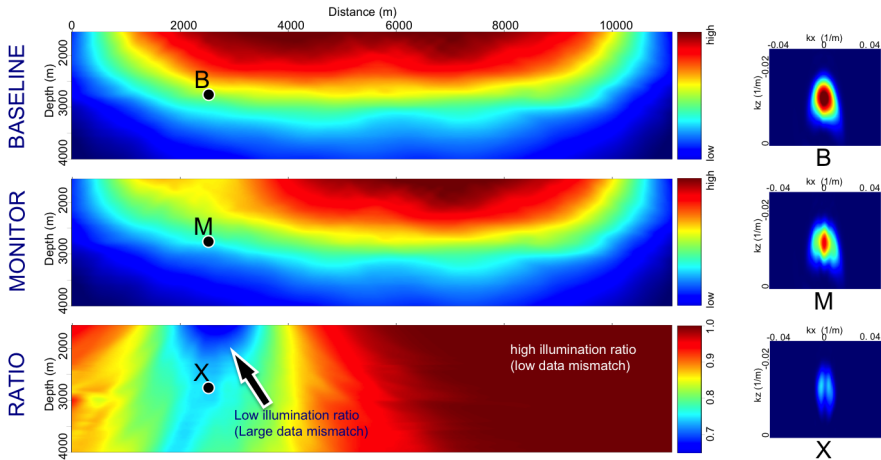
Illumination ratio: monitor/base



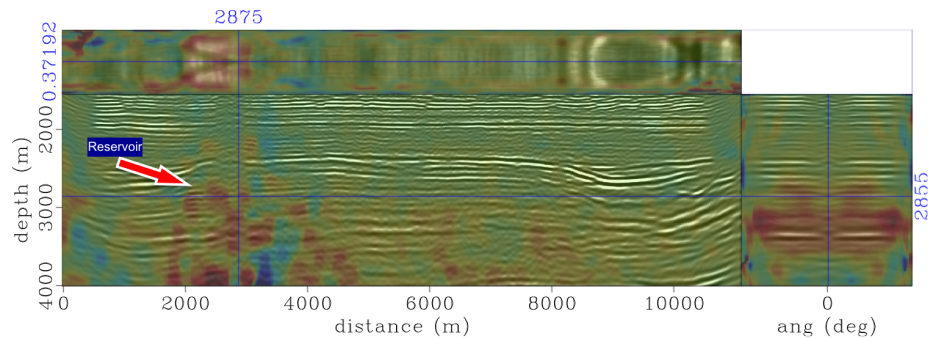
Illumination ratio: monitor/base



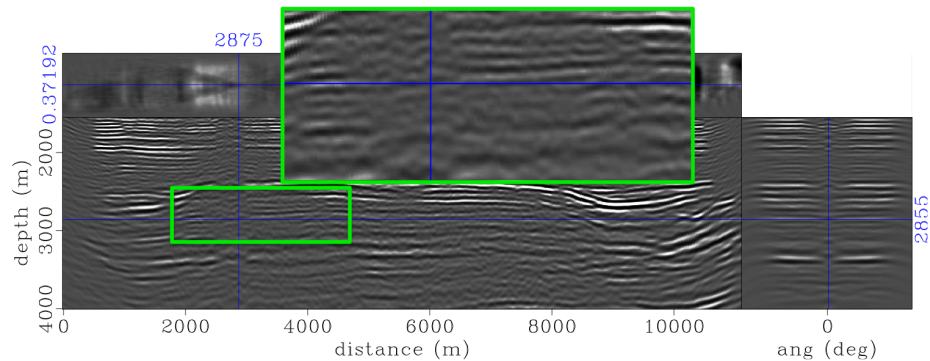
Illumination ratio: monitor/base



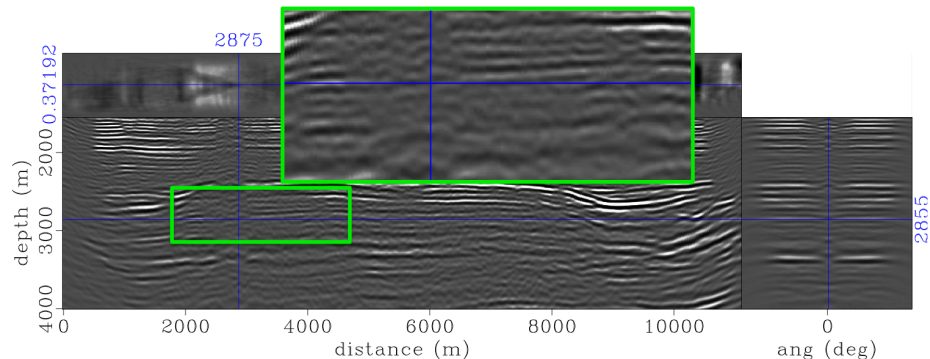
Prestack warping: vertical components



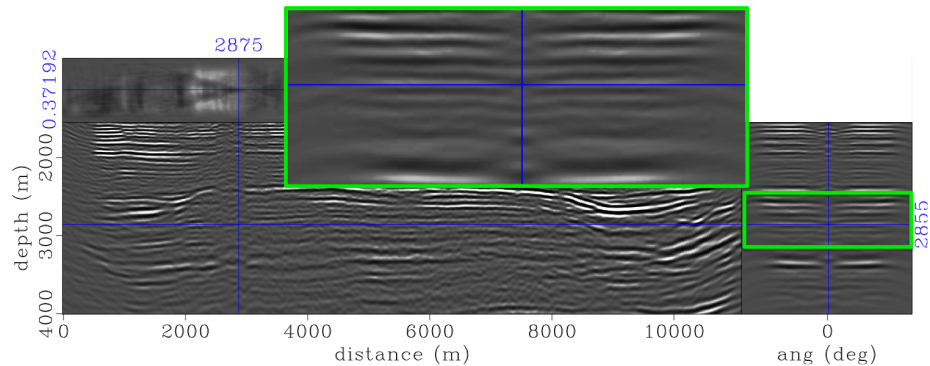
Prestack warping: before



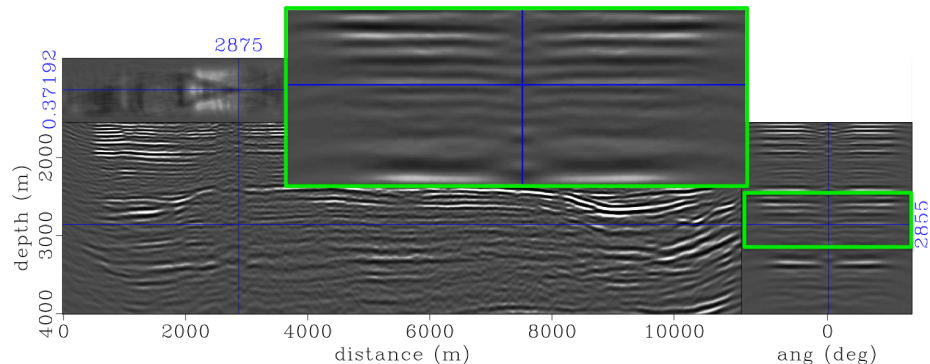
Prestack warping: after



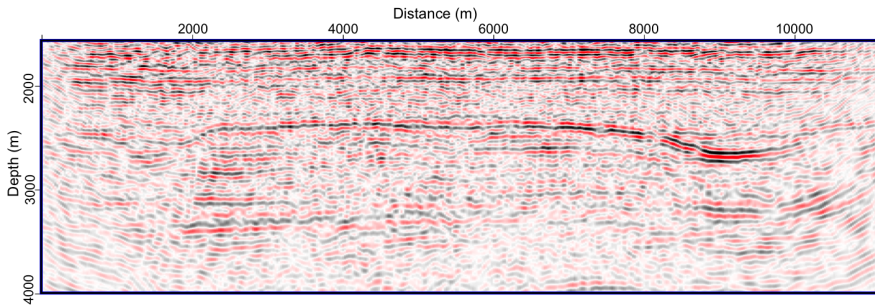
Prestack warping: before



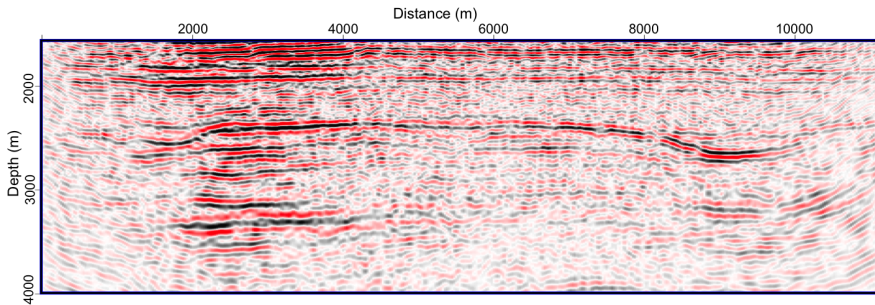
Prestack warping: after



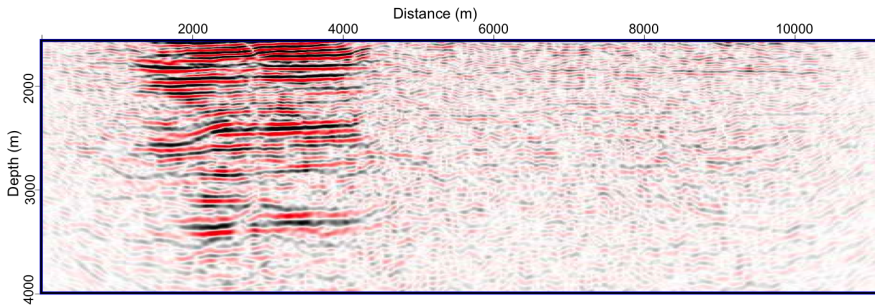
Time-lapse image: raw (complete data)



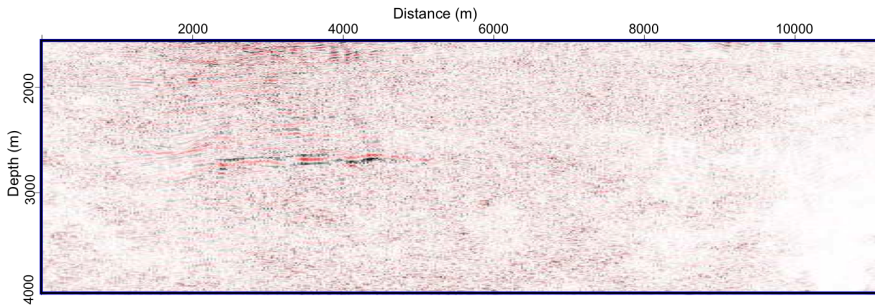
Time-lapse image: raw (gapped data)



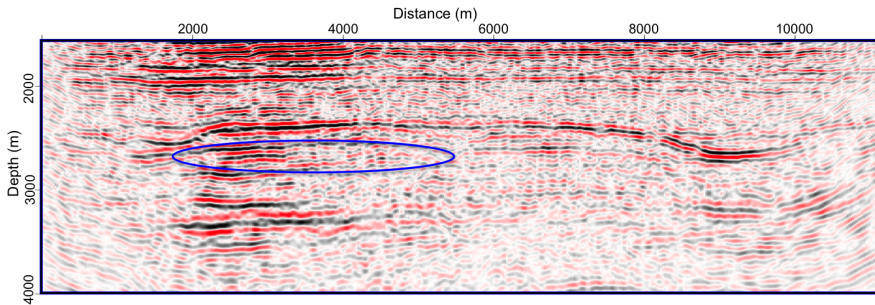
Time-lapse image: after all processing (gapped data)



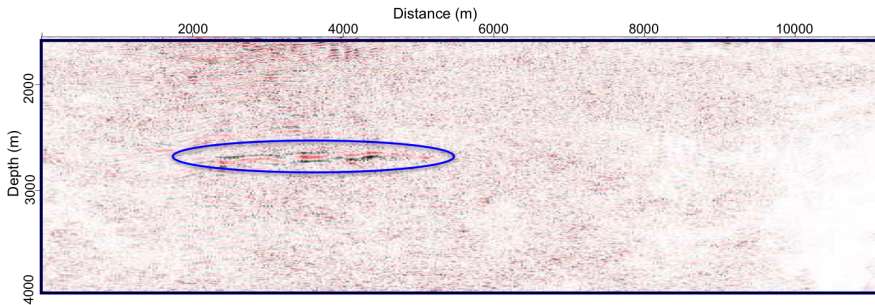
Time-lapse image: after WE inversion (gapped data)



Time-lapse image: raw (gapped data)



Time-lapse image: after WE inversion (gapped data)



Conclusions

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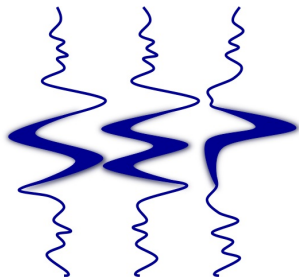
Acknowledgements

- We thank Statoil and partners (ENI and Petoro) for donating the data sets through the NTNU/IO CENTRE.
- We thank the Stanford Center for Computational Earth & Environmental Science for providing the computer resources used in this study.

Ongoing work

- Application to full-azimuth 3D time-lapse data set

Thanks



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