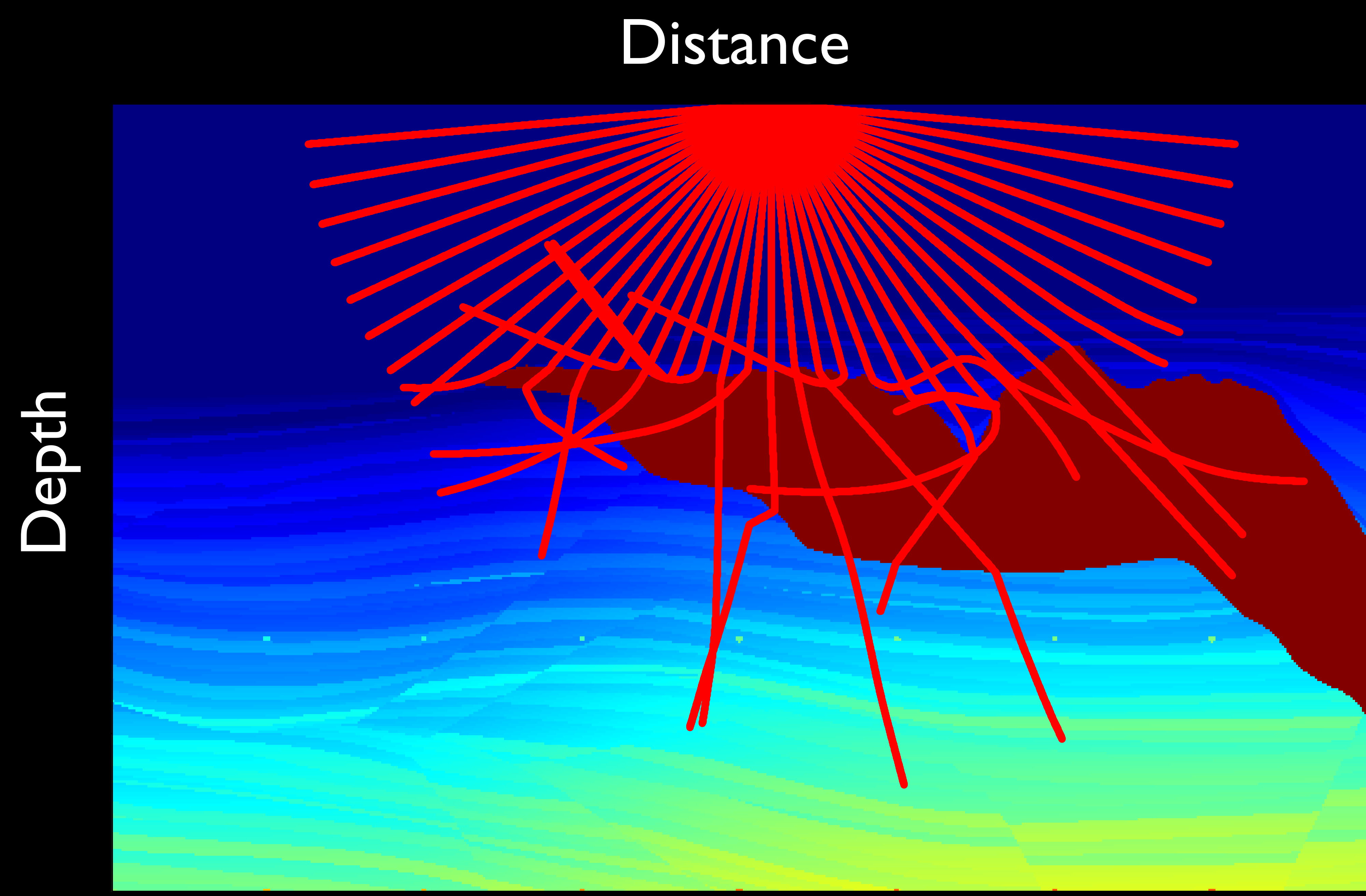
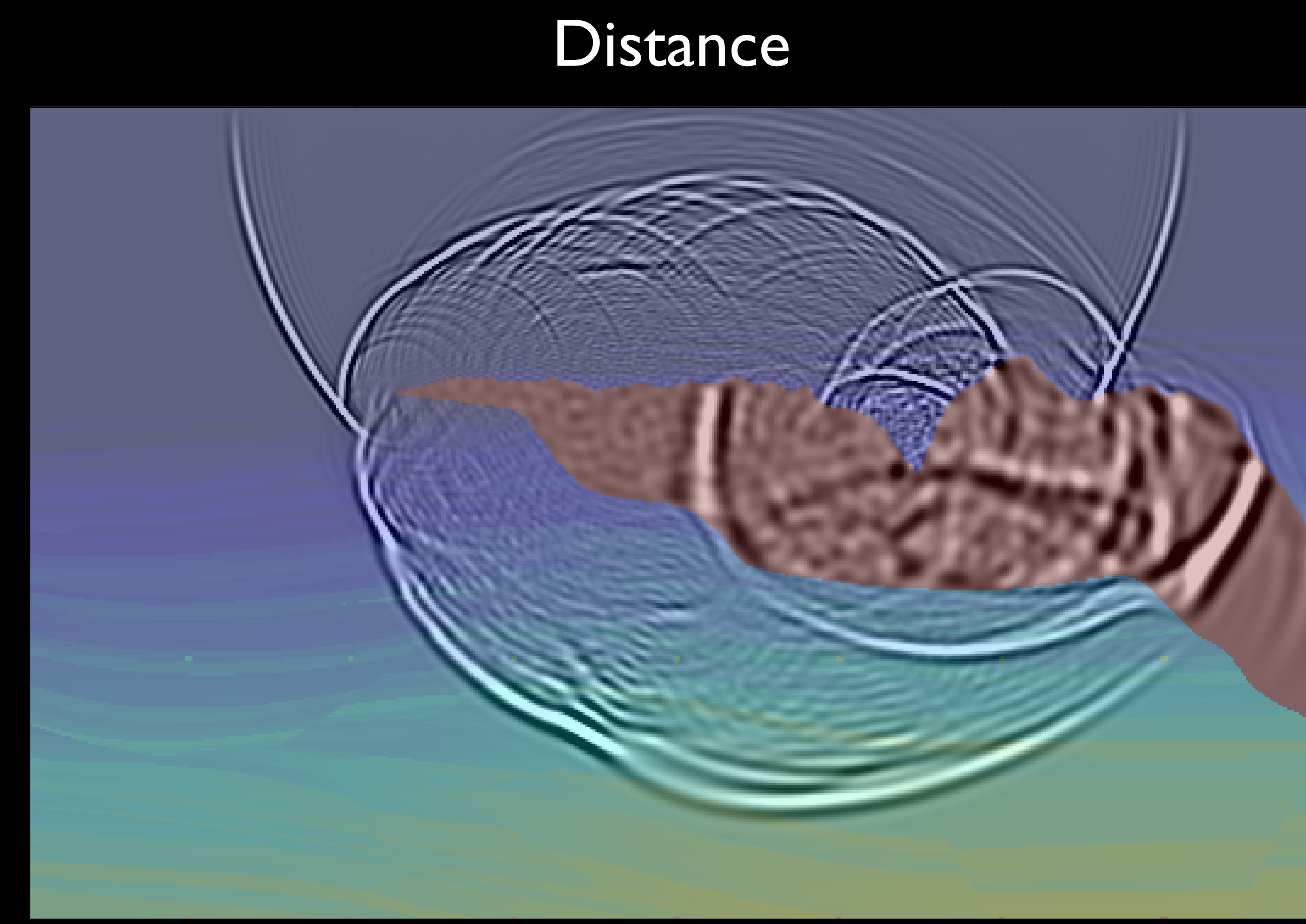
Angle-domain illumination gathers by wave-equation-based methods

Yaxun Tang and Biondo Biondi

SEP-140, pp. 67

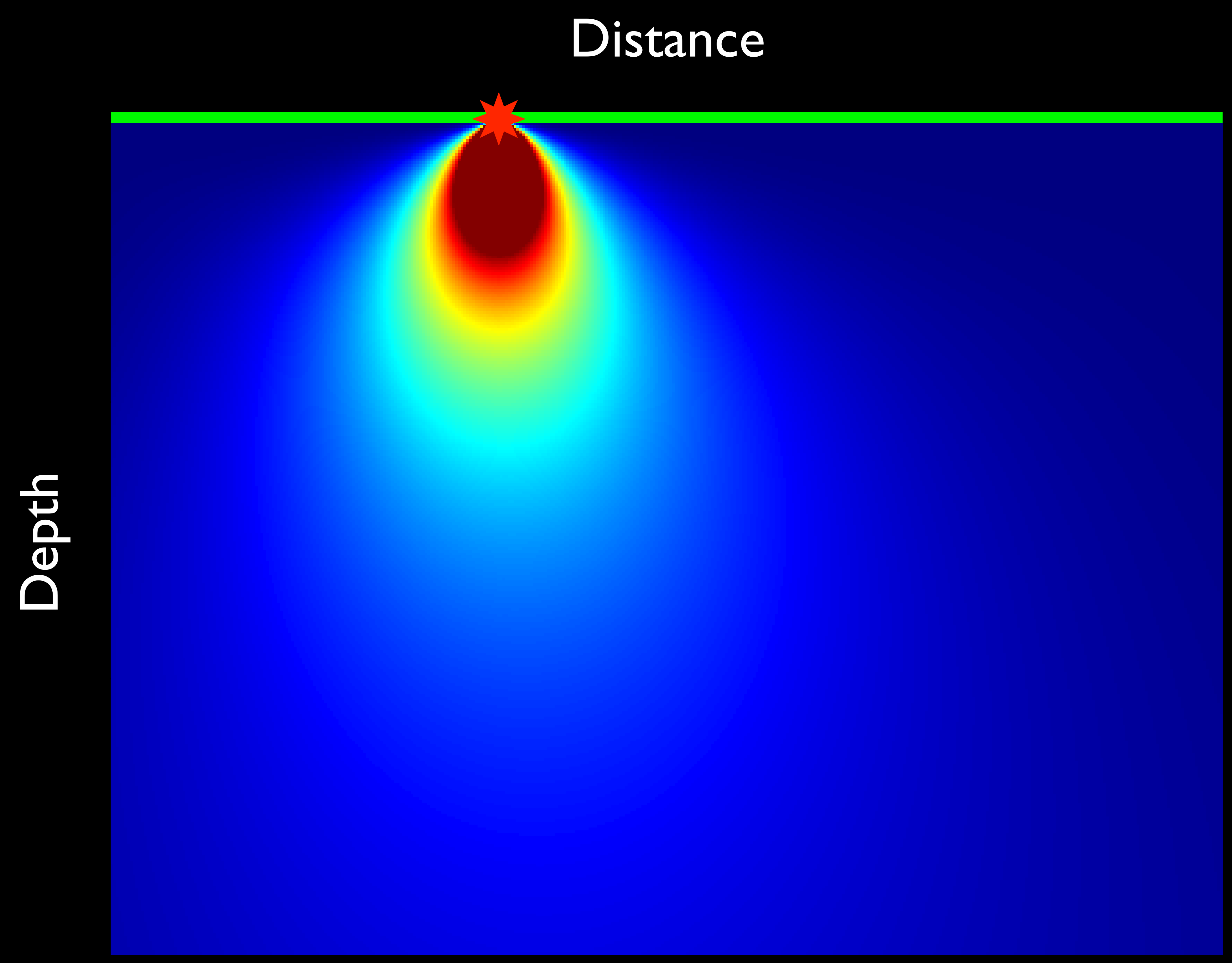
Rays vs. waves





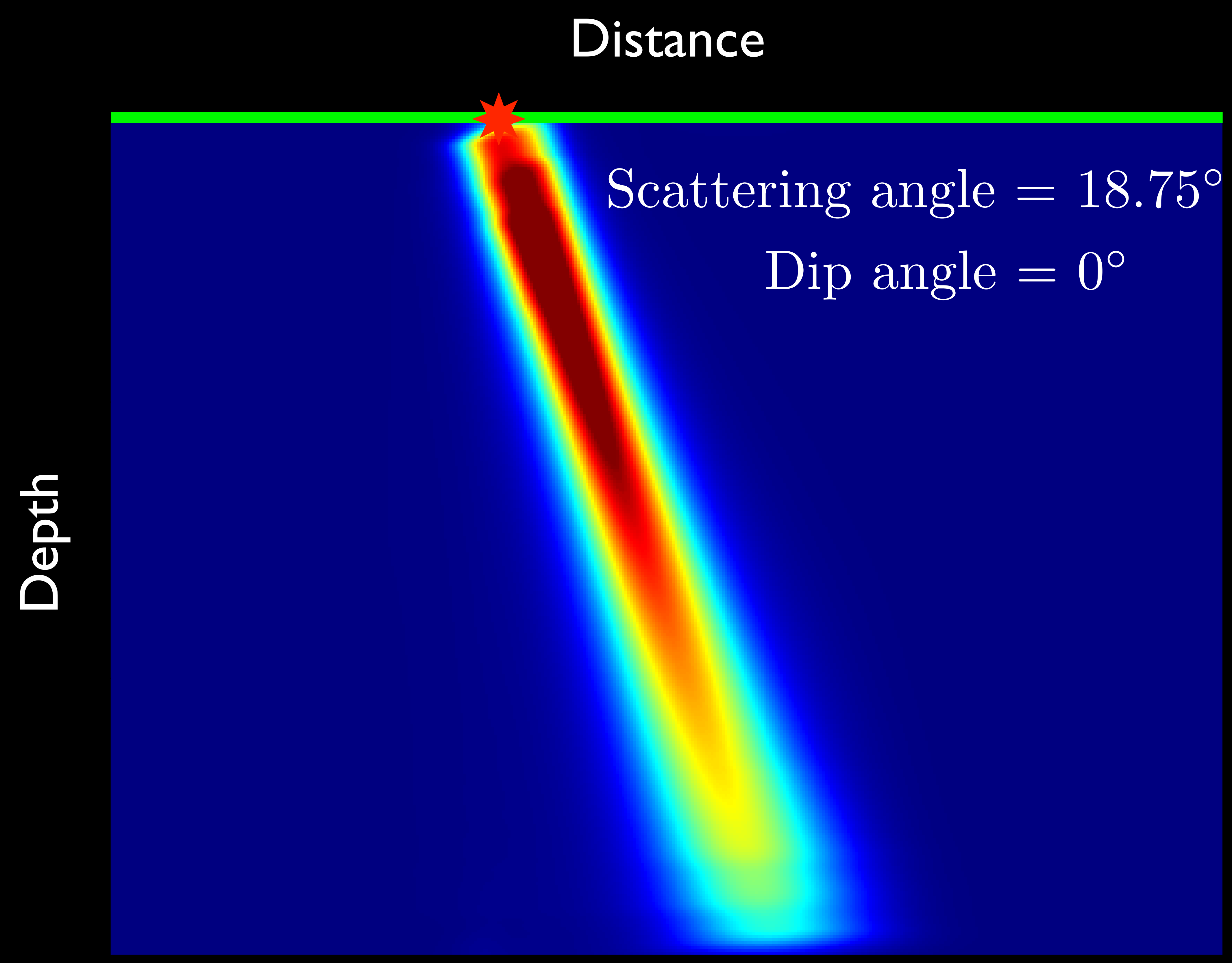
Wavefield-based methods become necessary for complex geologies

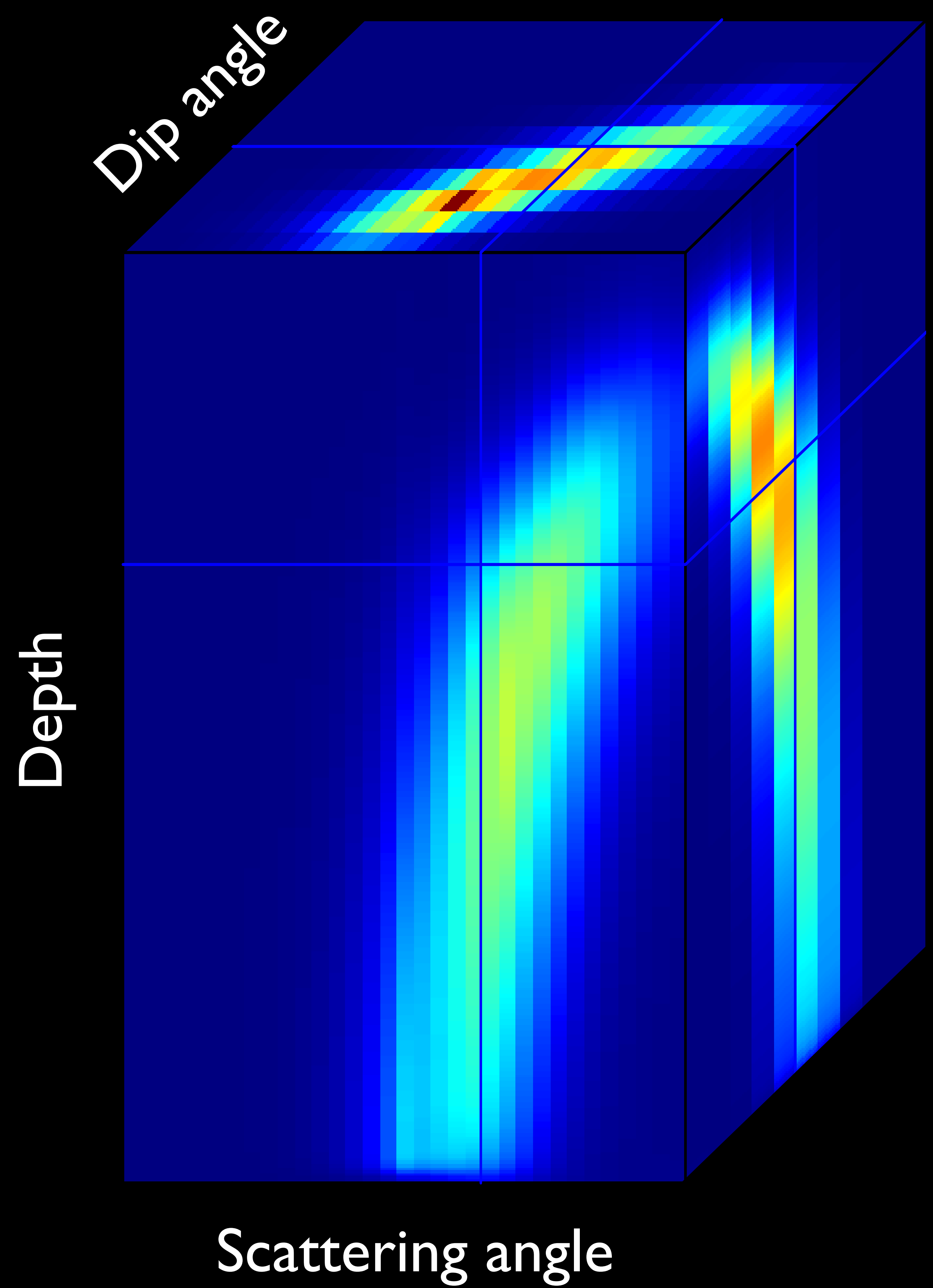
Conventional wave-equation illumination analysis



Stacked illumination

Wave-equation angle-domain illumination analysis





Potential applications

- Accurate AVA analysis
- Robust residual parameter estimation
- Optimum seismic survey planning
- ...

Agenda

- Motivation
- Subsurface-offset-domain illumination
- Angle-domain illumination
- Phase-encoded angle-domain illumination
- Examples

Linearized wave equation

Prestack Born modeling:

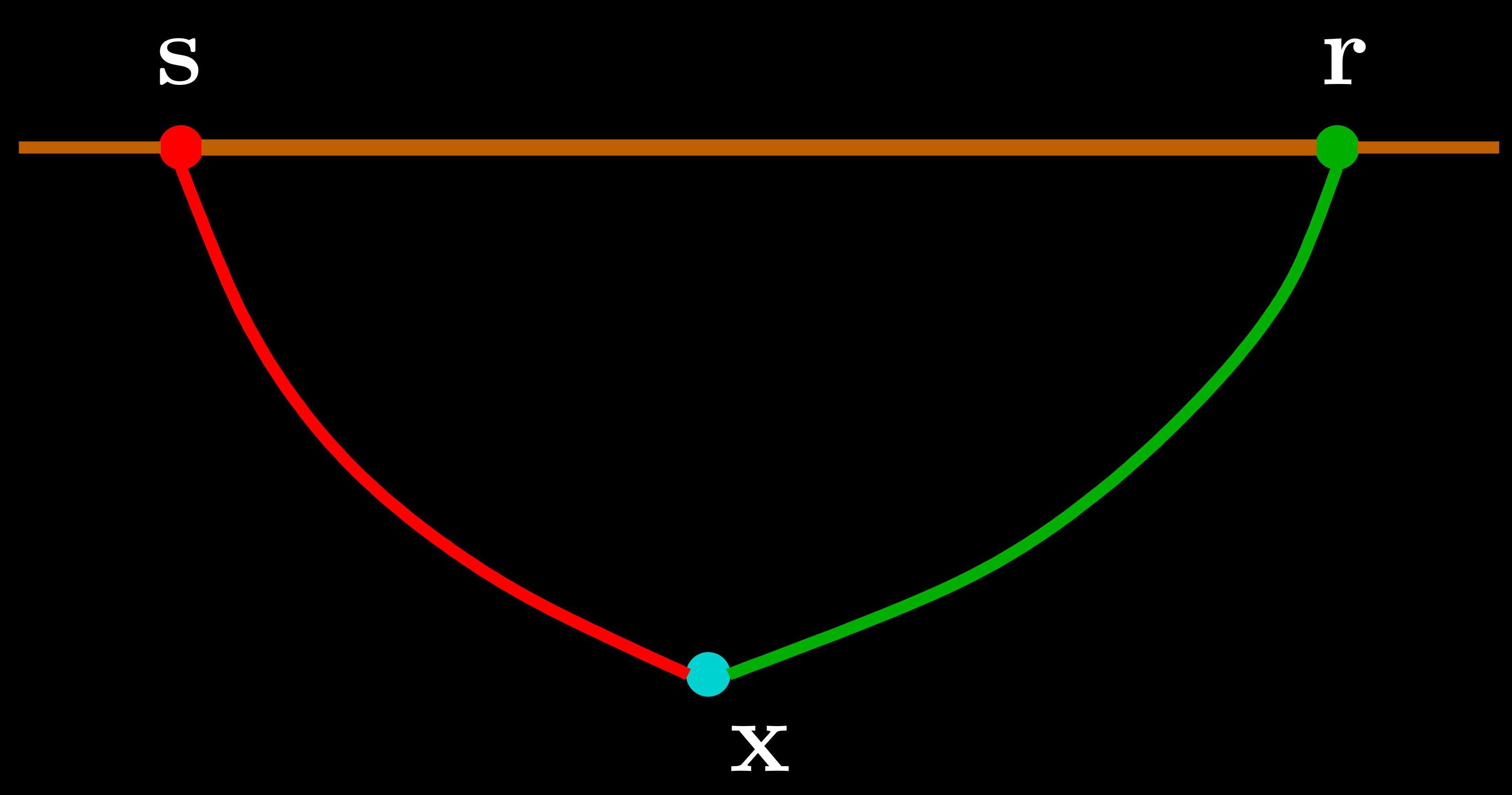
$$d(\mathbf{r},\mathbf{s},\omega) = \sum_{\mathbf{x}} \sum_{\mathbf{h}} L_h(\mathbf{x},\mathbf{h},\mathbf{r},\mathbf{s},\omega) m_h(\mathbf{x},\mathbf{h})$$

$$L_h(\mathbf{x},\mathbf{h},\mathbf{r},\mathbf{s},\omega) = G(\mathbf{x}-\mathbf{h},\mathbf{s},\omega)G(\mathbf{x}+\mathbf{h},\mathbf{r},\omega)$$

d is the modeled data

m_h is the reflectivity

L_h is the sensitivity kernel of the reflectivity w.r.t. the data



Subsurface-offset-domain image and illumination

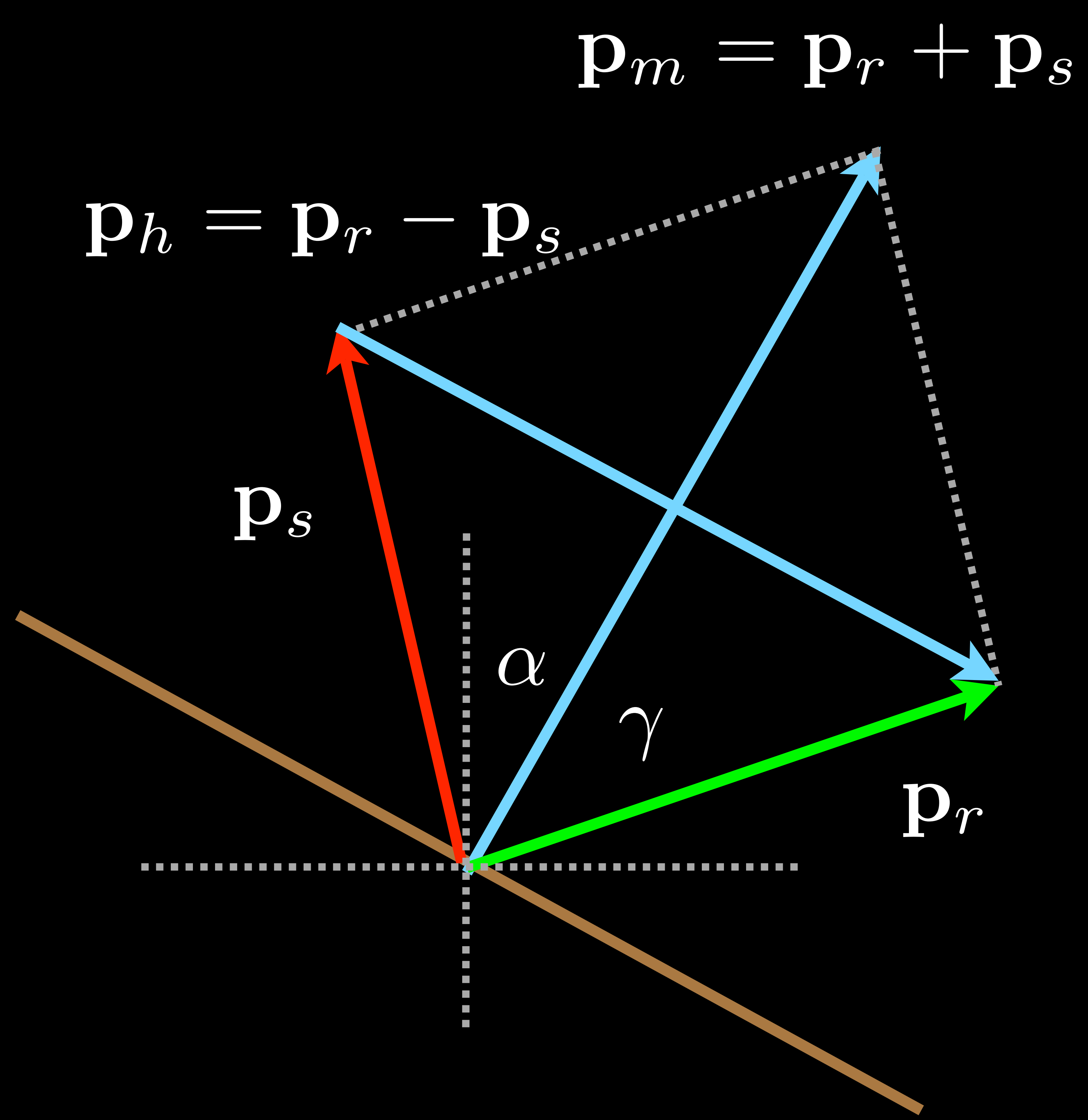
Migrated prestack image:

$$\widehat{m}_h(\mathbf{x},\mathbf{h}) = \sum_{\omega}\sum_{\mathbf{s}}\sum_{\mathbf{r}} L_h^*(\mathbf{x},\mathbf{h},\mathbf{r},\mathbf{s},\omega)d_{\text{obs}}(\mathbf{r},\mathbf{s},\omega)$$

Subsurface-offset-domain illumination:

$$H_h(\mathbf{x},\mathbf{h}) = \sum_{\omega}\sum_{\mathbf{s}}\sum_{\mathbf{r}} |L_h(\mathbf{x},\mathbf{h},\mathbf{r},\mathbf{s},\omega)|^2$$

Geometric relations in 2-D



A locally constant slowness medium

$$\mathbf{p}_m = \begin{pmatrix} p_{m_x} \\ p_{m_z} \end{pmatrix} = \begin{pmatrix} 2s\cos\gamma\sin\alpha \\ -2s\cos\gamma\cos\alpha \end{pmatrix}$$

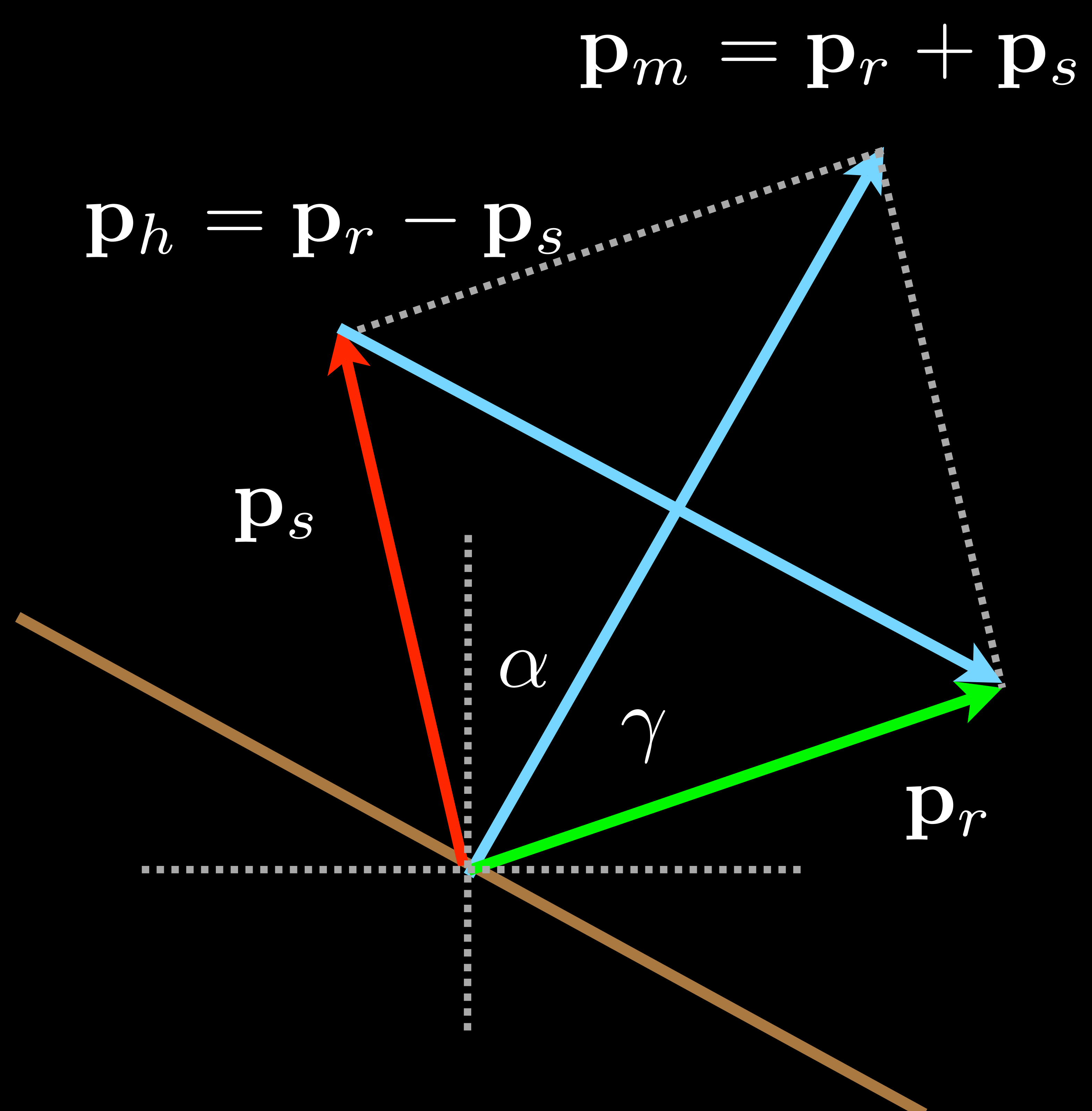
$$\mathbf{p}_h = \begin{pmatrix} p_{h_x} \\ p_{h_z} \end{pmatrix} = \begin{pmatrix} 2s\sin\gamma\cos\alpha \\ 2s\sin\gamma\sin\alpha \end{pmatrix}$$

γ is the scattering angle

α is the dip angle

s is the local slowness

Geometric relations



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$$\mathbf{p}_h = \begin{pmatrix} p_{h_x} \\ p_{h_z} \end{pmatrix} = \begin{pmatrix} 2s\sin\gamma\cos\alpha \\ 2s\sin\gamma\sin\alpha \end{pmatrix}$$

$$\tan\gamma = -\frac{p_{h_x}}{p_{m_z}} = -\frac{k_{h_x}}{k_{m_z}}$$

Sava and Fomel, 2003

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega)$$

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega) \xrightarrow{\text{2D FT}} L_h(x, k_z, k_{h_x}, \mathbf{r}, \mathbf{s}, \omega)$$

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega) \xrightarrow{\text{2D FT}} L_h(x, k_z, k_{h_x}, \mathbf{r}, \mathbf{s}, \omega)$$

$$\downarrow \quad \tan\gamma = -\frac{k_{h_x}}{k_z}$$

$$L_\gamma(x, k_z, \gamma, \mathbf{r}, \mathbf{s}, \omega)$$

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega) \xrightarrow{\text{2D FT}} L_h(x, k_z, k_{h_x}, \mathbf{r}, \mathbf{s}, \omega)$$

$$\downarrow \quad \tan\gamma = -\frac{k_{h_x}}{k_z}$$

$$L_\gamma(x, z, \gamma, \mathbf{r}, \mathbf{s}, \omega) \xleftarrow{\text{1D IFT}} L_\gamma(x, k_z, \gamma, \mathbf{r}, \mathbf{s}, \omega)$$

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega) \xrightarrow{\text{2D FT}} L_h(x, k_z, k_{h_x}, \mathbf{r}, \mathbf{s}, \omega)$$

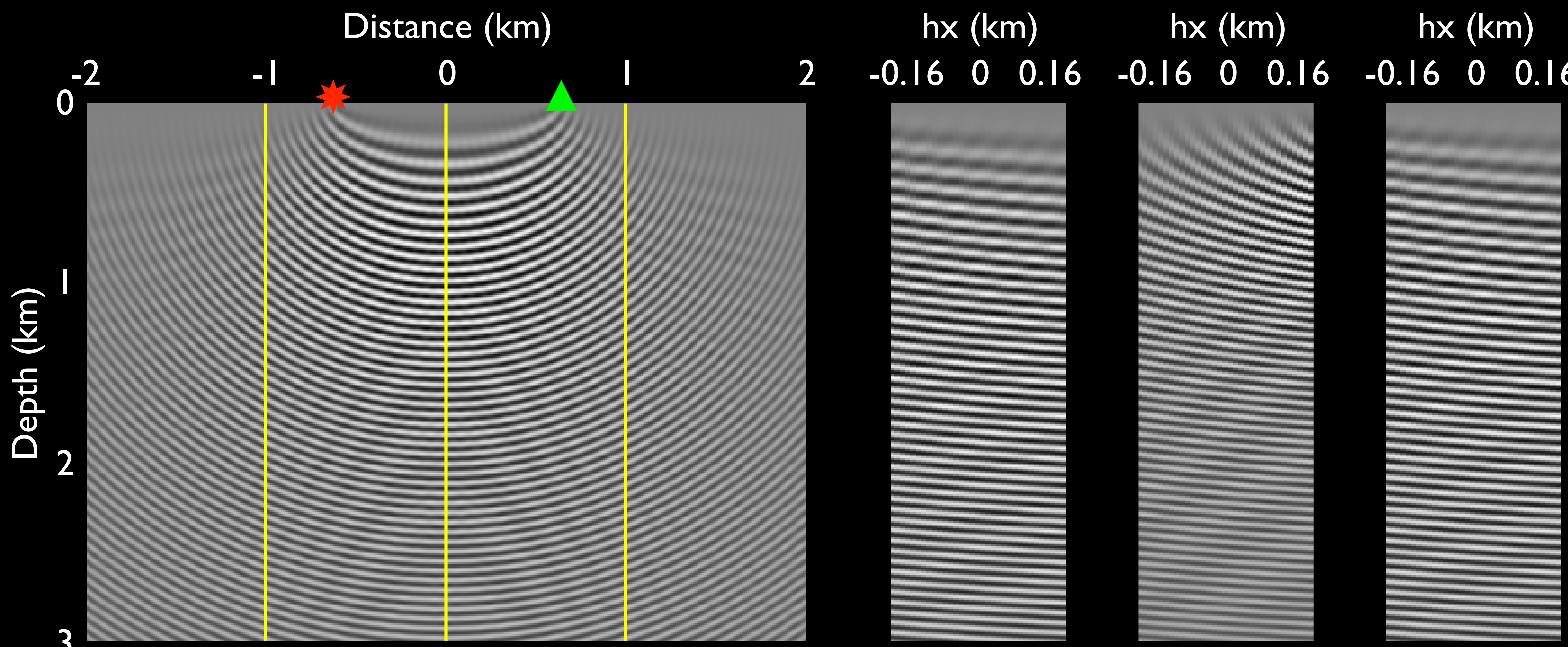
$$\downarrow \quad \tan\gamma = -\frac{k_{h_x}}{k_z}$$

$$L_\gamma(x, z, \gamma, \mathbf{r}, \mathbf{s}, \omega) \xleftarrow{\text{1D IFT}} L_\gamma(x, k_z, \gamma, \mathbf{r}, \mathbf{s}, \omega)$$

$$\downarrow$$

$$H_\gamma(x, z, \gamma) = \sum_\omega \sum_\mathbf{s} \sum_\mathbf{r} |L_\gamma(x, z, \gamma, \mathbf{r}, \mathbf{s}, \omega)|^2$$

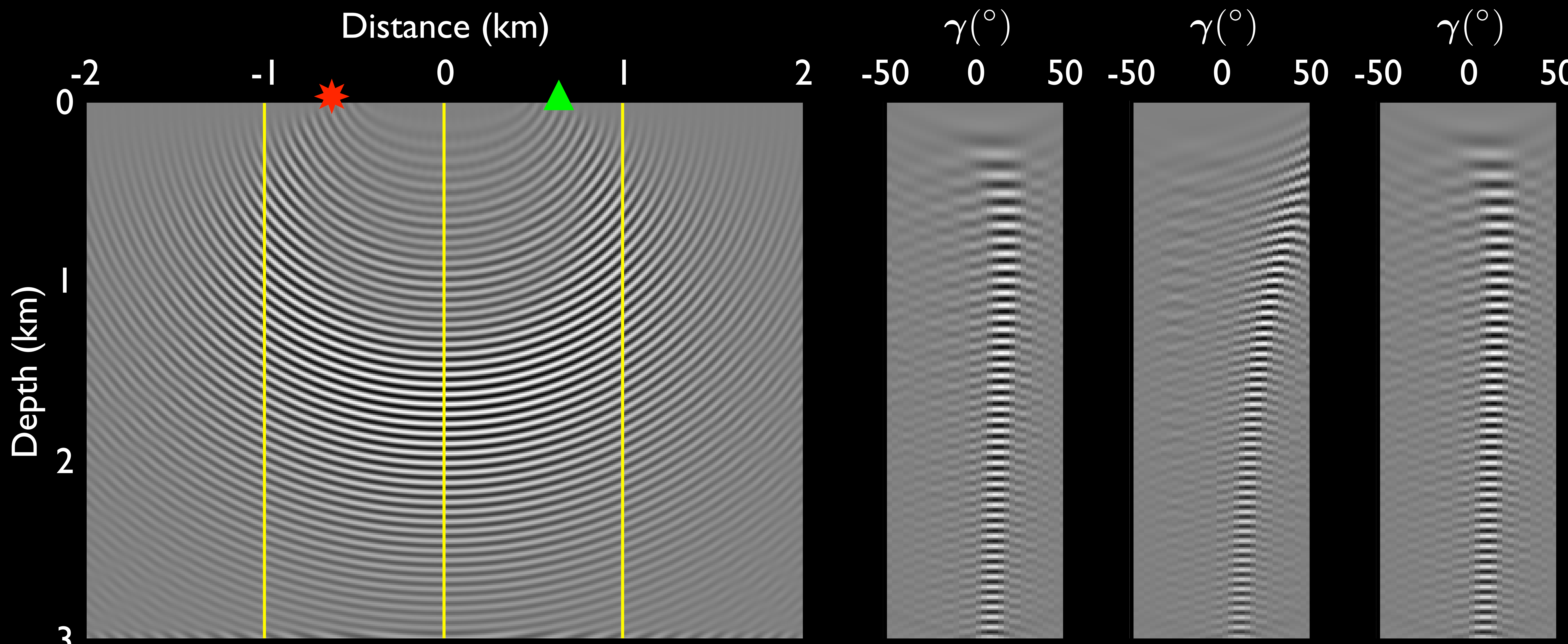
Subsurface-offset-domain sensitivity kernel



$h_x = 0$

Real part

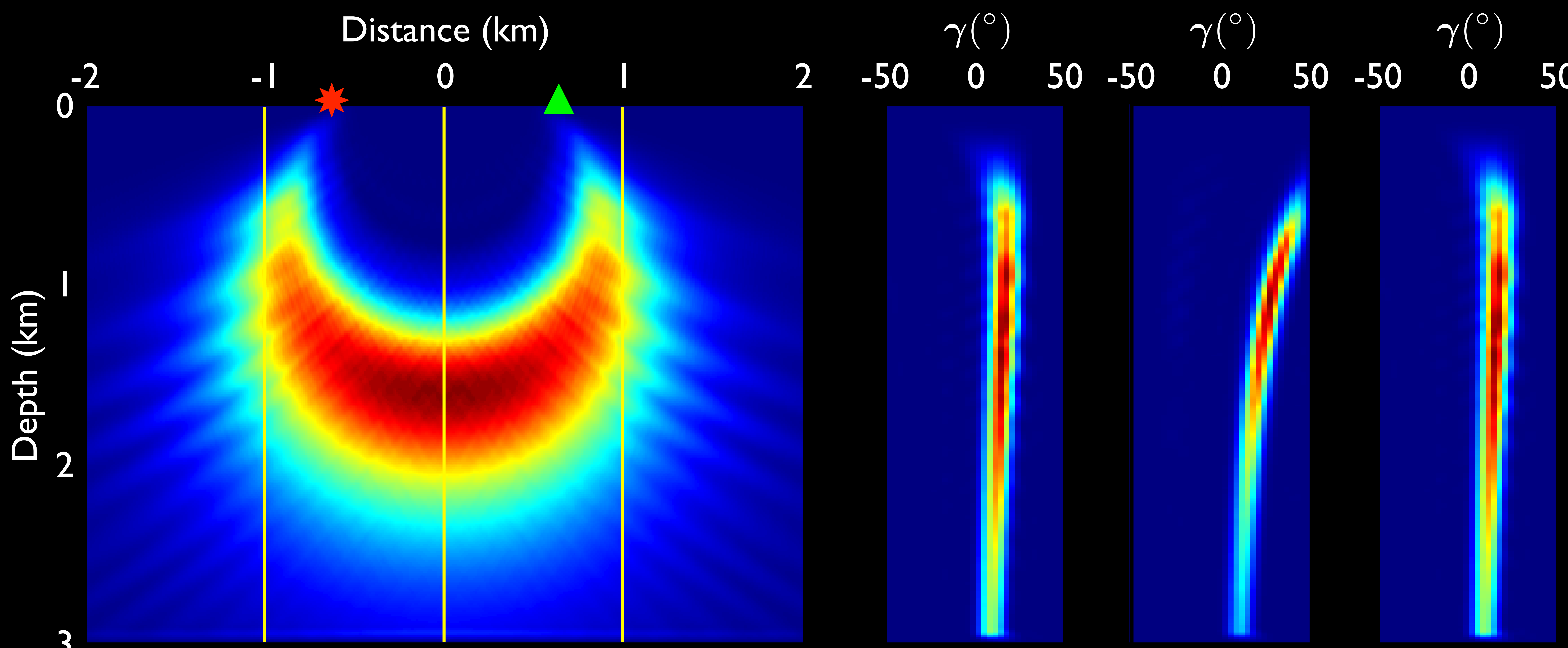
Scattering-angle-domain sensitivity kernel



$\gamma = 18.75^\circ$

Real part

Scattering-angle-domain illumination



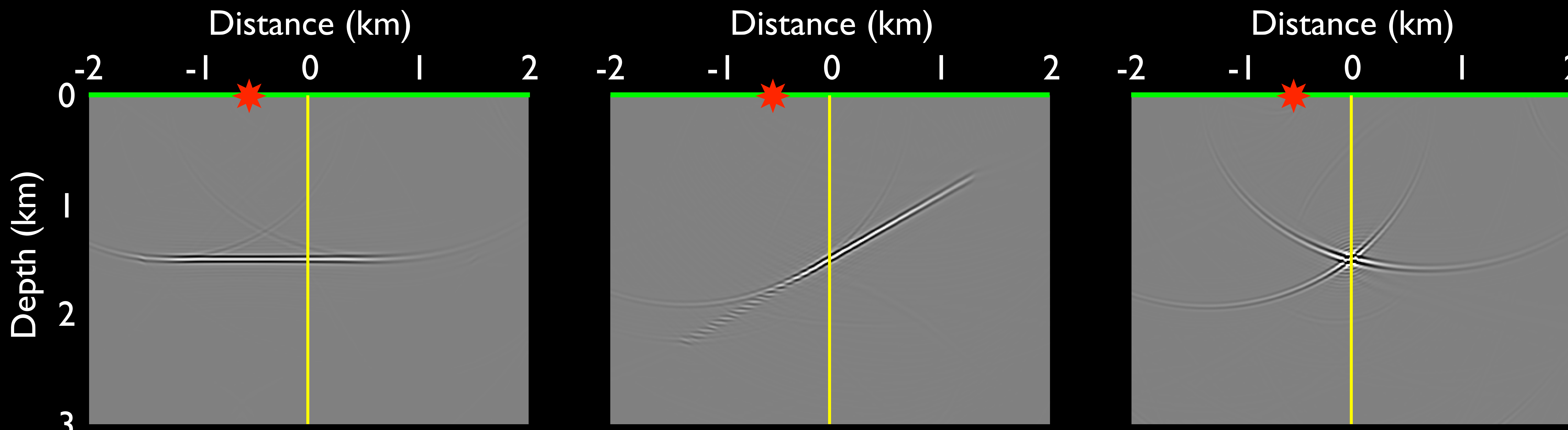
$$\gamma = 18.75^\circ$$

Limitation of scattering-angle illumination

$$\tan\gamma = -\frac{p_{h_x}}{p_{m_z}} = -\frac{k_{h_x}}{k_{m_z}}$$

- The illumination assumes point scatters, it has no dip discrimination
- Scattering-angle-domain illumination implicitly averages over dips
- It cannot accurately predict illumination strength for planar reflectors

Reflectivity images

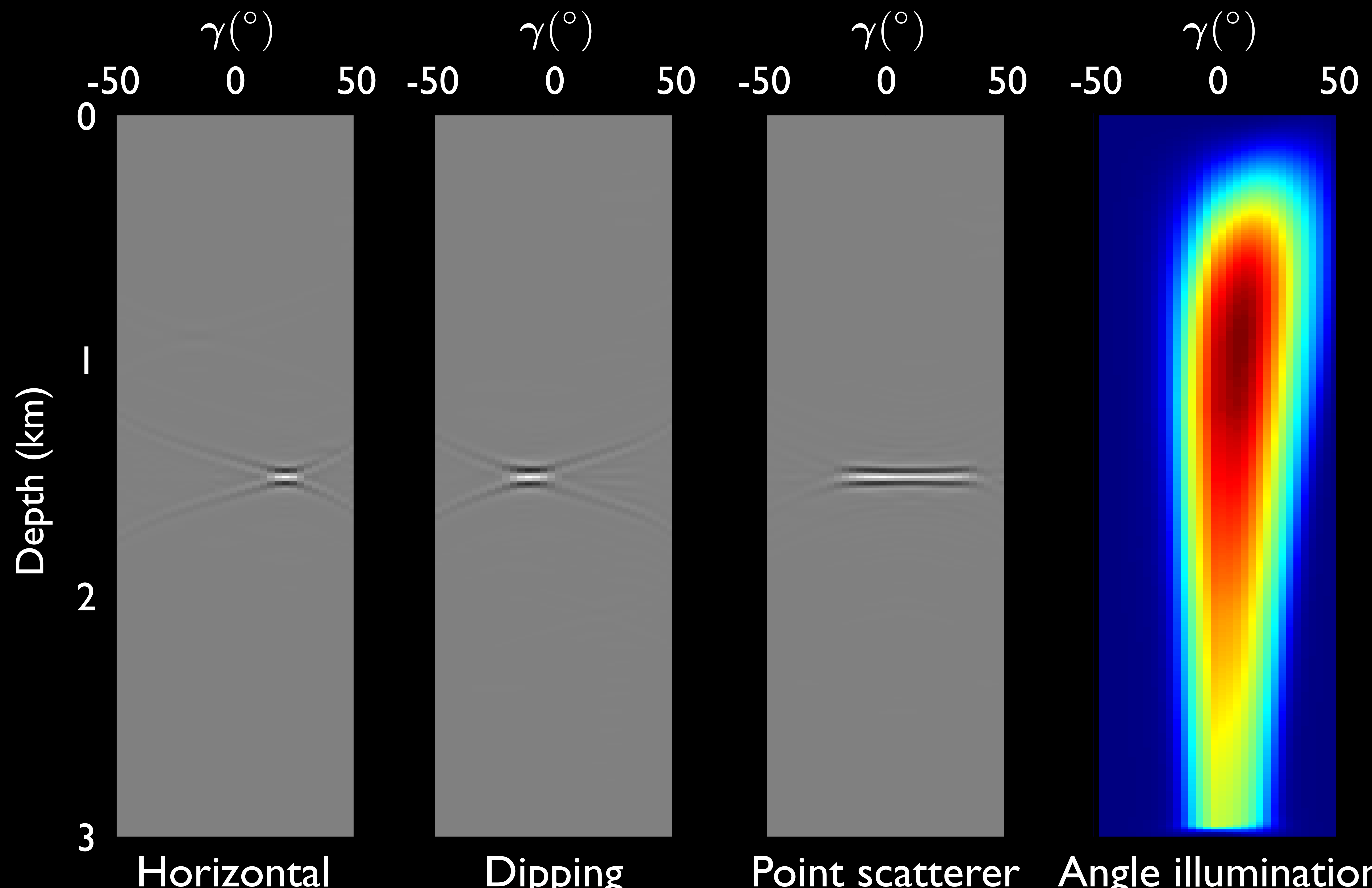


Horizontal reflector

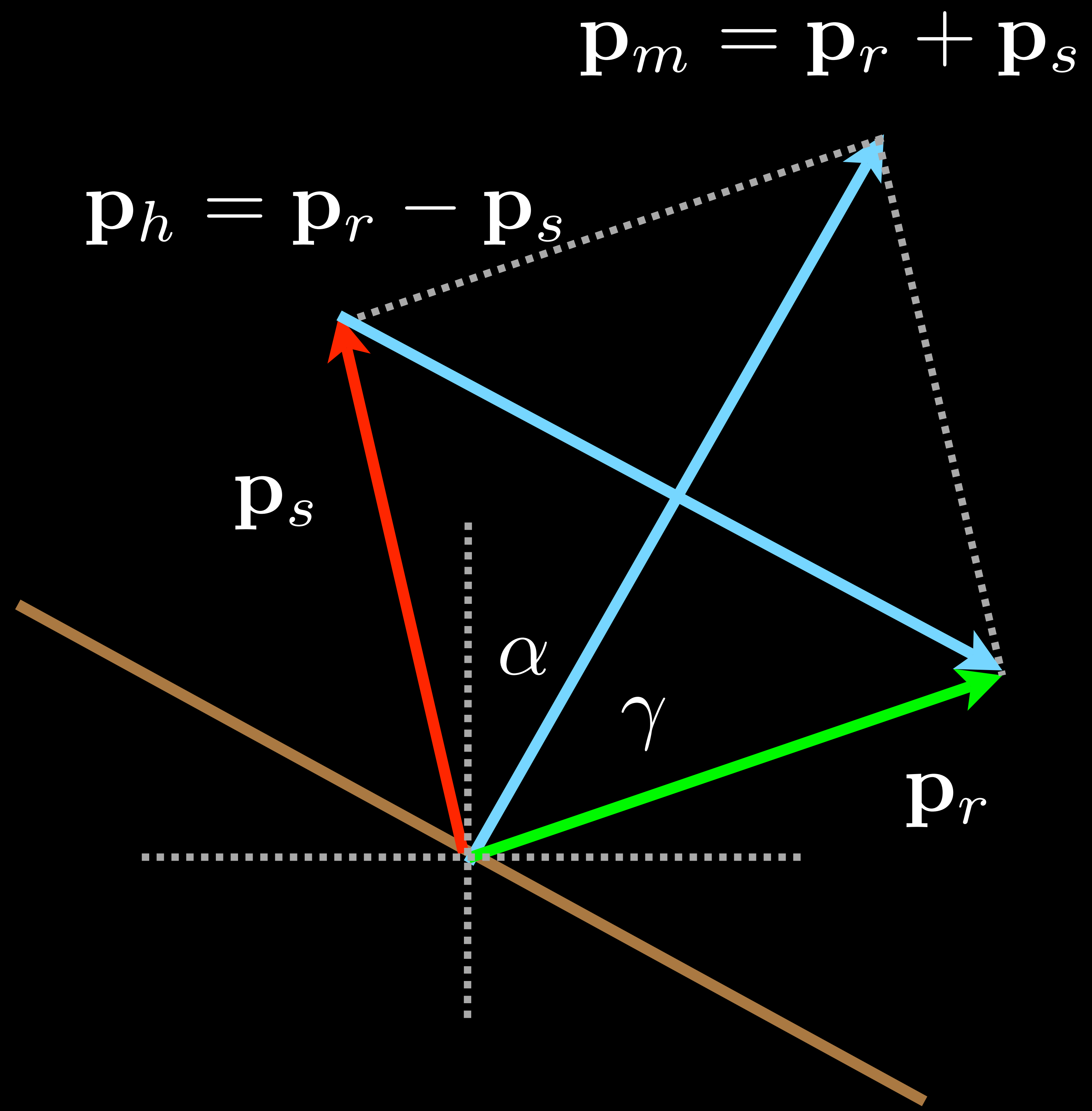
-30° dipping reflector

Point scatterer

Scattering-angle-domain images and illumination



Towards dip-dependent decomposition



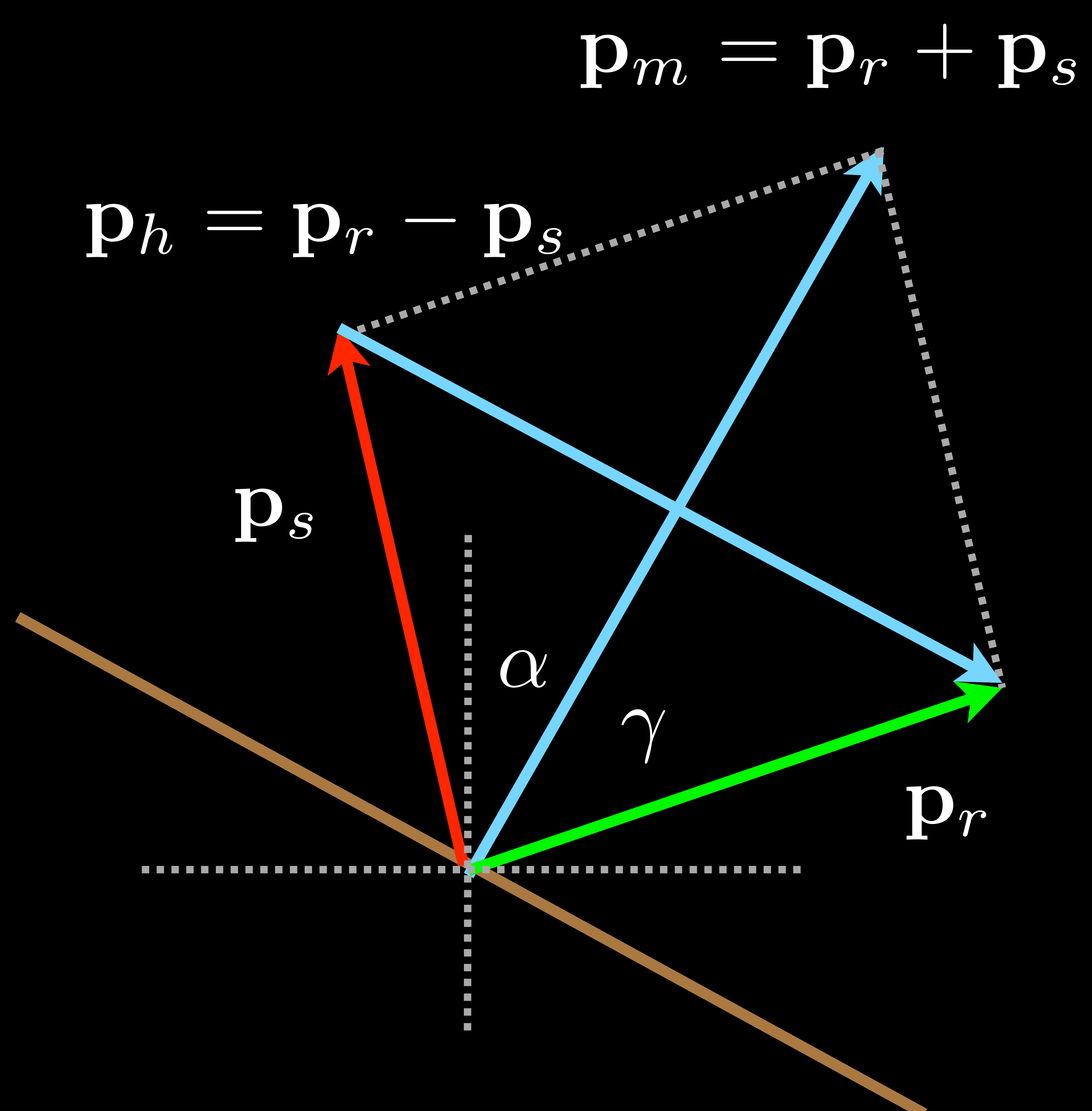
A locally constant slowness medium

$$\mathbf{p}_m = \begin{pmatrix} p_{m_x} \\ p_{m_z} \end{pmatrix} = \begin{pmatrix} 2s\cos\gamma\sin\alpha \\ -2s\cos\gamma\cos\alpha \end{pmatrix}$$

$$\mathbf{p}_h = \begin{pmatrix} p_{h_x} \\ p_{h_z} \end{pmatrix} = \begin{pmatrix} 2s\sin\gamma\cos\alpha \\ 2s\sin\gamma\sin\alpha \end{pmatrix}$$

$$\tan\gamma = -\frac{p_{h_x}}{p_{m_z}} = -\frac{k_{h_x}}{k_{m_z}}$$

Towards dip-dependent decomposition



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$$\tan\gamma = -\frac{p_{h_x}}{p_{m_z}} = -\frac{k_{h_x}}{k_{m_z}}$$

$$\tan\alpha = -\frac{p_{m_x}}{p_{m_z}} = -\frac{k_{m_x}}{k_{m_z}}$$

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega)$$

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega) \xrightarrow{\text{3D FT}} L_h(k_x, k_z, k_{h_x}, \mathbf{r}, \mathbf{s}, \omega)$$

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega) \xrightarrow{\text{3D FT}} L_h(k_x, k_z, k_{h_x}, \mathbf{r}, \mathbf{s}, \omega)$$

$$\tan\alpha = -\frac{k_x}{k_z} \downarrow \downarrow \tan\gamma = -\frac{k_{h_x}}{k_z}$$

$$L_{\alpha,\gamma}(k_x, k_z, \alpha, \gamma, \mathbf{r}, \mathbf{s}, \omega)$$

Workflow

$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega) \xrightarrow{\text{3D FT}} L_h(k_x, k_z, k_{h_x}, \mathbf{r}, \mathbf{s}, \omega)$$

$$\tan\alpha = -\frac{k_x}{k_z} \downarrow \downarrow \tan\gamma = -\frac{k_{h_x}}{k_z}$$

$$L_{\alpha,\gamma}(x, z, \alpha, \gamma, \mathbf{r}, \mathbf{s}, \omega) \xleftarrow{\text{2D IFT}} L_{\alpha,\gamma}(k_x, k_z, \alpha, \gamma, \mathbf{r}, \mathbf{s}, \omega)$$

Workflow

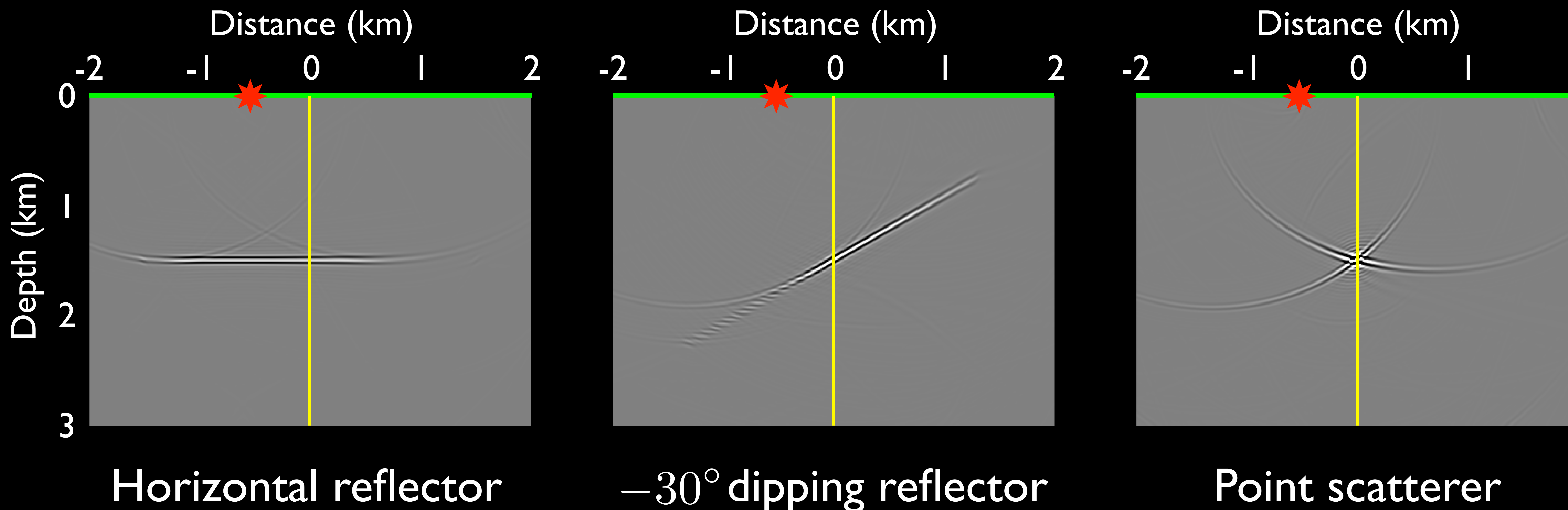
$$L_h(x, z, h_x, \mathbf{r}, \mathbf{s}, \omega) \xrightarrow{\text{3D FT}} L_h(k_x, k_z, k_{h_x}, \mathbf{r}, \mathbf{s}, \omega)$$

$$\tan\alpha = -\frac{k_x}{k_z} \qquad \tan\gamma = -\frac{k_{h_x}}{k_z}$$

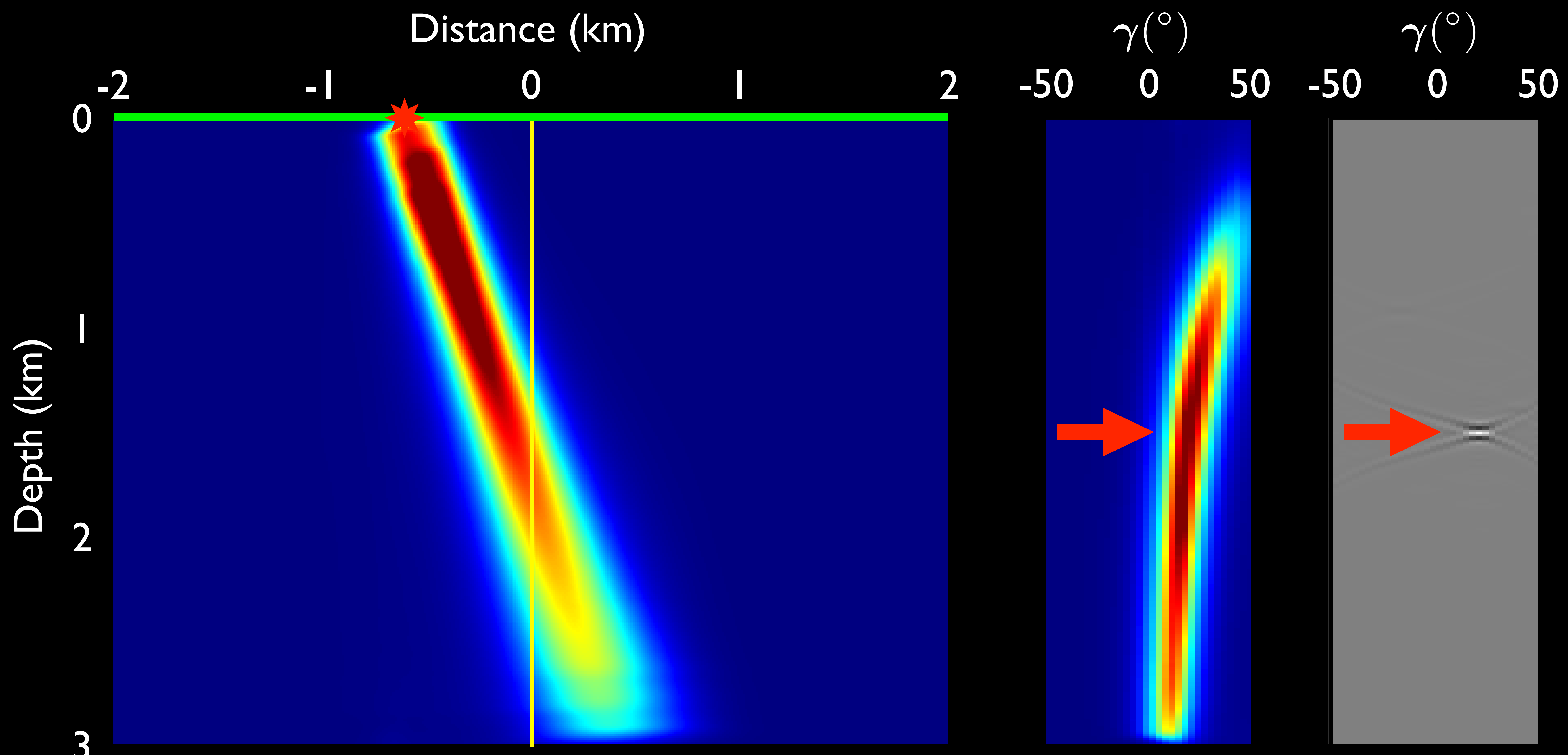
$$L_{\alpha,\gamma}(x, z, \alpha, \gamma, \mathbf{r}, \mathbf{s}, \omega) \xleftarrow{\text{2D IFT}} L_{\alpha,\gamma}(k_x, k_z, \alpha, \gamma, \mathbf{r}, \mathbf{s}, \omega)$$

$$H_{\alpha,\gamma}(x, z, \alpha, \gamma) = \sum_{\omega}\sum_{\mathbf{s}}\sum_{\mathbf{r}} |L_{\alpha,\gamma}(x, z, \alpha, \gamma, \mathbf{r}, \mathbf{s}, \omega)|^2$$

Reflectivity images

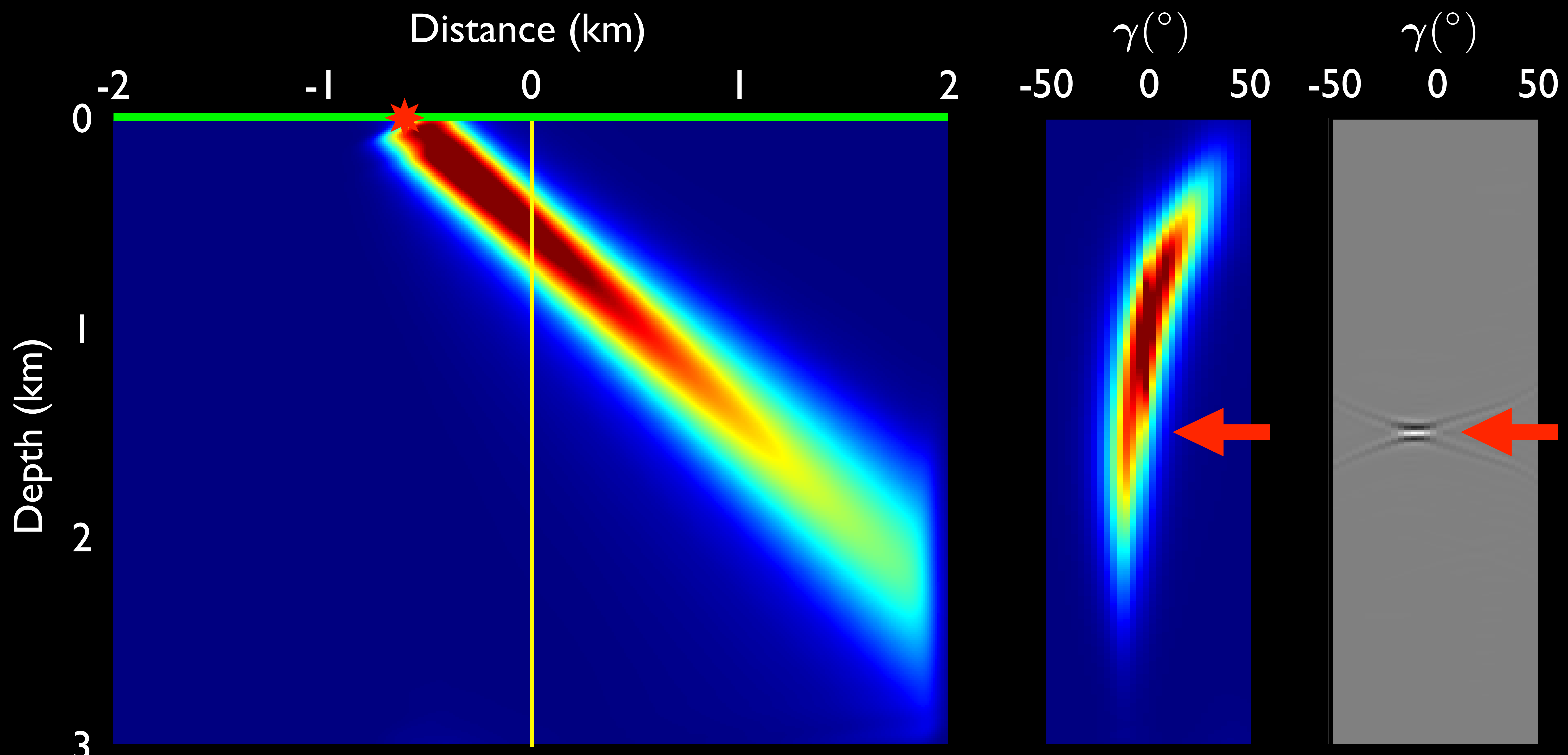


Dip and scattering-angle decomposed illumination $(\alpha = 0^\circ)$



$\gamma = 18.75^\circ$

Dip and scattering-angle decomposed illumination $(\alpha = -30^\circ)$



$\gamma = 18.75^\circ$

Huge cost...

Scattering-angle decomposed illumination:

$$H_\gamma(x,z,\gamma) = \sum_\omega \sum_{\mathbf{s}} \sum_{\mathbf{r}} |L_\gamma(x,z,\gamma,\mathbf{r},\mathbf{s},\omega)|^2$$

Dip and scattering-angle decomposed illumination:

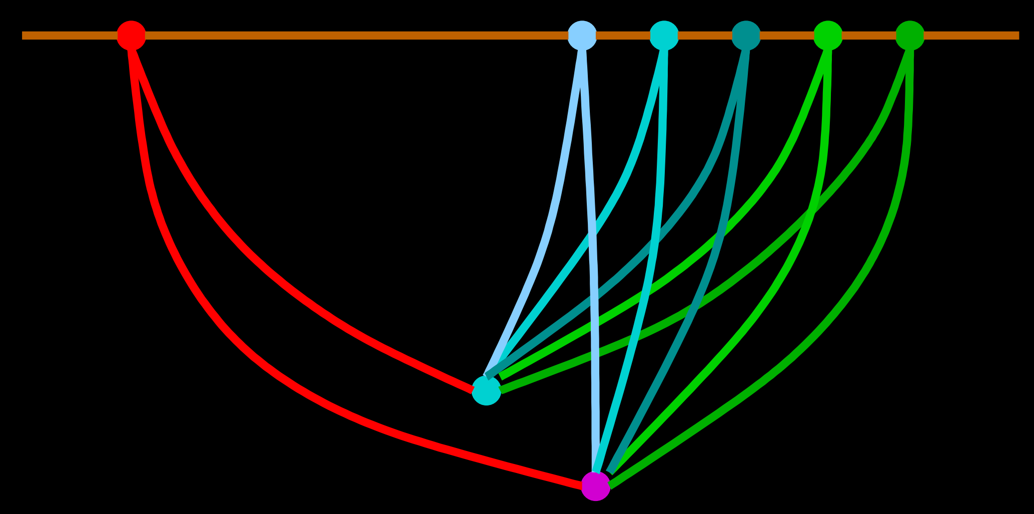
$$H_{\alpha,\gamma}(x,z,\alpha,\gamma) = \sum_\omega \sum_{\mathbf{s}} \sum_{\mathbf{r}} |L_{\alpha,\gamma}(x,z,\alpha,\gamma,\mathbf{r},\mathbf{s},\omega)|^2$$

$$\text{Cost} \propto N_\omega N_s N_r$$

Phase-encoded sensitivity kernel

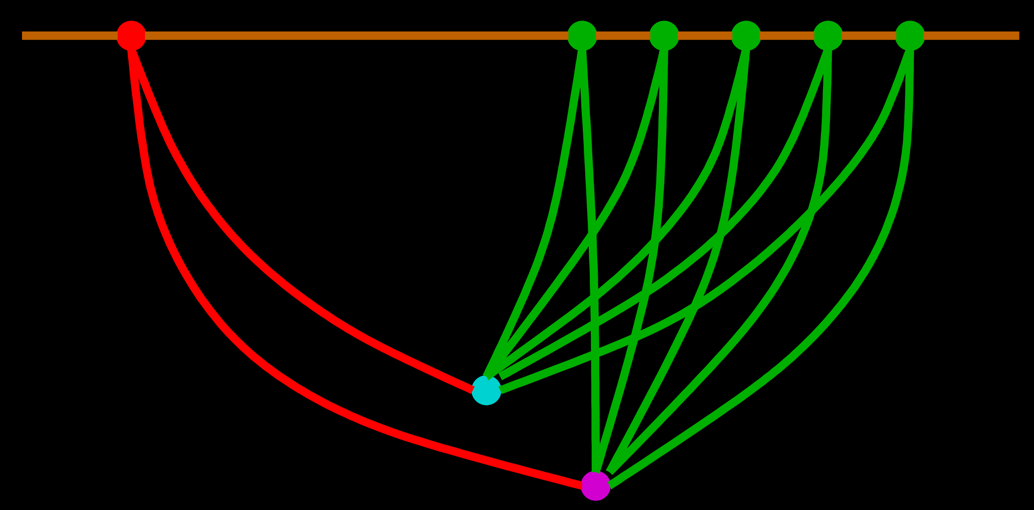
Original sensitivity kernel:

$$L_h(\mathbf{x}, \mathbf{h}, \mathbf{r}, \mathbf{s}, \omega) = G(\mathbf{x} - \mathbf{h}, \mathbf{s}, \omega) G(\mathbf{x} + \mathbf{h}, \mathbf{r}, \omega)$$



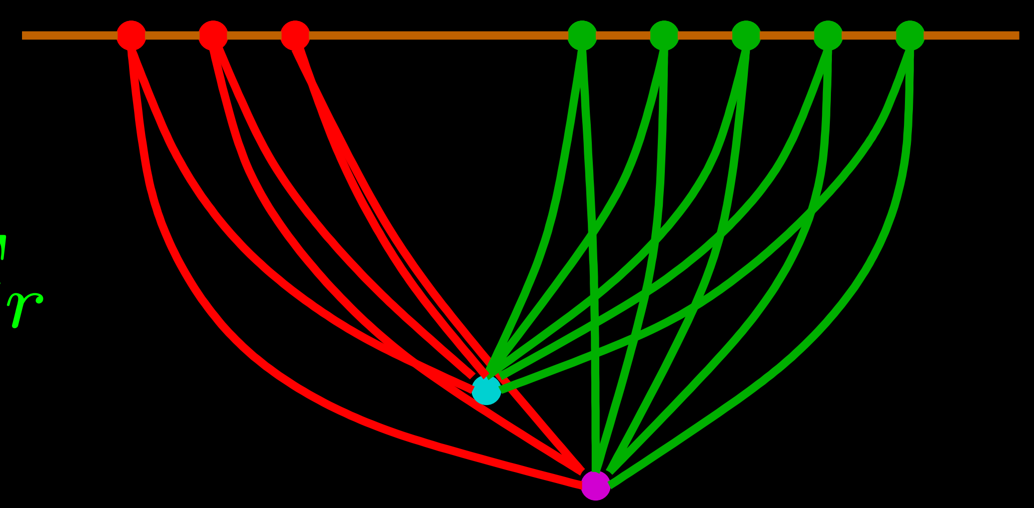
Encoding of receivers:

$$L_h^R(\mathbf{x}, \mathbf{h}, \mathbf{s}, \omega) = G(\mathbf{x} - \mathbf{h}, \mathbf{s}, \omega) \sum_{\mathbf{r}} G(\mathbf{x} + \mathbf{h}, \mathbf{r}, \omega) E_r$$



Encoding of sources and receivers:

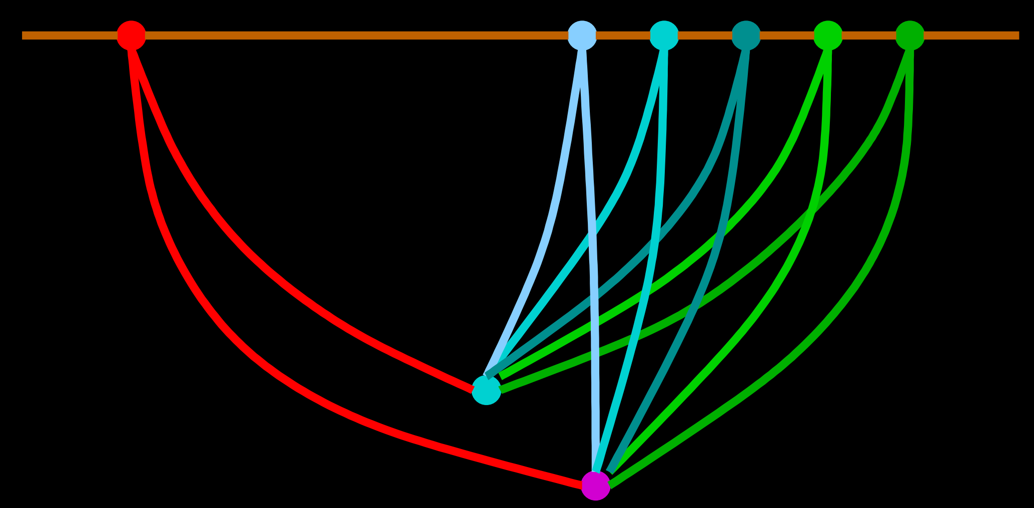
$$L_h^{SR}(\mathbf{x}, \mathbf{h}, \omega) = \sum_{\mathbf{s}} G(\mathbf{x} - \mathbf{h}, \mathbf{s}, \omega) E_s \sum_{\mathbf{r}} G(\mathbf{x} + \mathbf{h}, \mathbf{r}, \omega) E_r$$



Phase-encoded sensitivity kernel

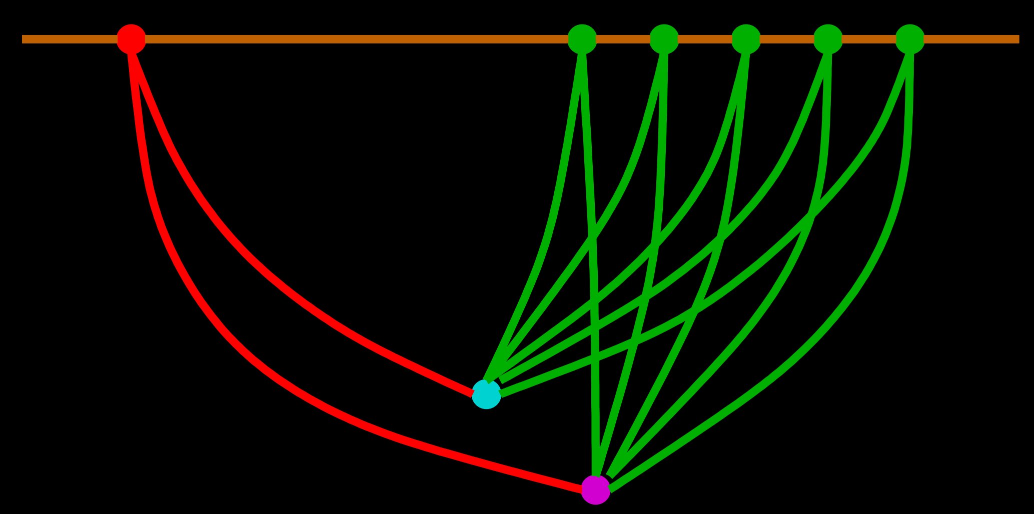
Original sensitivity kernel:

$$L_h(\mathbf{s},\mathbf{r},\omega) \longrightarrow L_{\alpha,\gamma}(\mathbf{s},\mathbf{r},\omega)$$



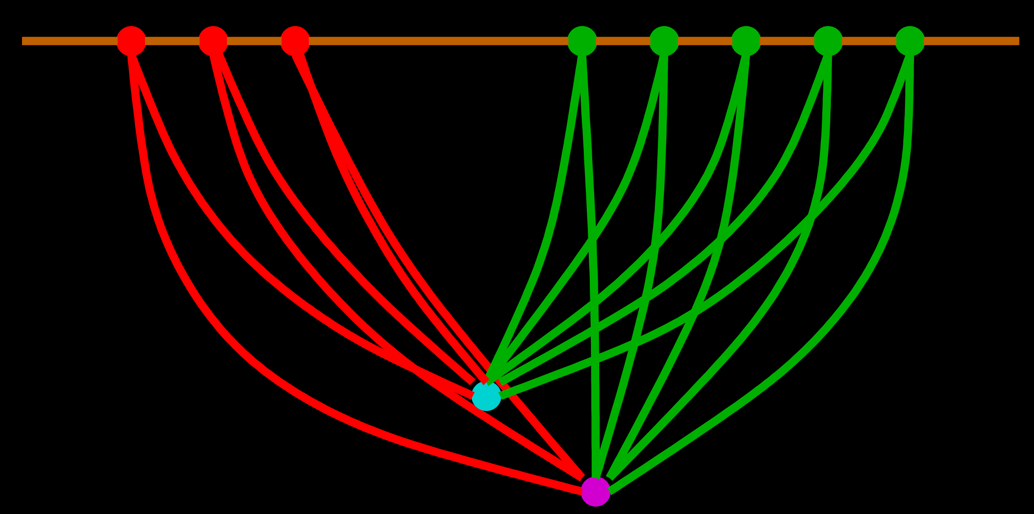
Encoding of receivers:

$$L_h^R(\mathbf{s},\omega) \longrightarrow L_{\alpha,\gamma}^R(\mathbf{s},\omega)$$



Encoding of sources and receivers:

$$L_h^{SR}(\omega) \longrightarrow L_{\alpha,\gamma}^{SR}(\omega)$$



Cost saving

Original illumination:

$$H_{\alpha,\gamma} = \sum_{\omega} \sum_{\mathbf{s}} \sum_{\mathbf{r}} |L_{\alpha,\gamma}(\mathbf{r}, \mathbf{s}, \omega)|^2 \qquad \text{Cost} \propto N_\omega N_\mathbf{s} N_\mathbf{r}$$

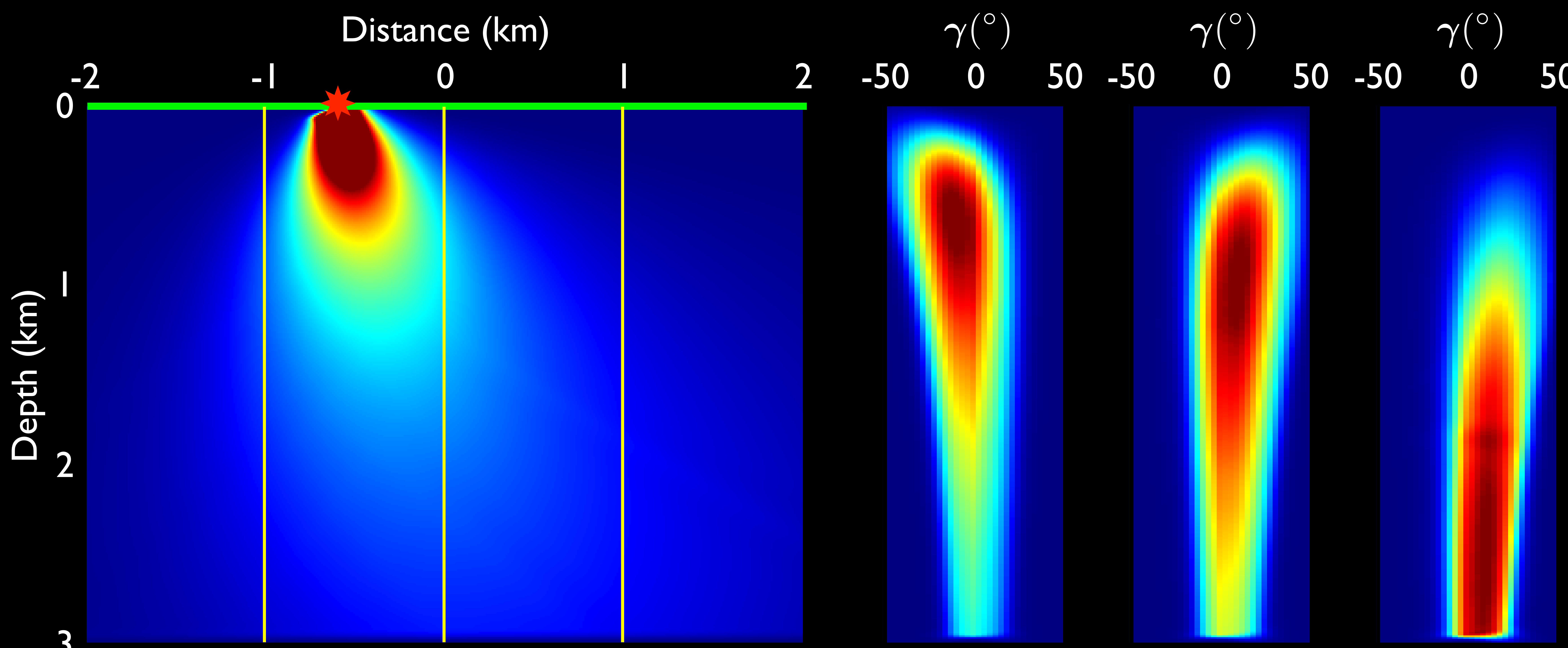
Encoding of receivers:

$$H^{\mathrm{R}}_{\alpha,\gamma} = \sum_{\omega} \sum_{\mathbf{s}} |L^{\mathrm{R}}_{\alpha,\gamma}(\mathbf{s}, \omega)|^2 \qquad \text{Cost} \propto N_\omega N_\mathbf{s}$$

Encoding of sources and receivers:

$$H^{\mathrm{SR}}_{\alpha,\gamma} = \sum_{\omega} |L^{\mathrm{SR}}_{\alpha,\gamma}(\omega)|^2 \qquad \text{Cost} \propto N_\omega$$

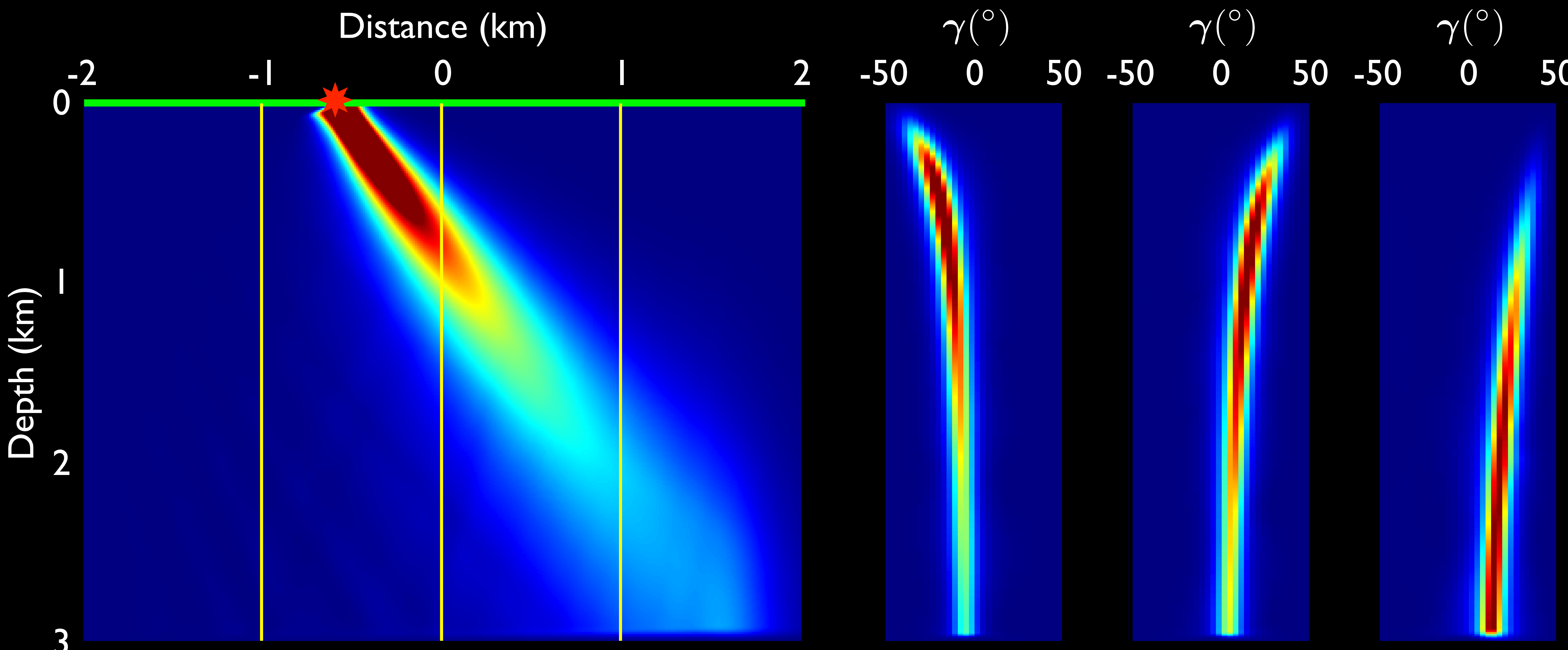
Scattering-angle-domain illumination



$\gamma = 18.75^\circ$

Exact

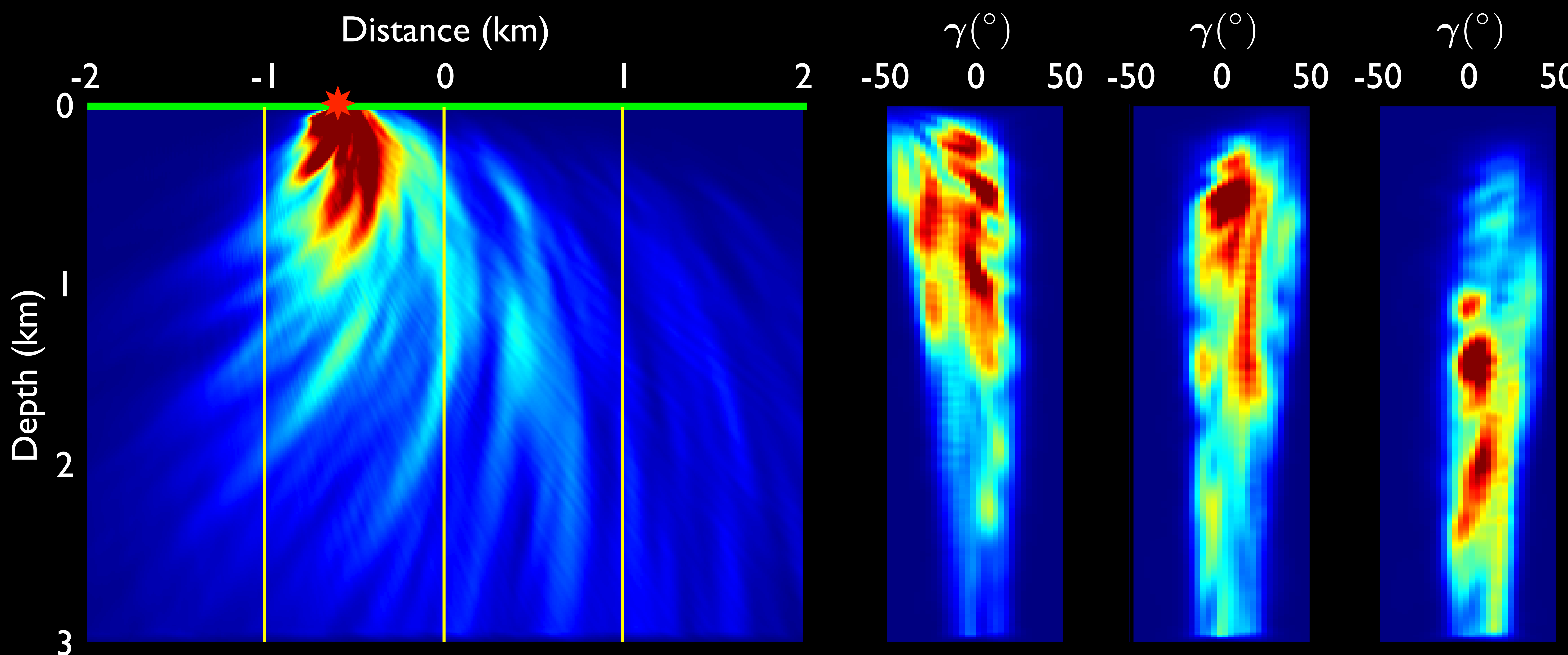
Scattering-angle-domain illumination



$\gamma = 18.75^\circ$

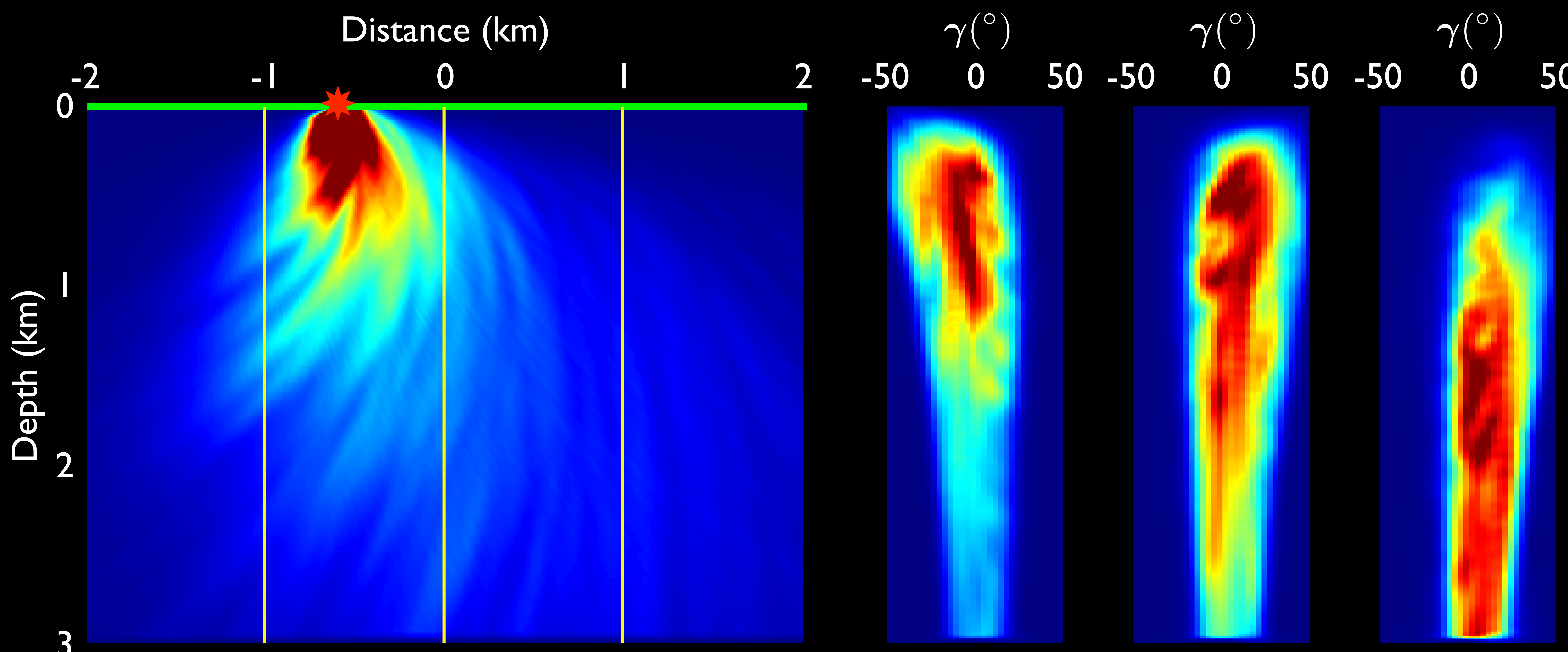
Crosstalk

Scattering-angle-domain illumination



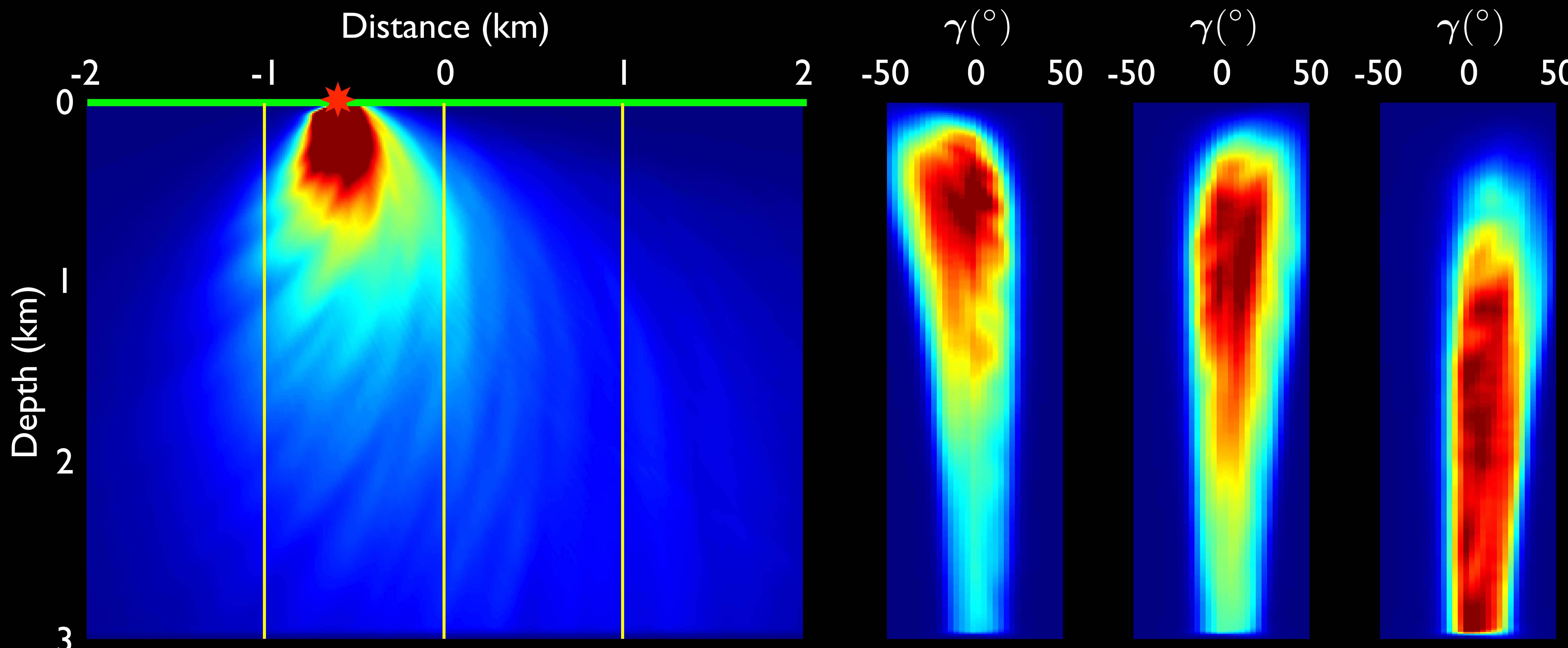
$\gamma = 18.75^\circ$ **1 realization**

Scattering-angle-domain illumination



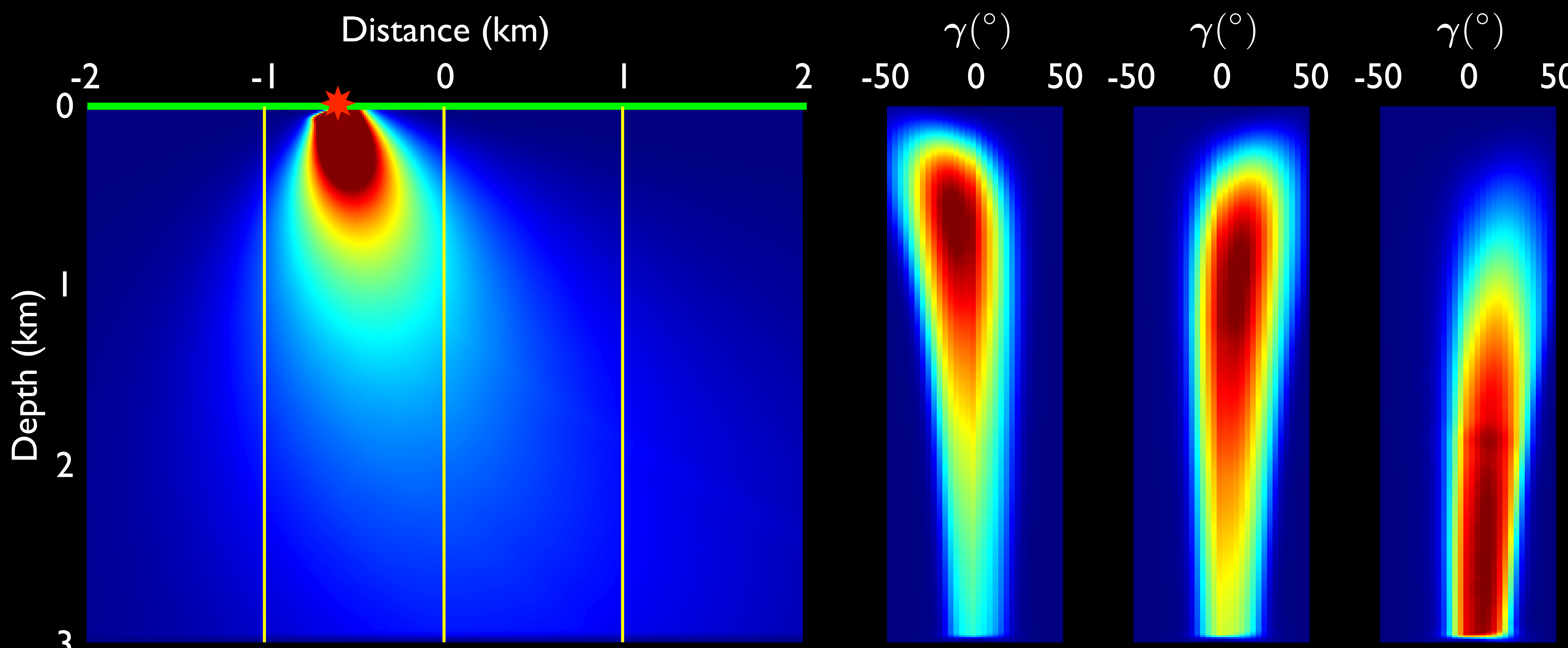
$\gamma = 18.75^\circ$ **5 realization**

Scattering-angle-domain illumination



$\gamma = 18.75^\circ$ **20 realization**

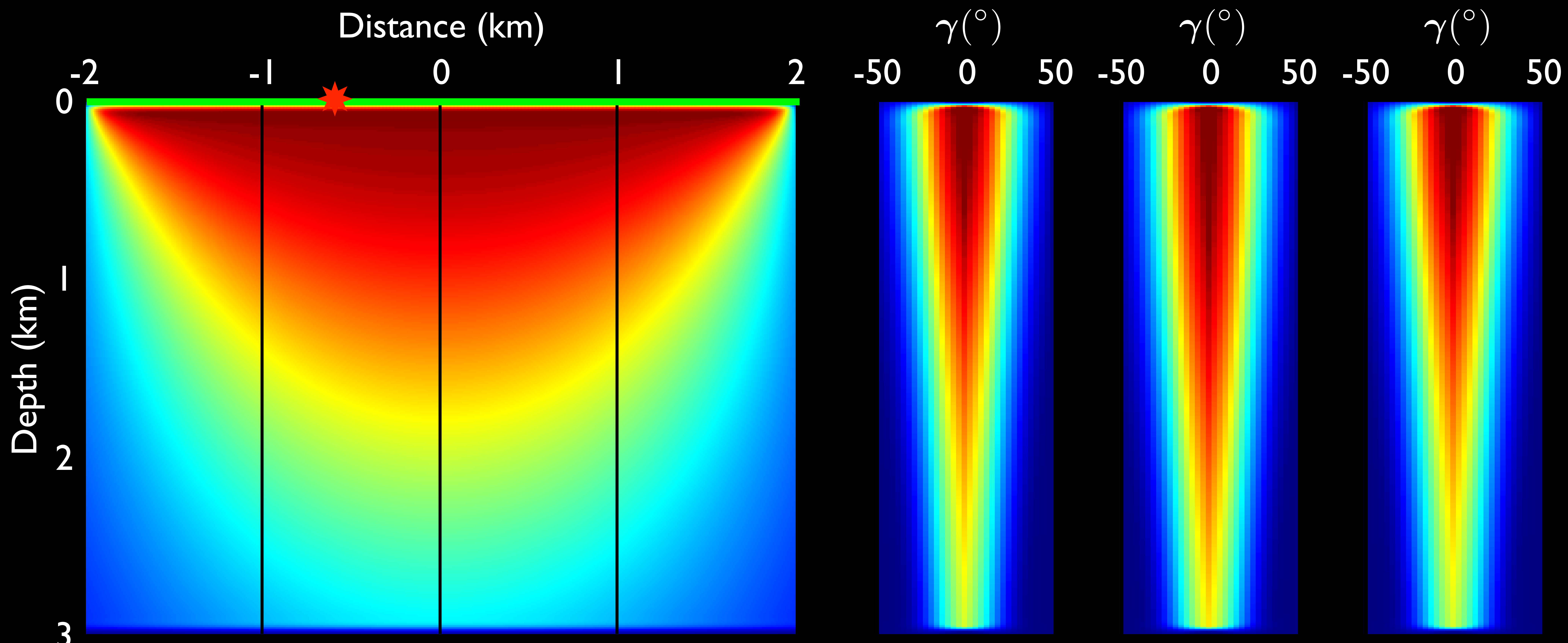
Scattering-angle-domain illumination



$$\gamma = 18.75^\circ$$

Exact

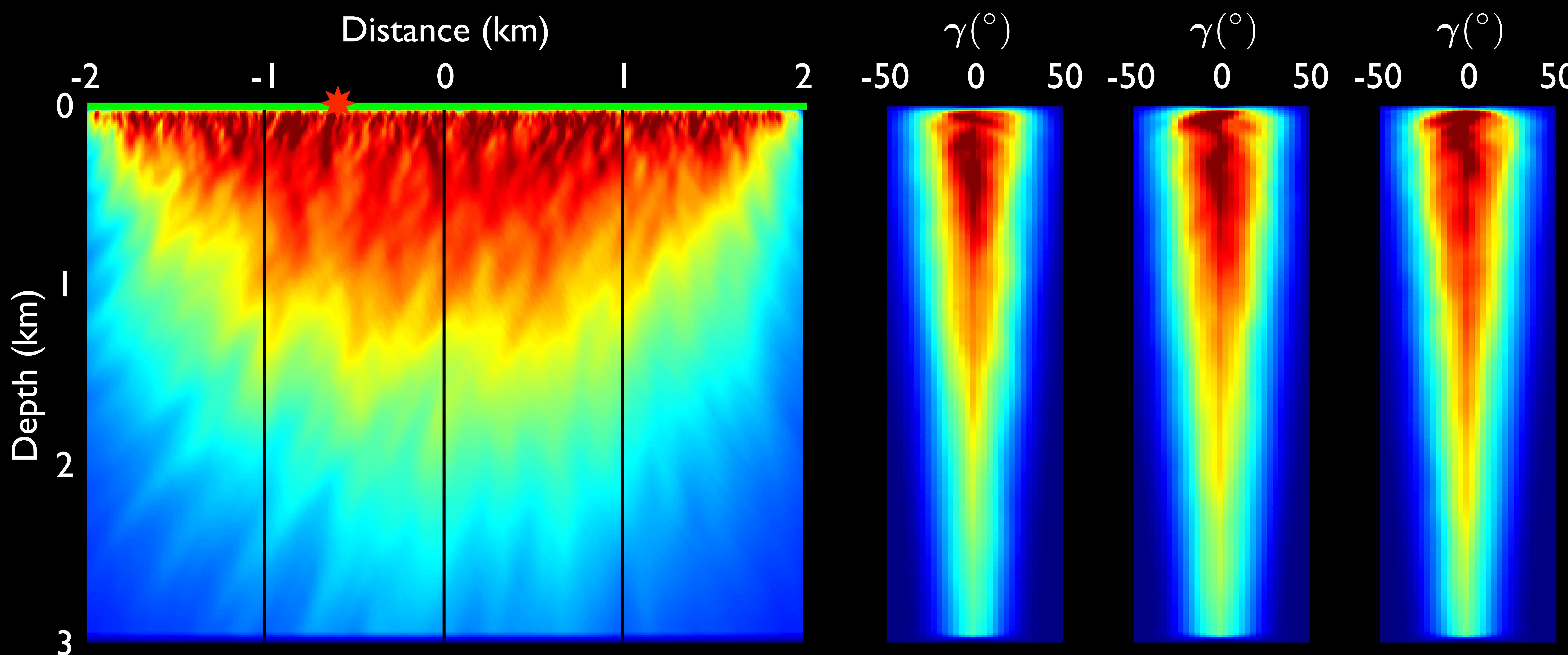
101 shots and 101 receivers



$\gamma = 18.75^\circ$

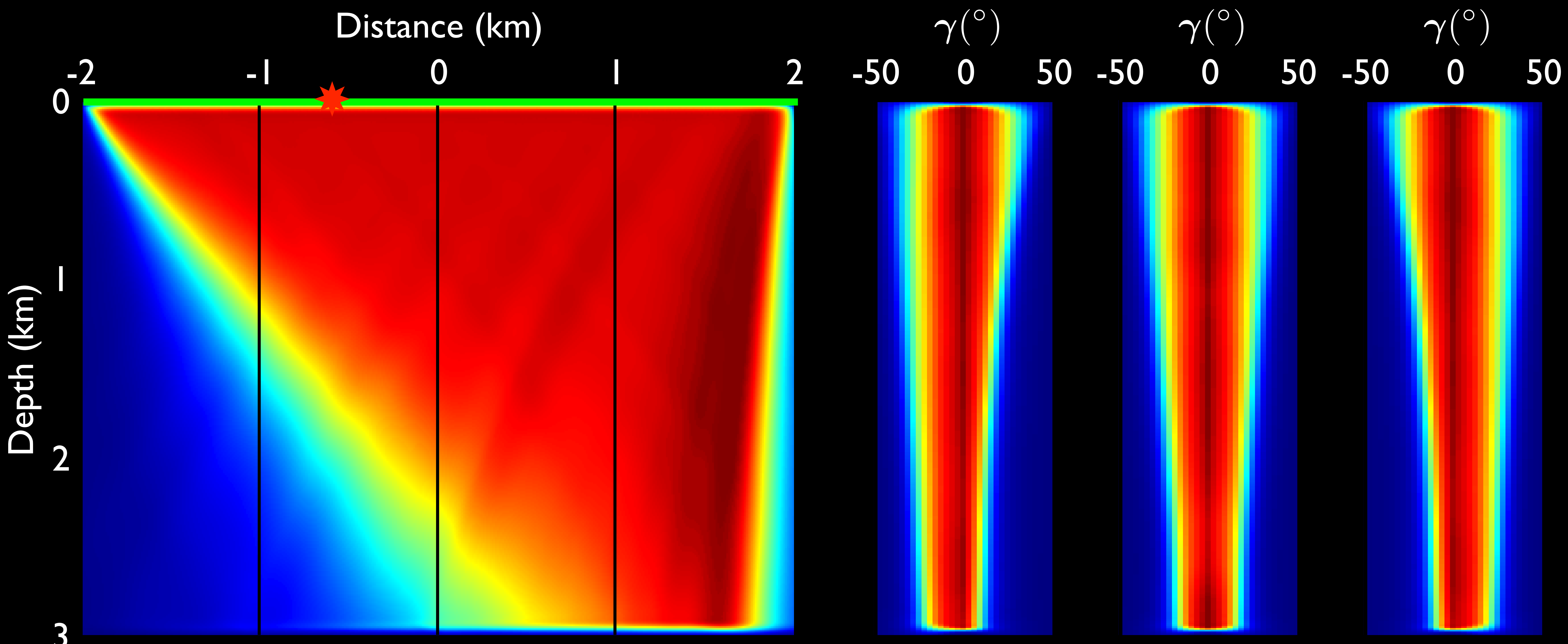
Exact

101 shots and 101 receivers



$\gamma = 18.75^\circ$ **1 realization**

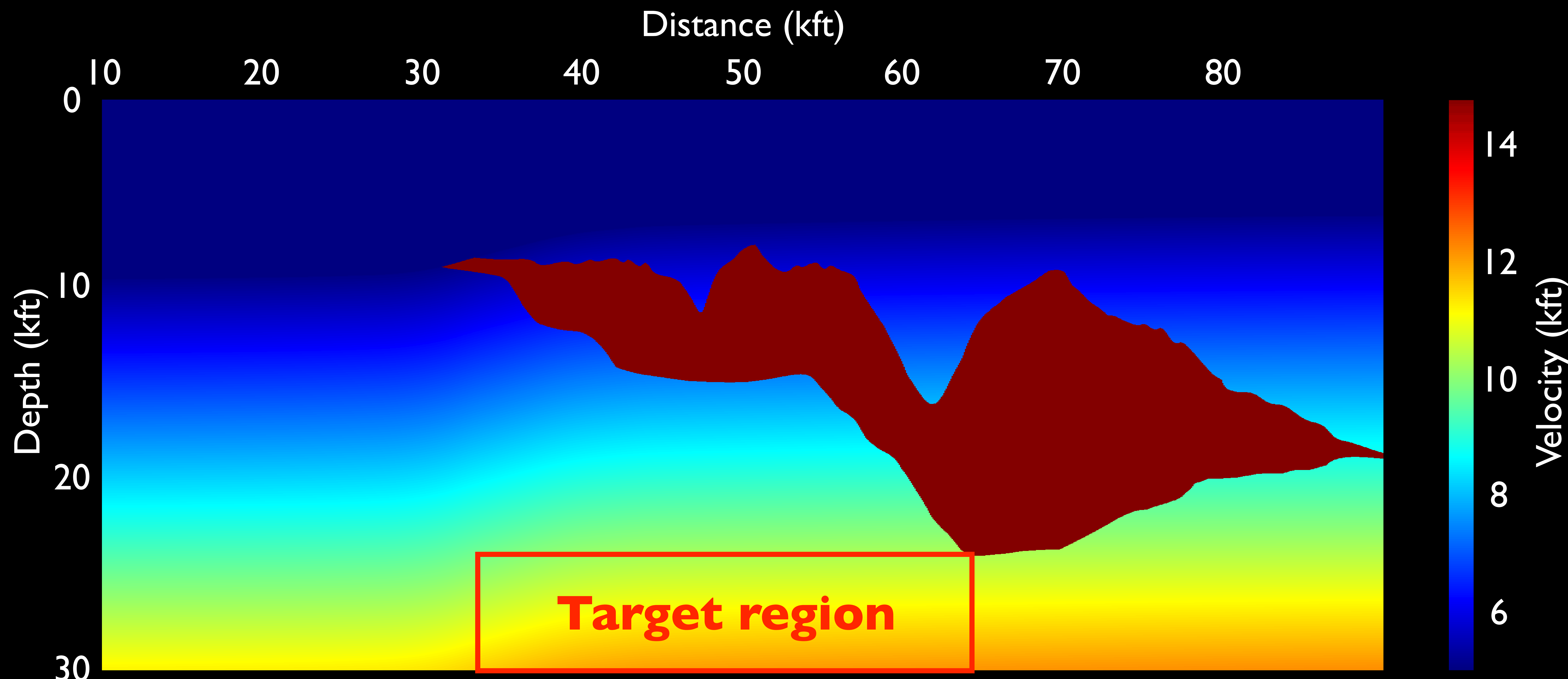
101 shots and 101 receivers



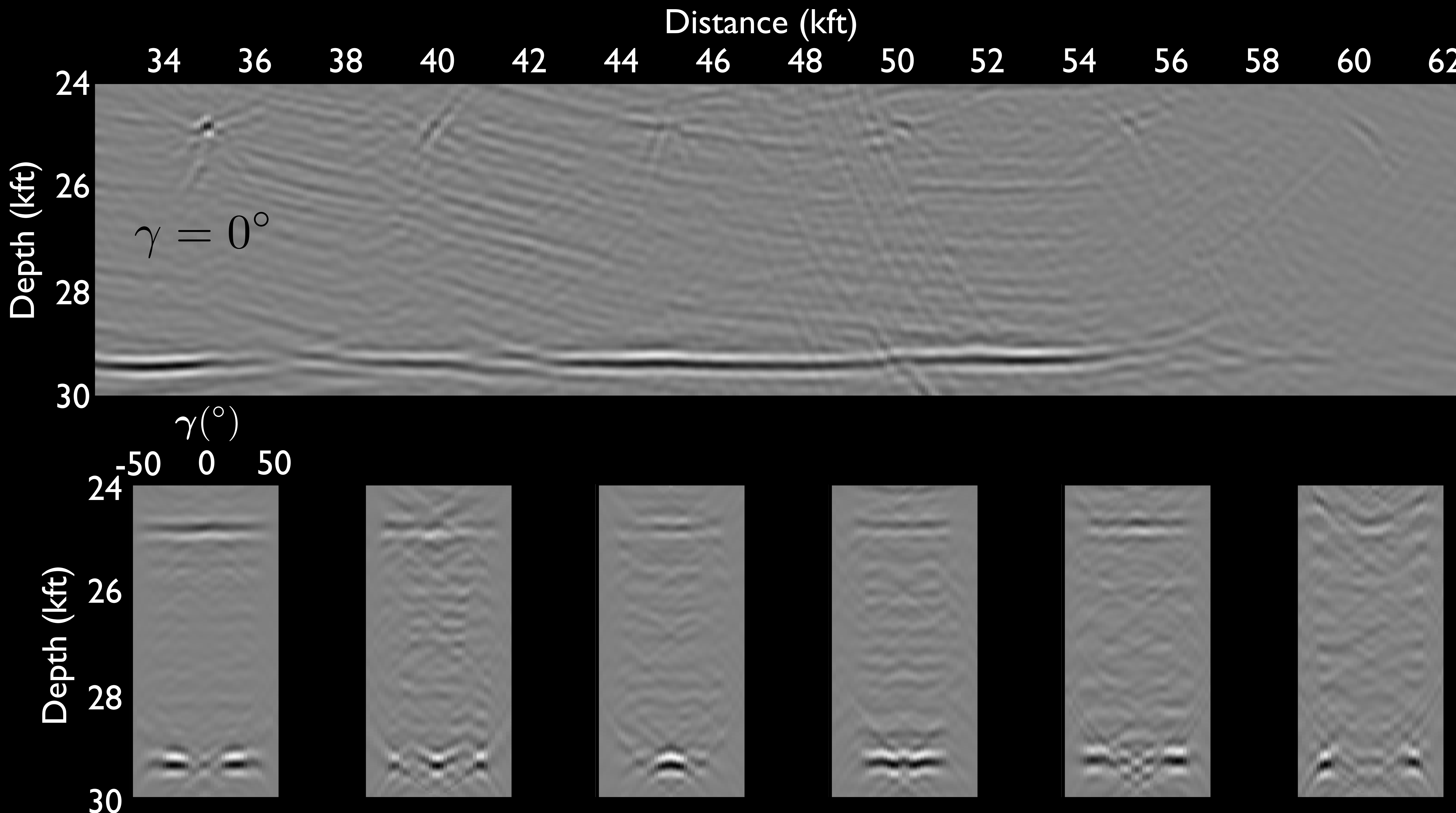
$\gamma = 18.75^\circ$

Crosstalk

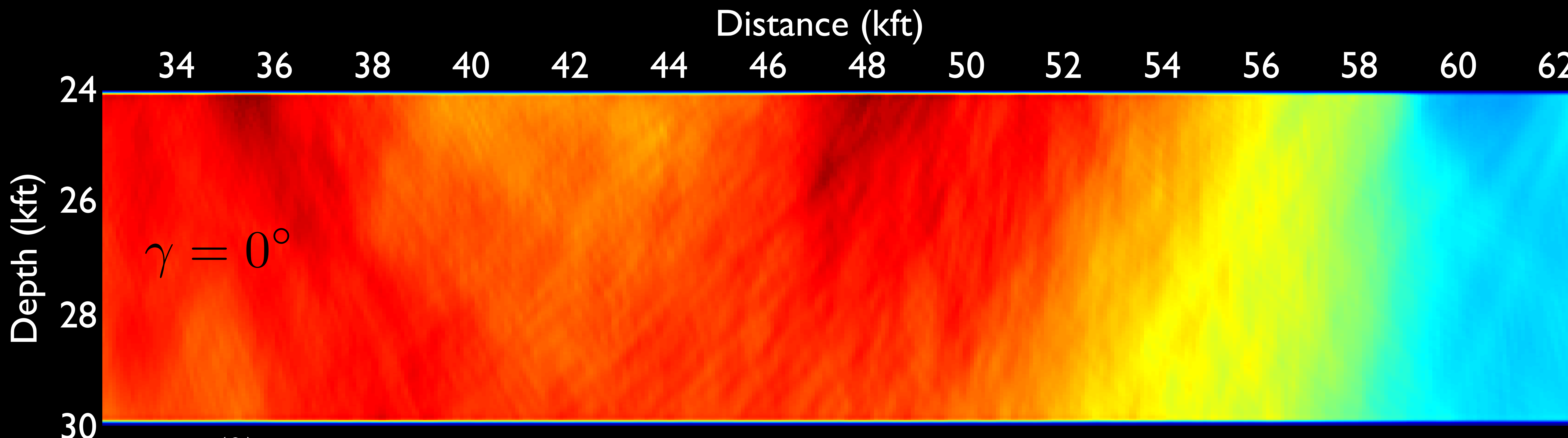
The Sigsbee2A velocity model

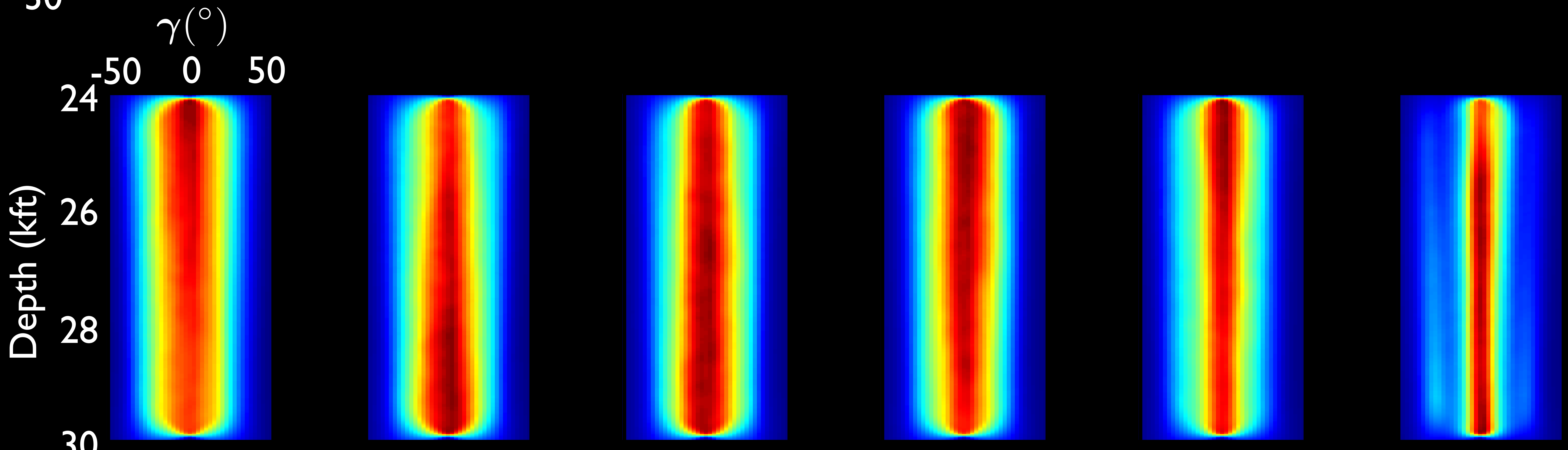


Scattering-angle decomposed image



Scattering-angle decomposed illumination





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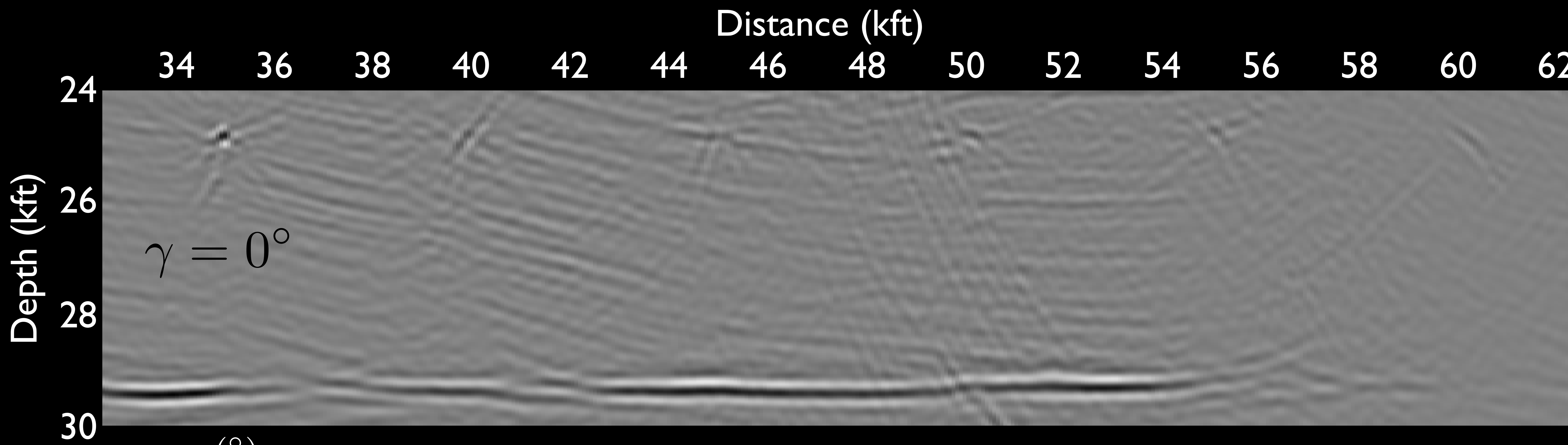

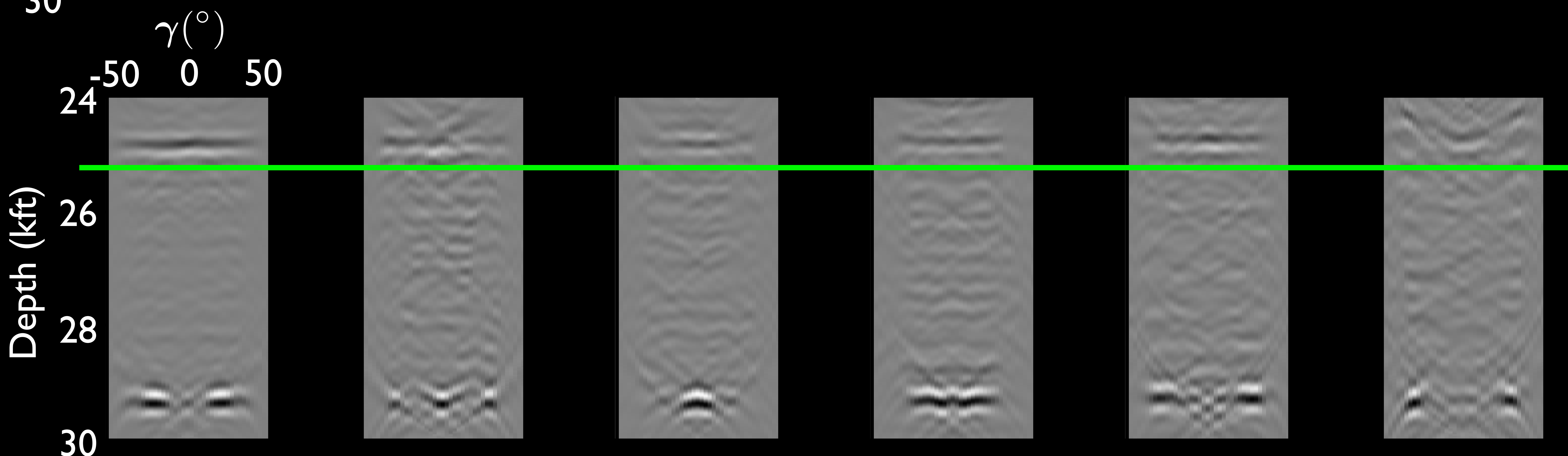

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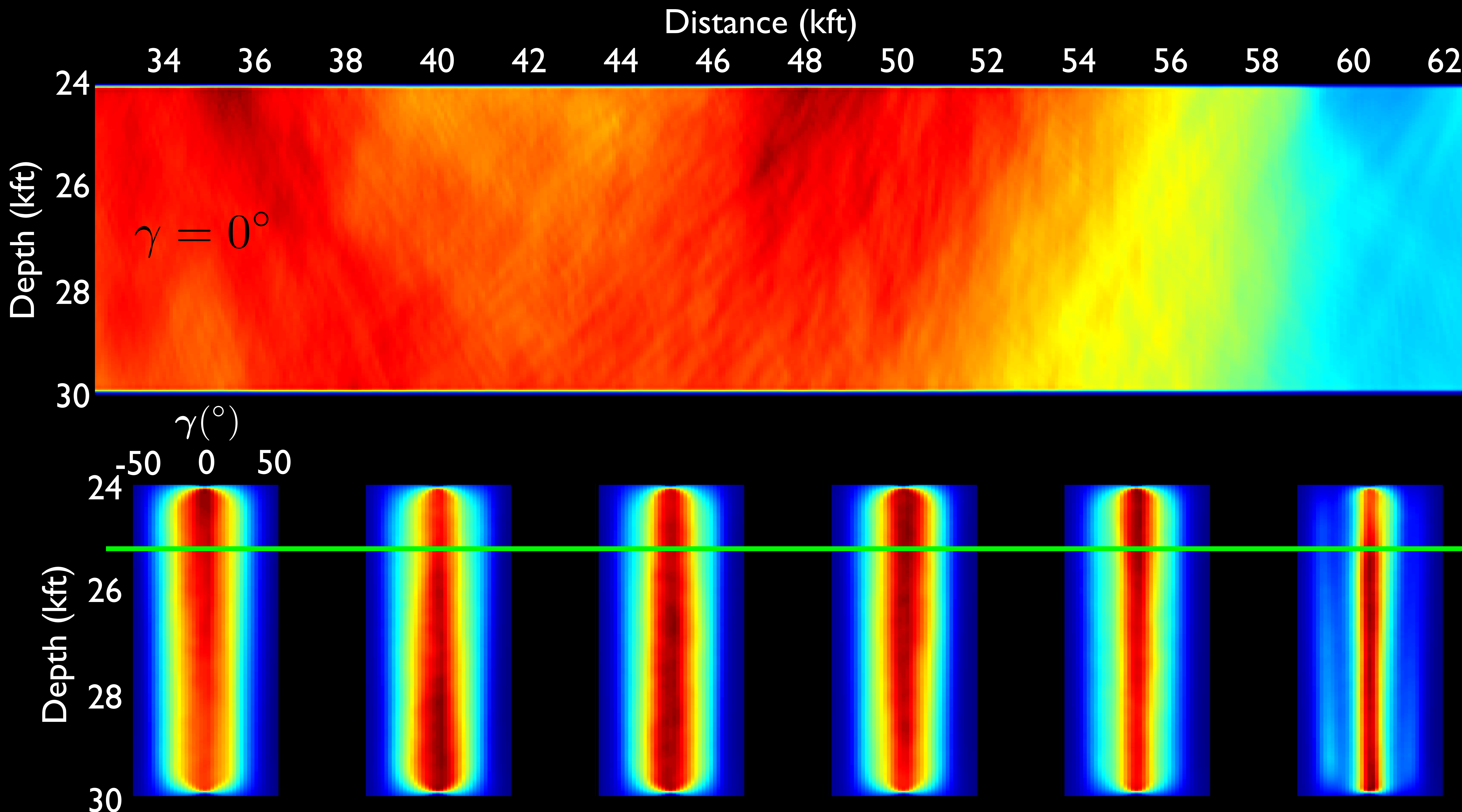
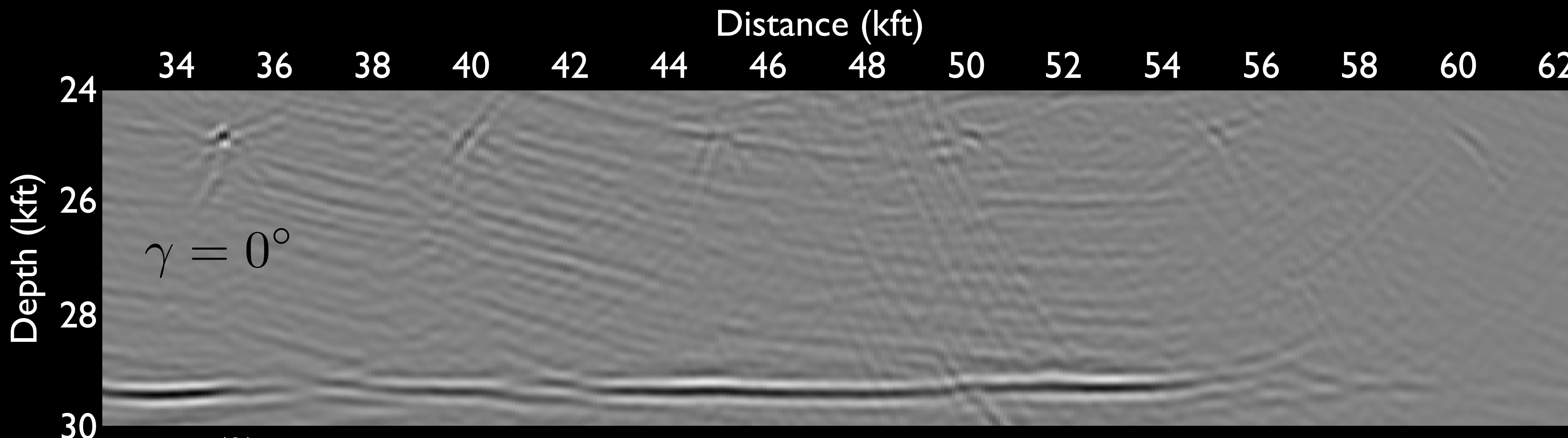
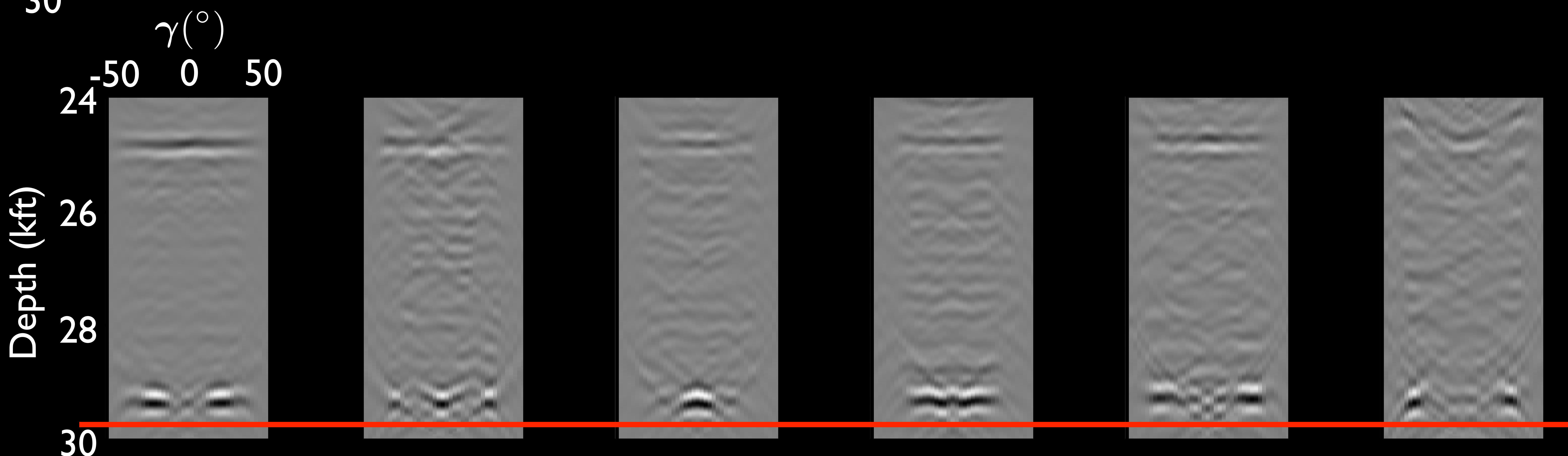
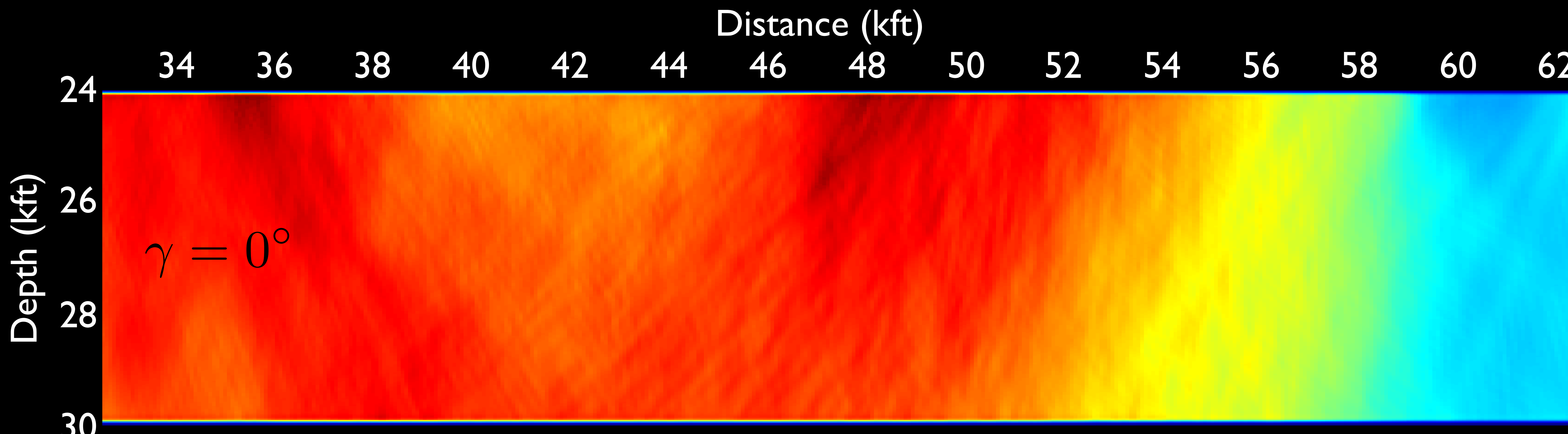


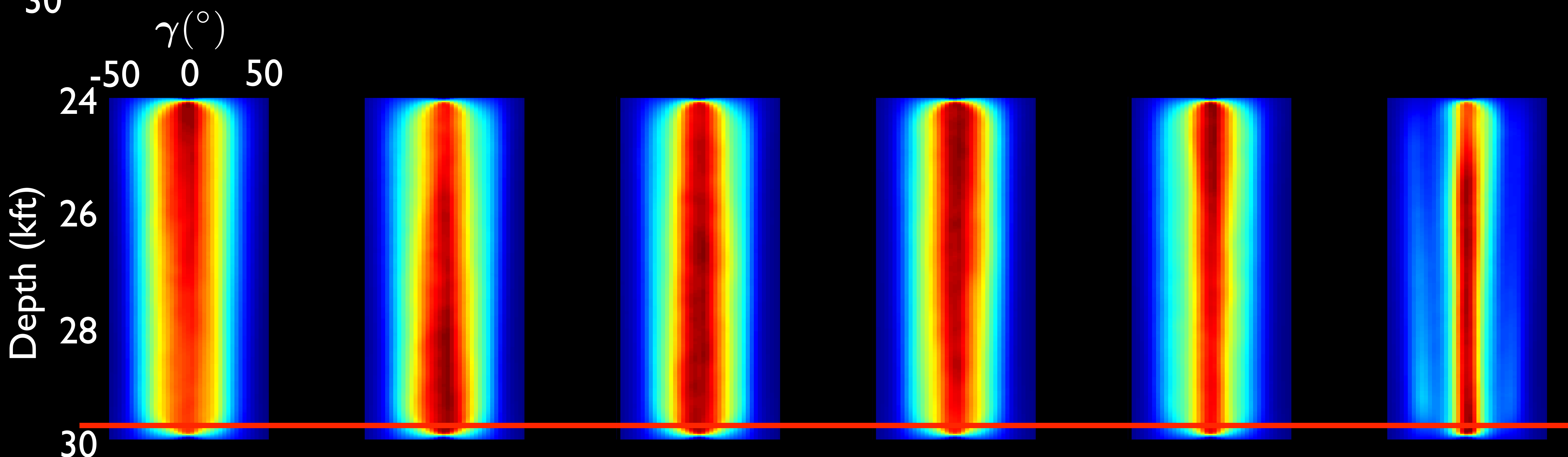
Scattering-angle decomposed image



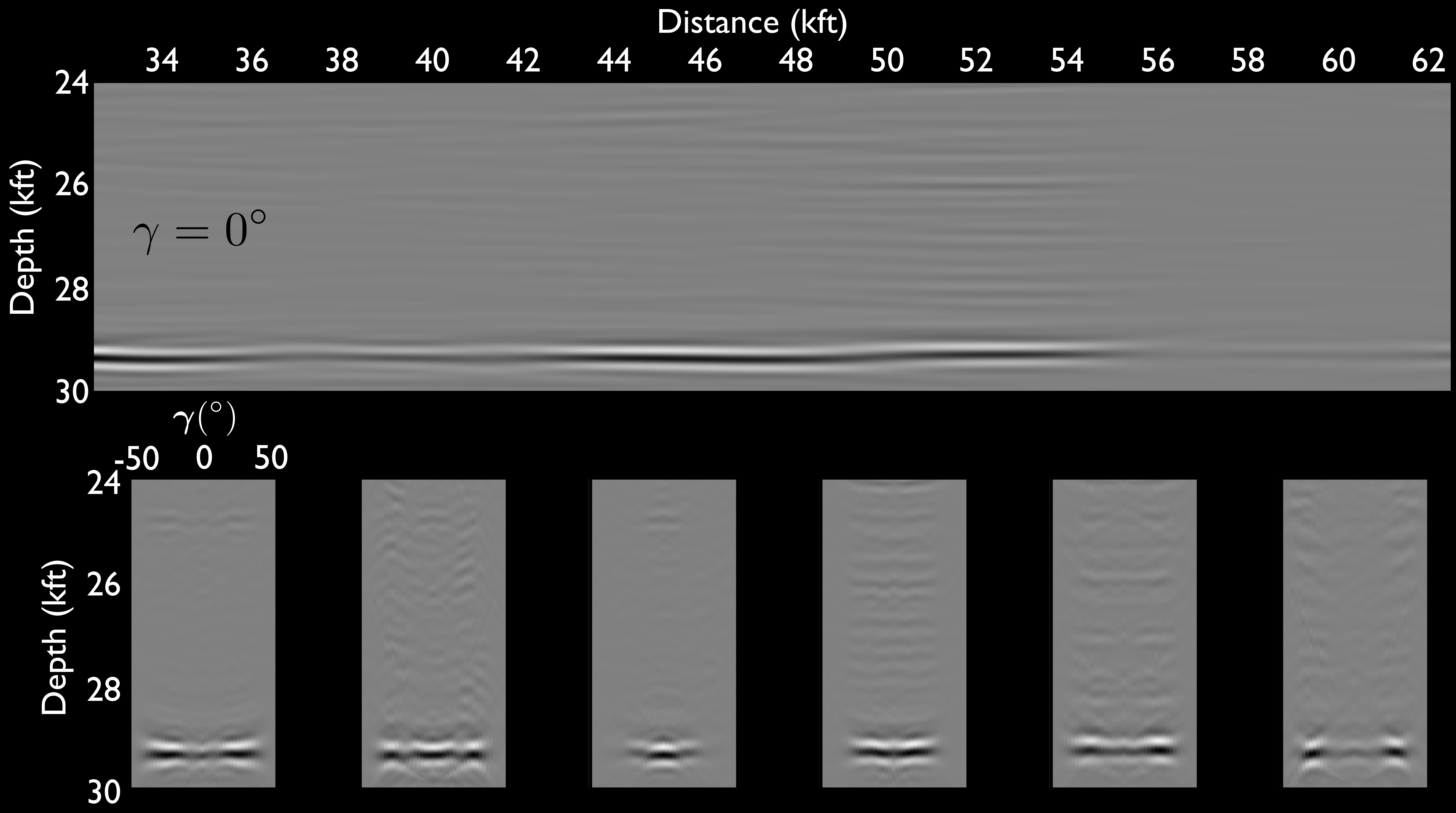


Scattering-angle decomposed illumination





Dip and aperture-angle decomposed image $(\alpha = 0^\circ)$



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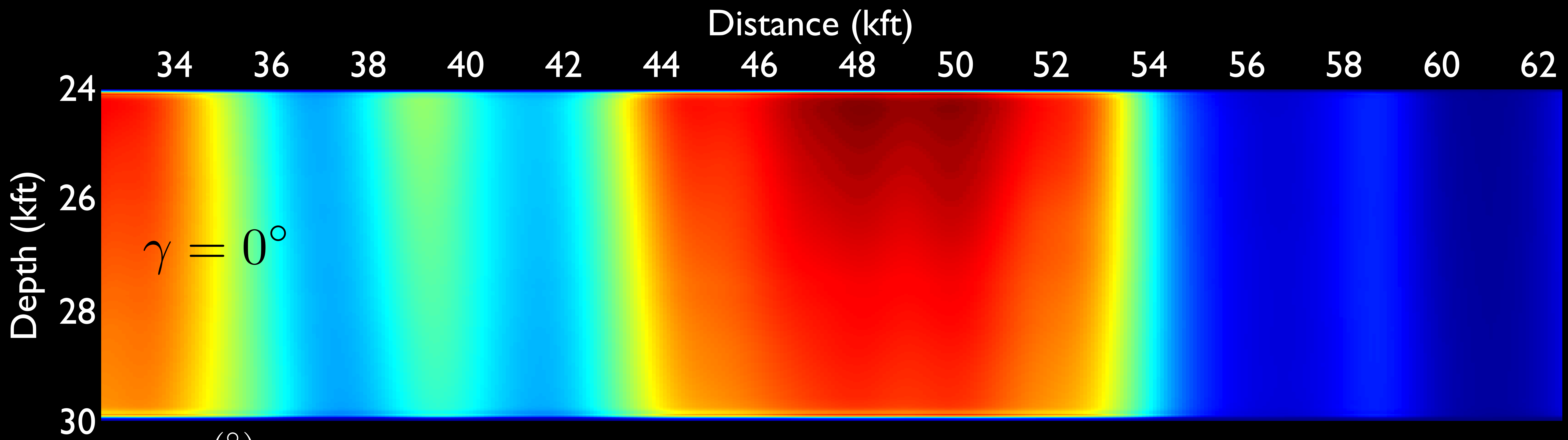
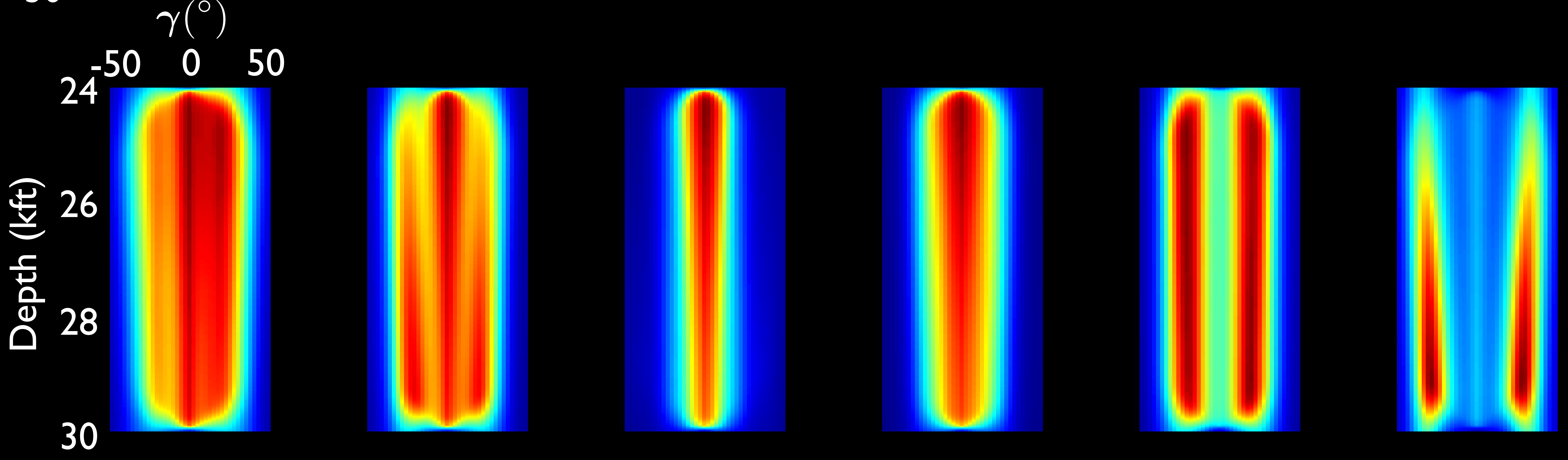
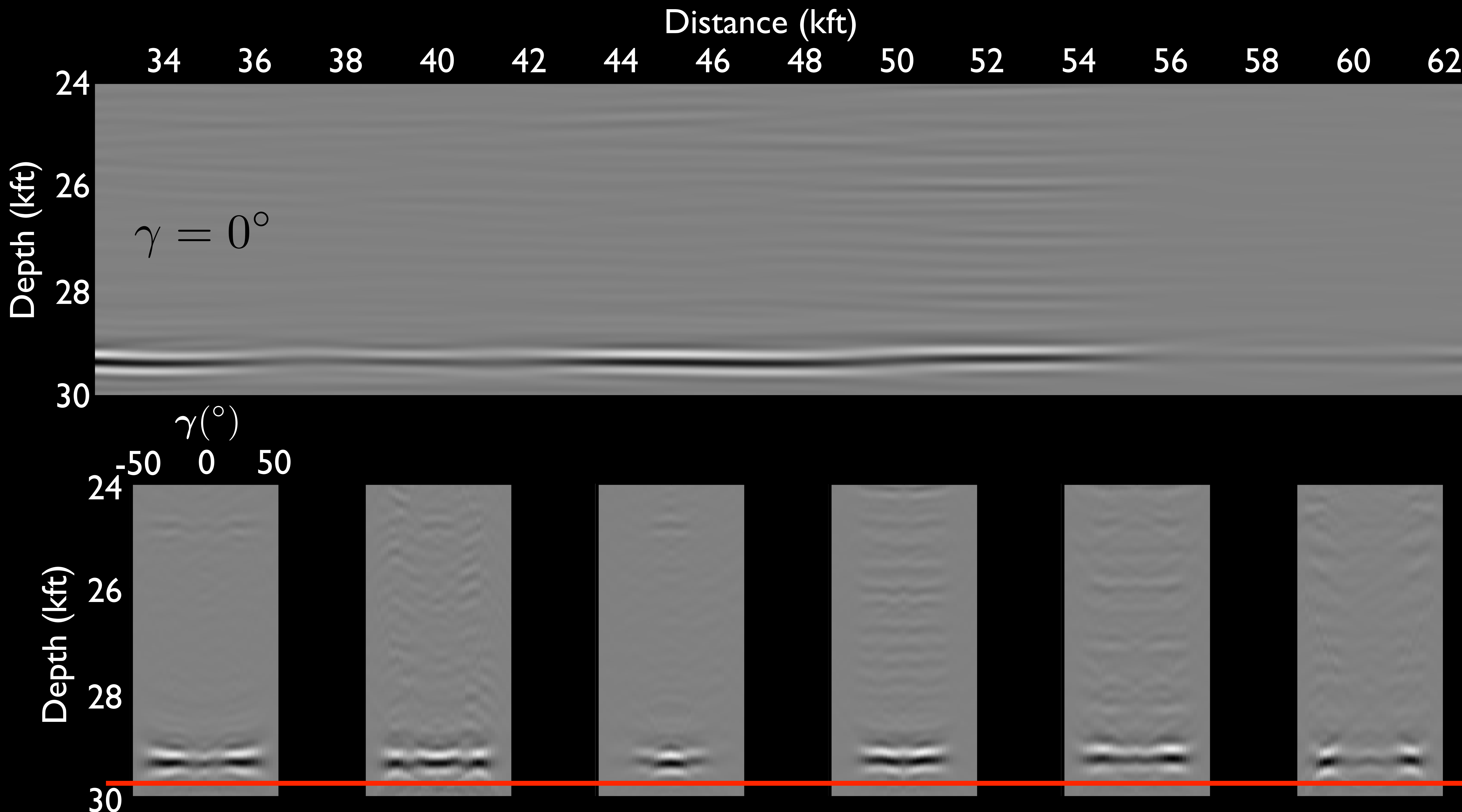
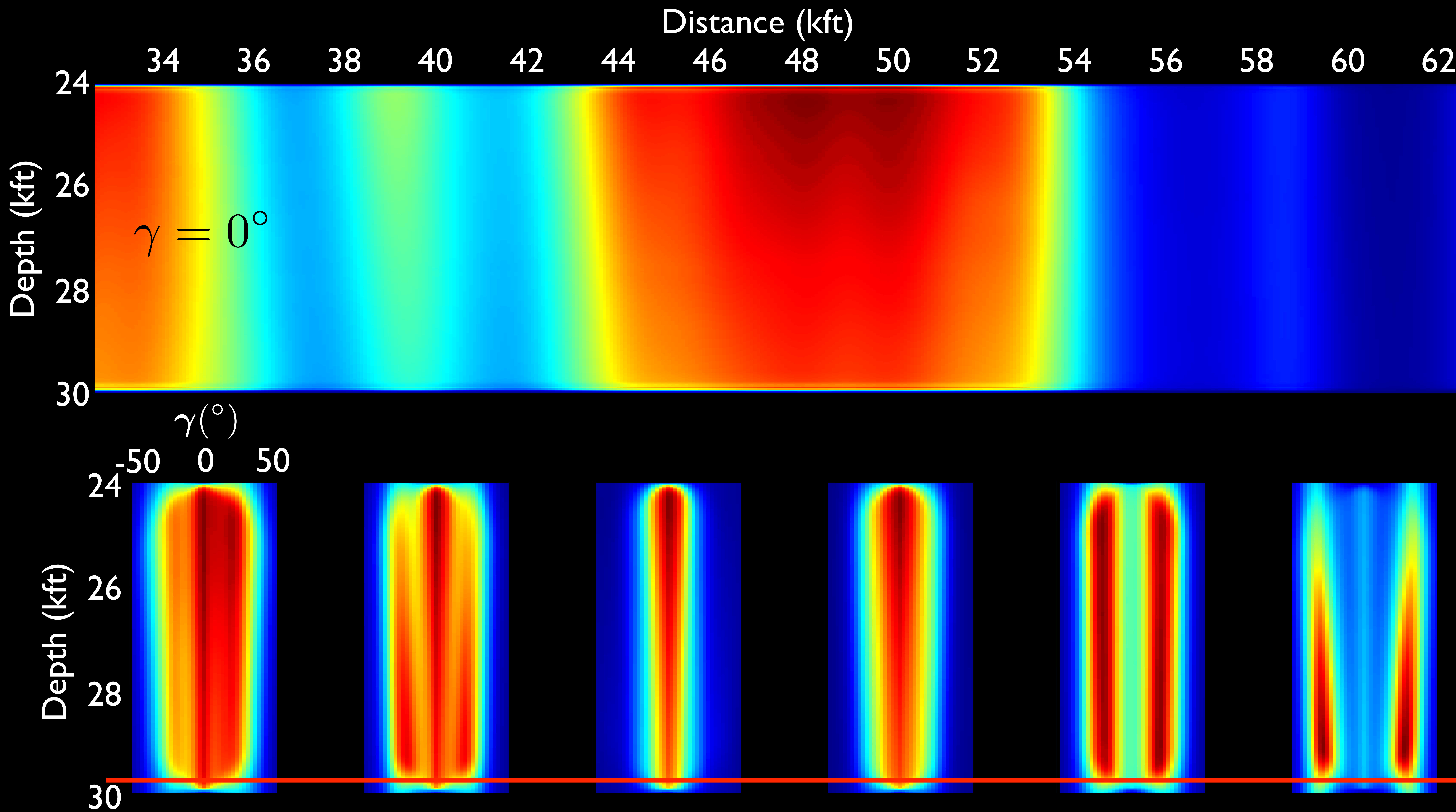




Dip and scattering-angle decomposed image $(\alpha = 0^\circ)$



Dip and scattering-angle decomposed illumination $(\alpha = 0^\circ)$



Conclusions

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Conclusions

- **Wave-equation angle-domain illumination can be computed using a simple Fourier-domain mapping**
- **Scattering-angle-domain illumination is accurate for point scatterers**
- **Dip-dependent scattering-angle-domain illumination is accurate for planar reflectors**
- **Phase-encoding significantly reduces the computational cost**

Acknowledgements

- SMAAT JV for the Sigsbee2A synthetic data set

Thanks !

Geometric relations

$$\mathbf{p}_m = \begin{pmatrix} p_{m_x} \\ p_{m_z} \end{pmatrix} = \begin{pmatrix} 2s\cos\gamma\sin\alpha \\ -2s\cos\gamma\cos\alpha \end{pmatrix}$$

$$\tan\alpha = -\frac{p_{m_x}}{p_{m_z}} = -\frac{k_{m_x}}{k_{m_z}}$$

$$\mathbf{p}_h = \begin{pmatrix} p_{h_x} \\ p_{h_z} \end{pmatrix} = \begin{pmatrix} 2s\sin\gamma\cos\alpha \\ 2s\sin\gamma\sin\alpha \end{pmatrix}$$

$$\tan\gamma = -\frac{p_{h_x}}{p_{m_z}} = -\frac{k_{h_x}}{k_{m_z}}$$

$$\tan\alpha = \frac{p_{h_z}}{p_{h_x}} = \frac{k_{h_z}}{k_{h_x}}$$

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$$H_\gamma(x,z,\gamma) = \sum_\omega \sum_\mathbf{s} \sum_\mathbf{r} |L_\gamma(x,z,\gamma,\mathbf{r},\mathbf{s},\omega)|^2$$

$$\tan\alpha = \frac{p_{h_z}}{p_{h_x}} = \frac{k_{h_z}}{k_{h_x}}$$

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$$H_{\alpha,\gamma}(x,z,\alpha,\gamma) = \sum_\omega \sum_\mathbf{s} \sum_\mathbf{r} |L_{\alpha,\gamma}(x,z,\alpha,\gamma,\mathbf{r},\mathbf{s},\omega)|^2$$

Subsurface-offset domain image and illumination

$$d(\mathbf{x}_r, \mathbf{x}_s, \omega) = \sum_{\mathbf{x}} \sum_{\mathbf{h}} L_h(\mathbf{x}, \mathbf{h}, \mathbf{x}_r, \mathbf{x}_s, \omega) m_h(\mathbf{x}, \mathbf{h})$$

$$L_h(\mathbf{x}, \mathbf{h}, \mathbf{x}_r, \mathbf{x}_s, \omega) = G(\mathbf{x} - \mathbf{h}, \mathbf{x}_s, \omega) G(\mathbf{x} + \mathbf{h}, \mathbf{x}_r, \omega)$$

$$\widehat{m}_h(\mathbf{x}, \mathbf{h}) = -\sum_{\omega} \sum_{\mathbf{x}_s} \sum_{\mathbf{x}_r} L_h^*(\mathbf{x}, \mathbf{h}, \mathbf{x}_r, \mathbf{x}_s, \omega) d_{\mathrm{obs}}(\mathbf{x}_r, \mathbf{x}_s, \omega)$$

$$H_h(\mathbf{x}, \mathbf{h}) = \sum_{\omega} \sum_{\mathbf{x}_s} \sum_{\mathbf{x}_r} |L_h(\mathbf{x}, \mathbf{h}, \mathbf{x}_r, \mathbf{x}_s, \omega)|^2$$

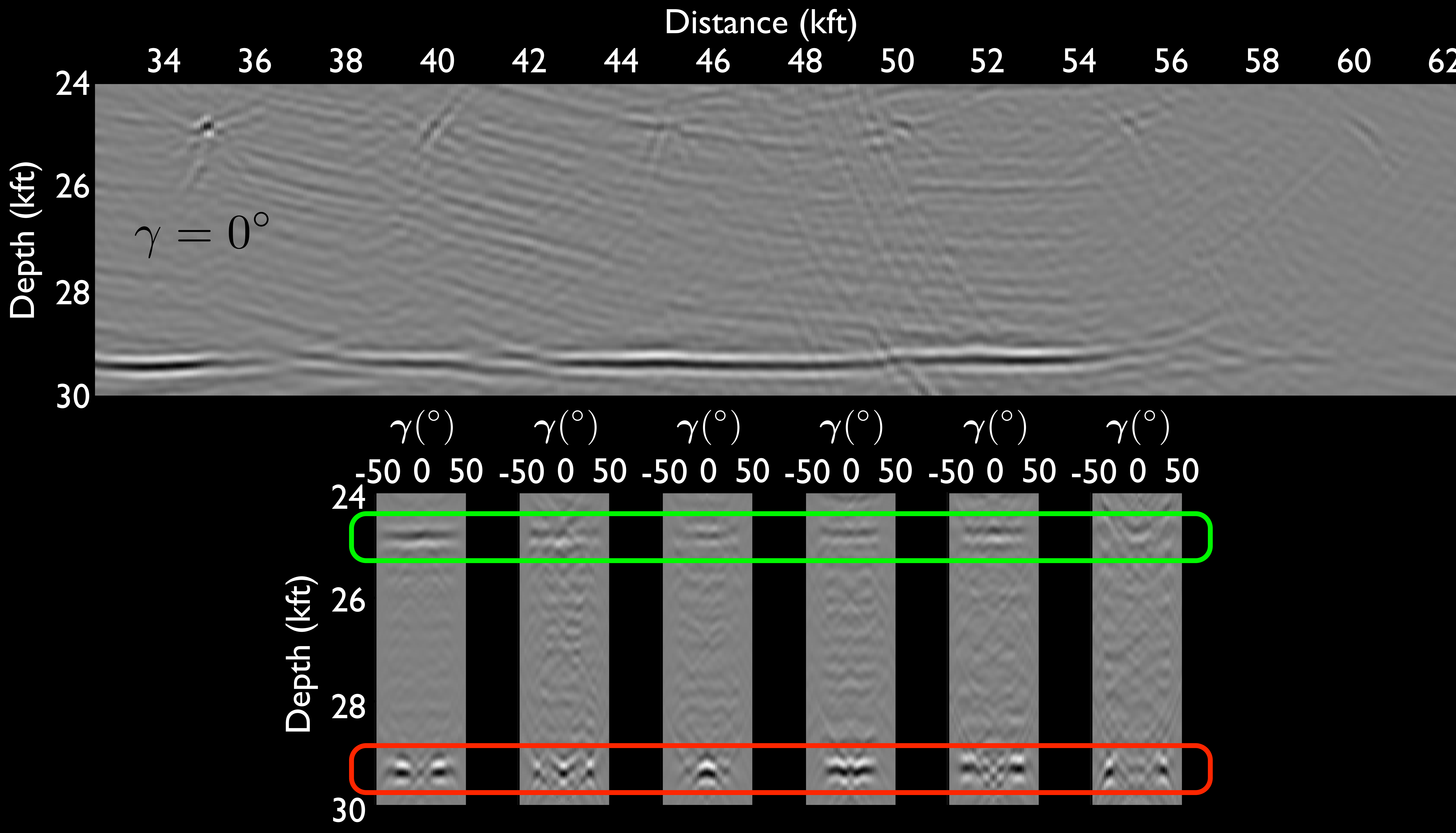
From Data to Model space

Show Sigsbee2A image to demonstrate the influence of the migration operator

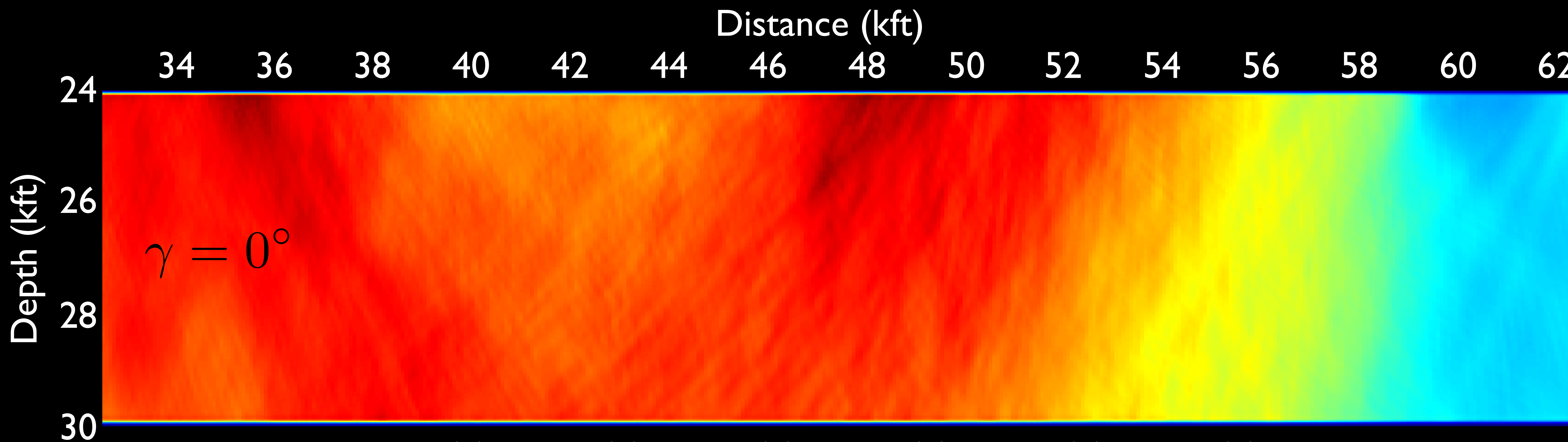
Stress the importance of analyzing the resolution of the imaging operator

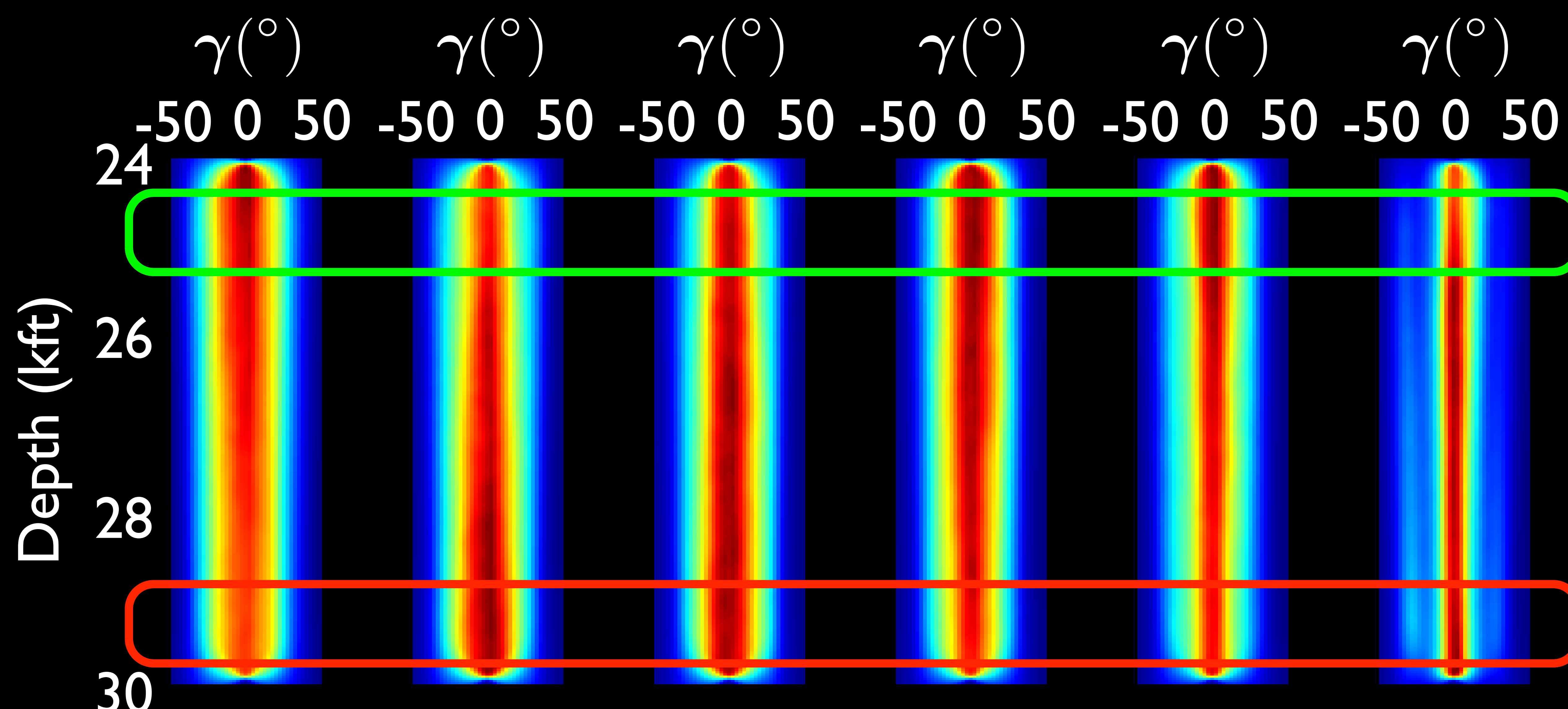
Conventional wave-equation-based resolution analysis is carried out in the CMP domain (show diagonal of Hessian). But we would like to produce angle gathers for various reasons, so analyzing angle-domain resolution becomes necessary

Scattering-angle decomposed image

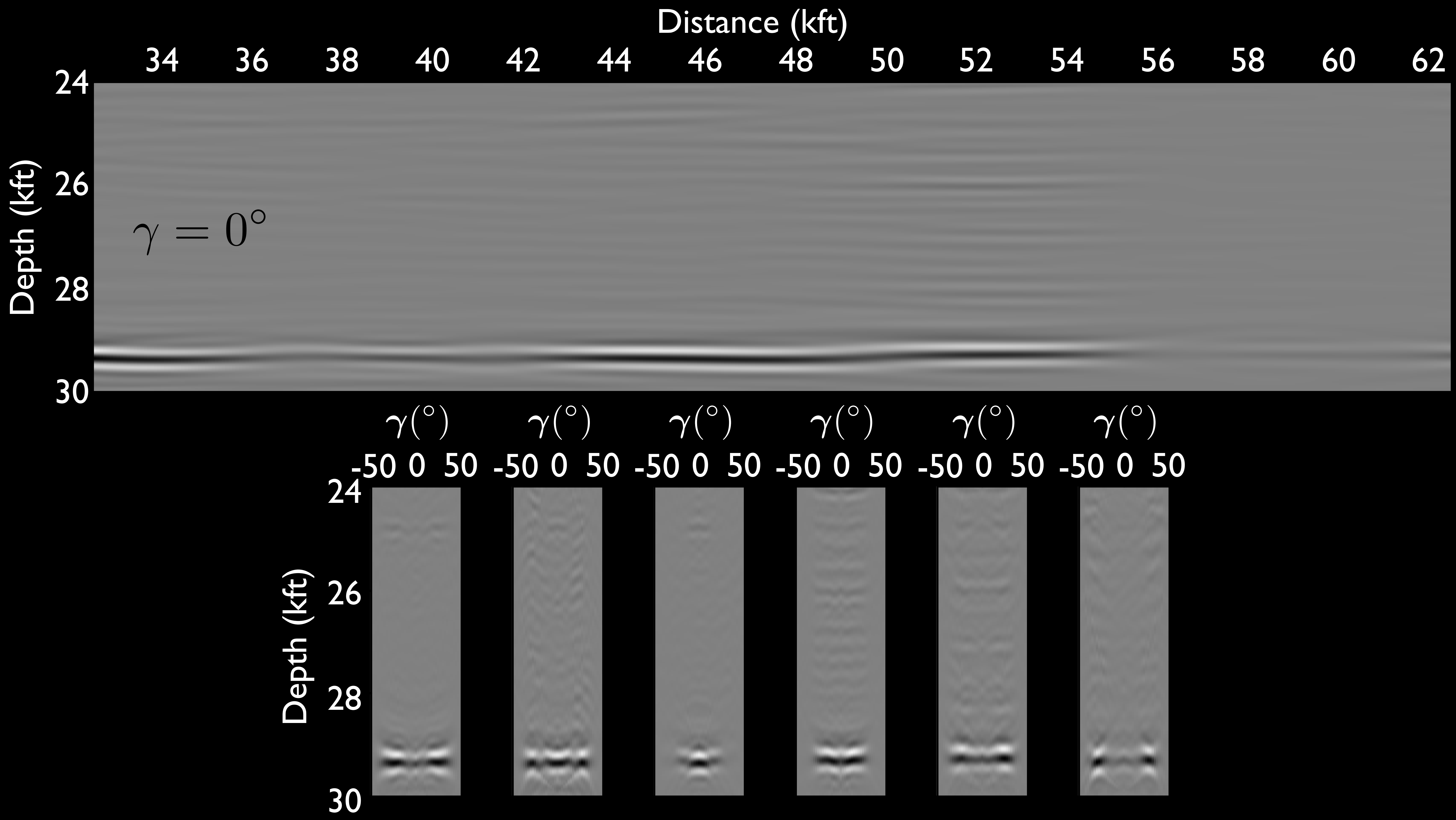


Scattering-angle decomposed illumination

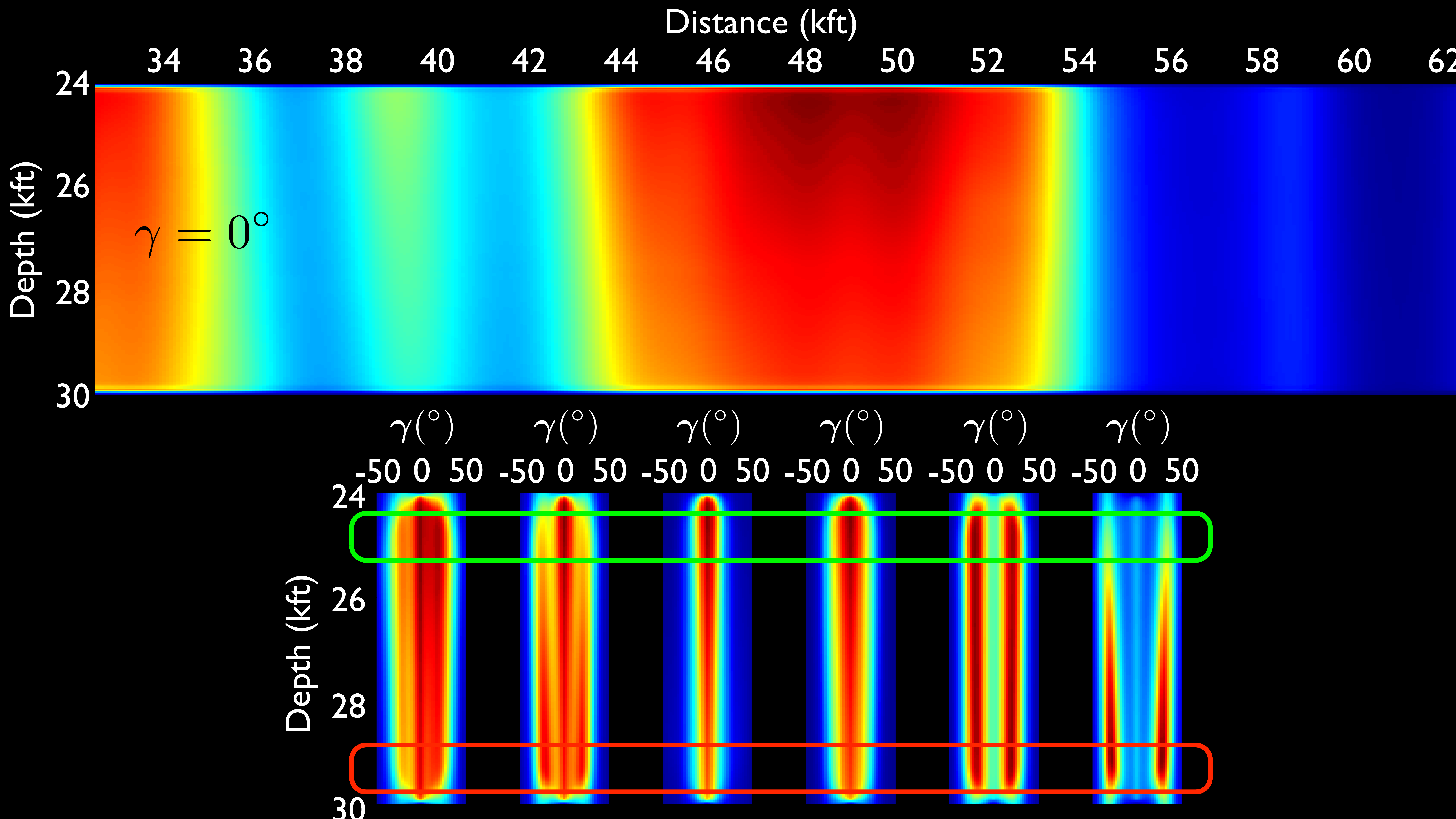




Dip and scattering-angle decomposed image $(\alpha = 0^\circ)$



Dip and reflection-angle decomposed illumination $(\alpha = 0°)$



Sigsbee2A example