Target-oriented least-squares migration/ inversion with sparseness constraints

Yaxun Tang

SEP-136, pp. 97
SEP-138, pp. 171

The adjoint Born modeling operator

Migration = adjoint of the Born modeling operator:

$$\mathbf{m}_{\mathrm{mig}} = \mathbf{L}'\mathbf{d}_{\mathrm{obs}}$$

$\mathbf{L}$: Born modeling operator

$\mathbf{d}_{\mathrm{obs}}$: Observed data

$\mathbf{m}$: Reflectivity

Standard data-space inversion

$$F(\mathbf{m}) = ||\mathbf{Lm} - \mathbf{d}_{\text{obs}}||_2^2$$

- Gradient-based optimization (Nemeth, 1999, Clapp, 2005)
- Explicit Hessian is not required
- Full domain migration/demigration at each iteration

$\mathbf{L}$: Born modeling operator

$\mathbf{d}_{\text{obs}}$: Observed data

$\mathbf{m}$: Reflectivity

Issues with least-squares migration/inversion

$$F(\mathbf{m}) = ||\mathbf{L}\mathbf{m} - \mathbf{d}_{\mathrm{obs}}||_2^2$$

- Computational cost
- Operator mismatch
- Non-unique solution

Issues with least-squares migration/inversion

$$F(\mathbf{m}) = ||\mathbf{L}\mathbf{m} - \mathbf{d}_{\mathrm{obs}}||_2^2$$

$$J(\mathbf{m}) = ||\mathbf{H}\mathbf{m} - \mathbf{m}_{\mathrm{mig}}||_2^2$$

- Computational cost
- Operator mismatch
- Non-unique solution

➡ Target-oriented inversion with phase-encoded Hessian

➡ Regularization that accurately estimate the inverse of the model covariance

Agenda

- Target-oriented inversion
- Hessian by phase encoding
- Regularization that promotes sparsity
- Synthetic data examples
- Conclusions

Model-space two-step inversion

Step 1: $$\mathbf{H} = \frac{\partial^2 F(\mathbf{m})}{\partial \mathbf{m}^2} = \mathbf{L}'\mathbf{L}$$

Step 2: $$J(\mathbf{m}) = ||\mathbf{H}\mathbf{m} - \mathbf{m}_{\mathrm{mig}}||_2^2$$

- Explicit Hessian is required
- Target-oriented inversion (Valenciano, 2008)
- Different regularization can be easily tested without extra cost

Explicit Hessian formula

$$H(\mathbf{x},\mathbf{y}) = \sum_{\omega}\sum_{\mathbf{x}_s} G(\mathbf{x},\mathbf{x}_s,\omega)G'(\mathbf{y},\mathbf{x}_s,\omega) \times \sum_{\mathbf{x}_r} w(\mathbf{x}_r,\mathbf{x}_s)G(\mathbf{x},\mathbf{x}_r,\omega)G'(\mathbf{y},\mathbf{x}_r,\omega)$$

Explicit Hessian formula

$$H(\mathbf{x},\mathbf{y}) = \sum_{\omega}\sum_{\mathbf{x}_s} G(\mathbf{x},\mathbf{x}_s,\omega)G'(\mathbf{y},\mathbf{x}_s,\omega)$$
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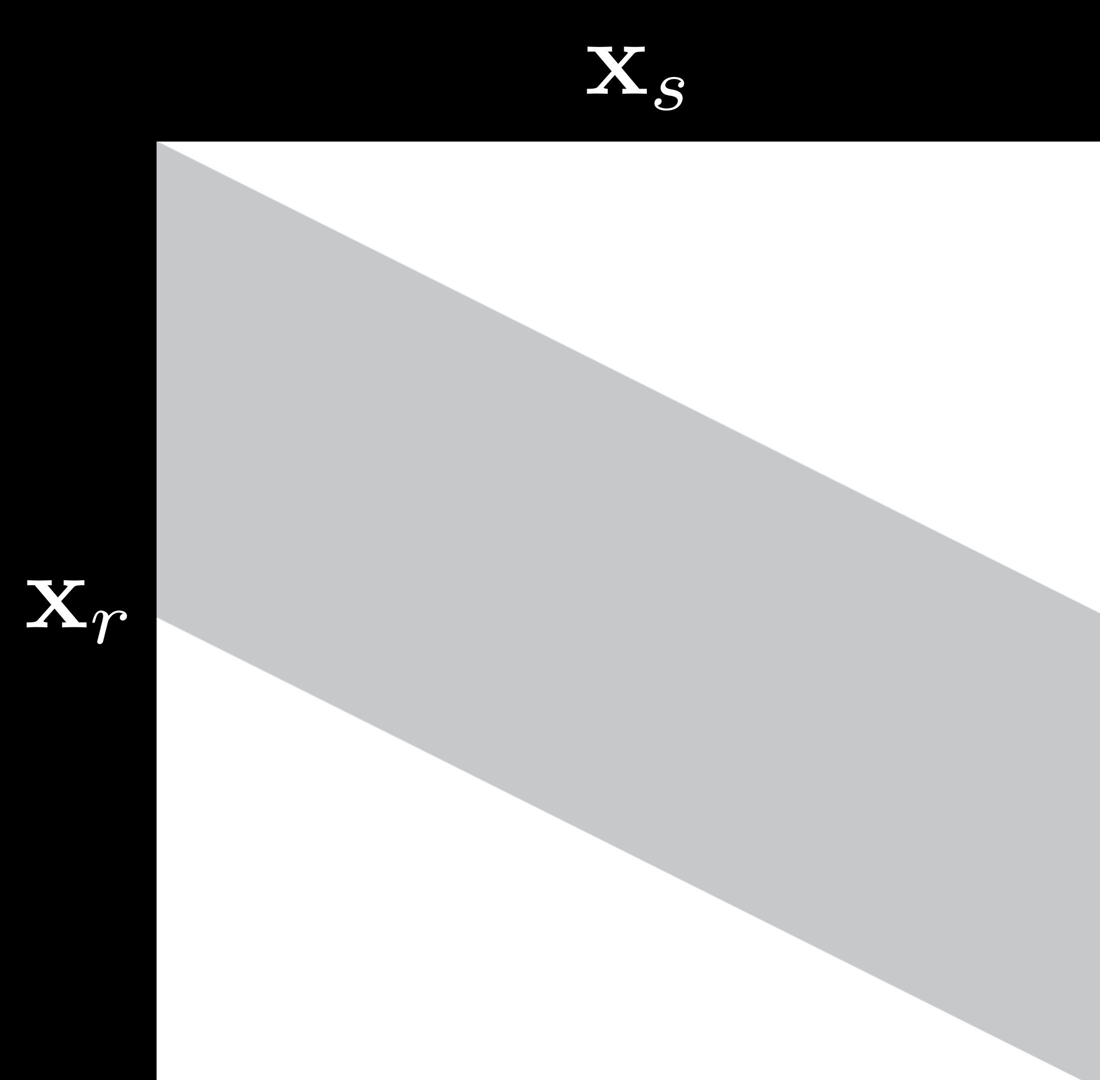
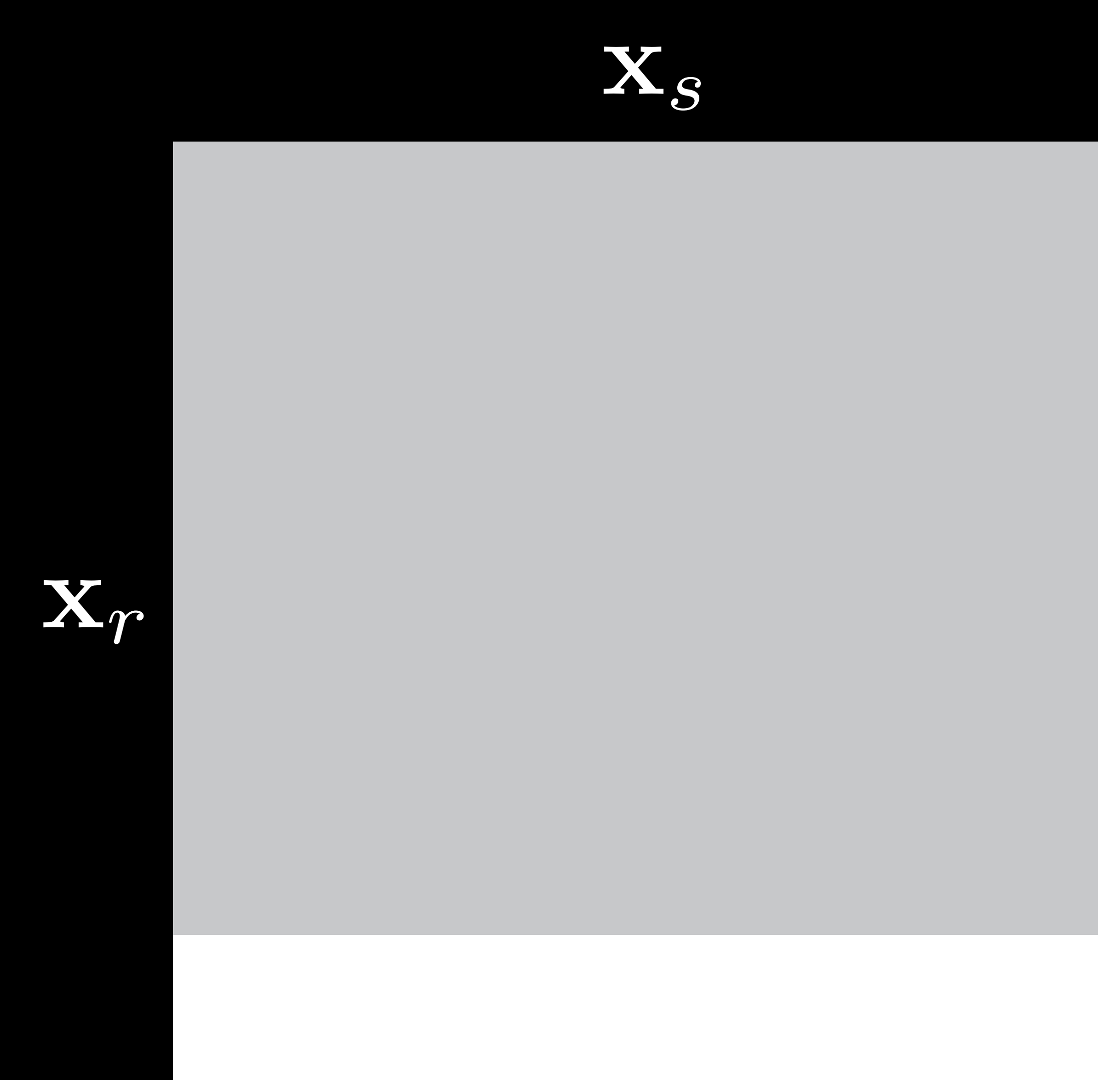


2-D marine



2-D OBS

Explicit Hessian formula

$$H(\mathbf{x},\mathbf{y}) = \sum_{\omega}\sum_{\mathbf{x}_s} \boxed{G(\mathbf{x},\mathbf{x}_s,\omega)G'(\mathbf{y},\mathbf{x}_s,\omega)}$$

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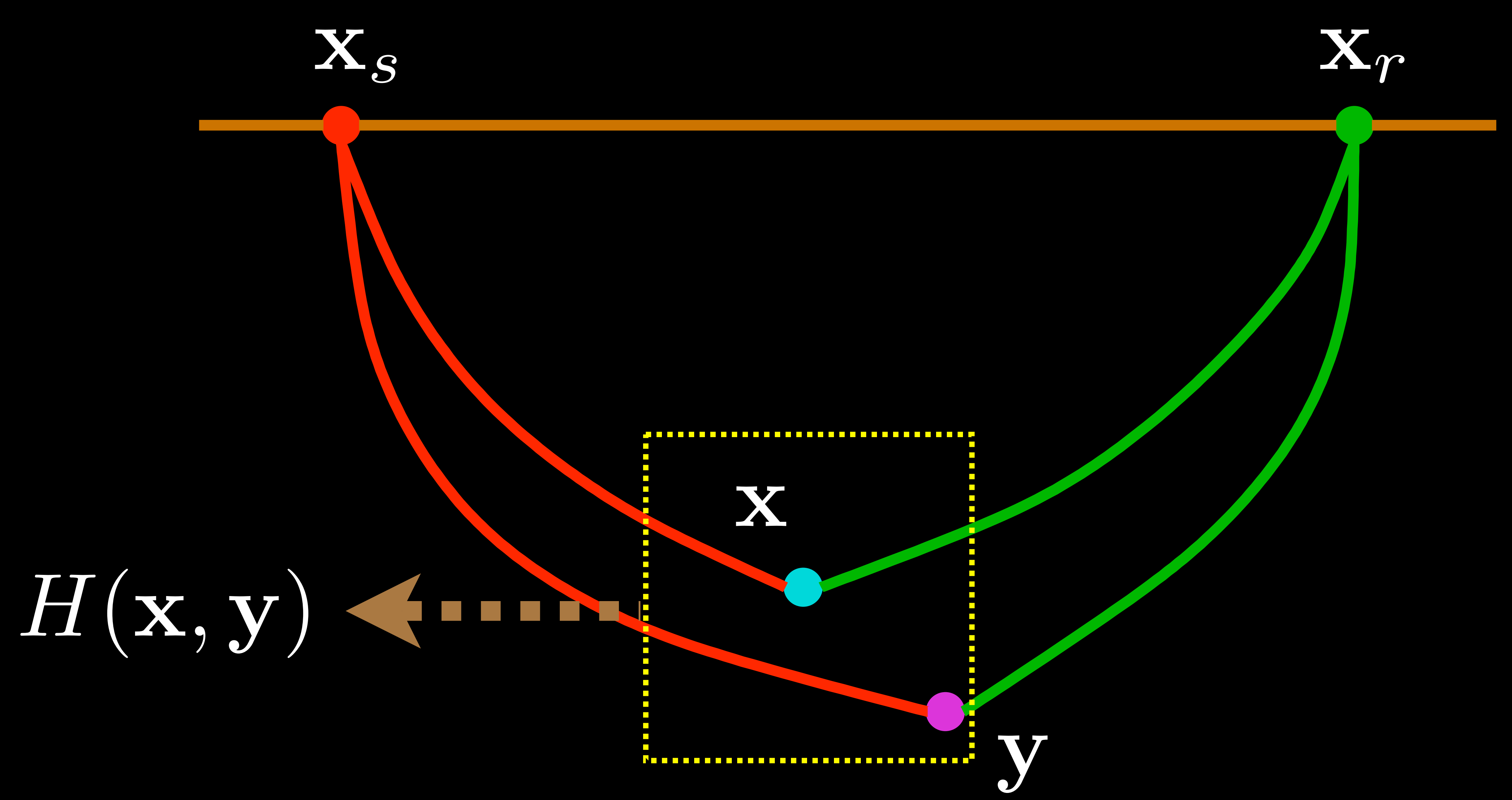

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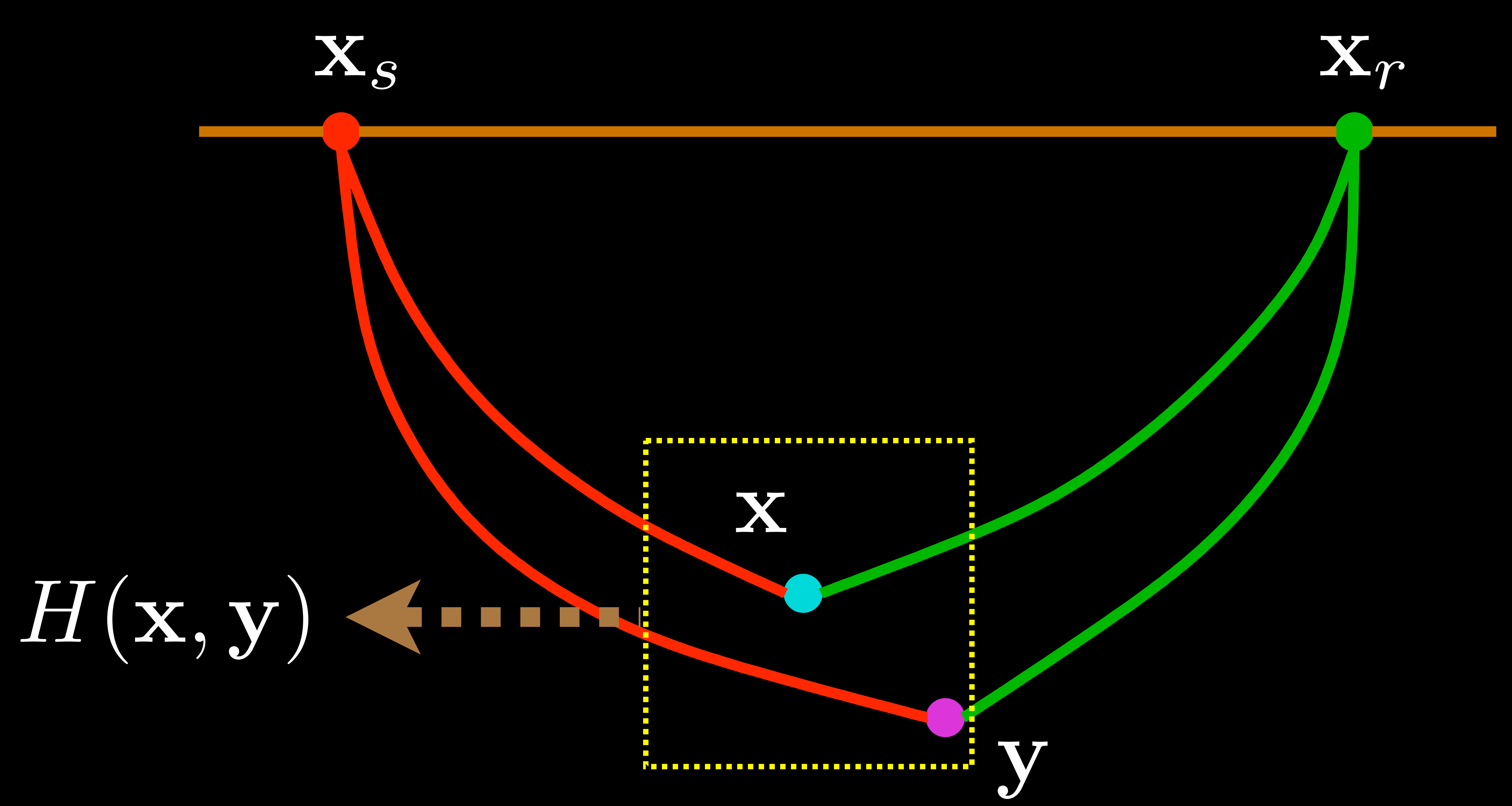

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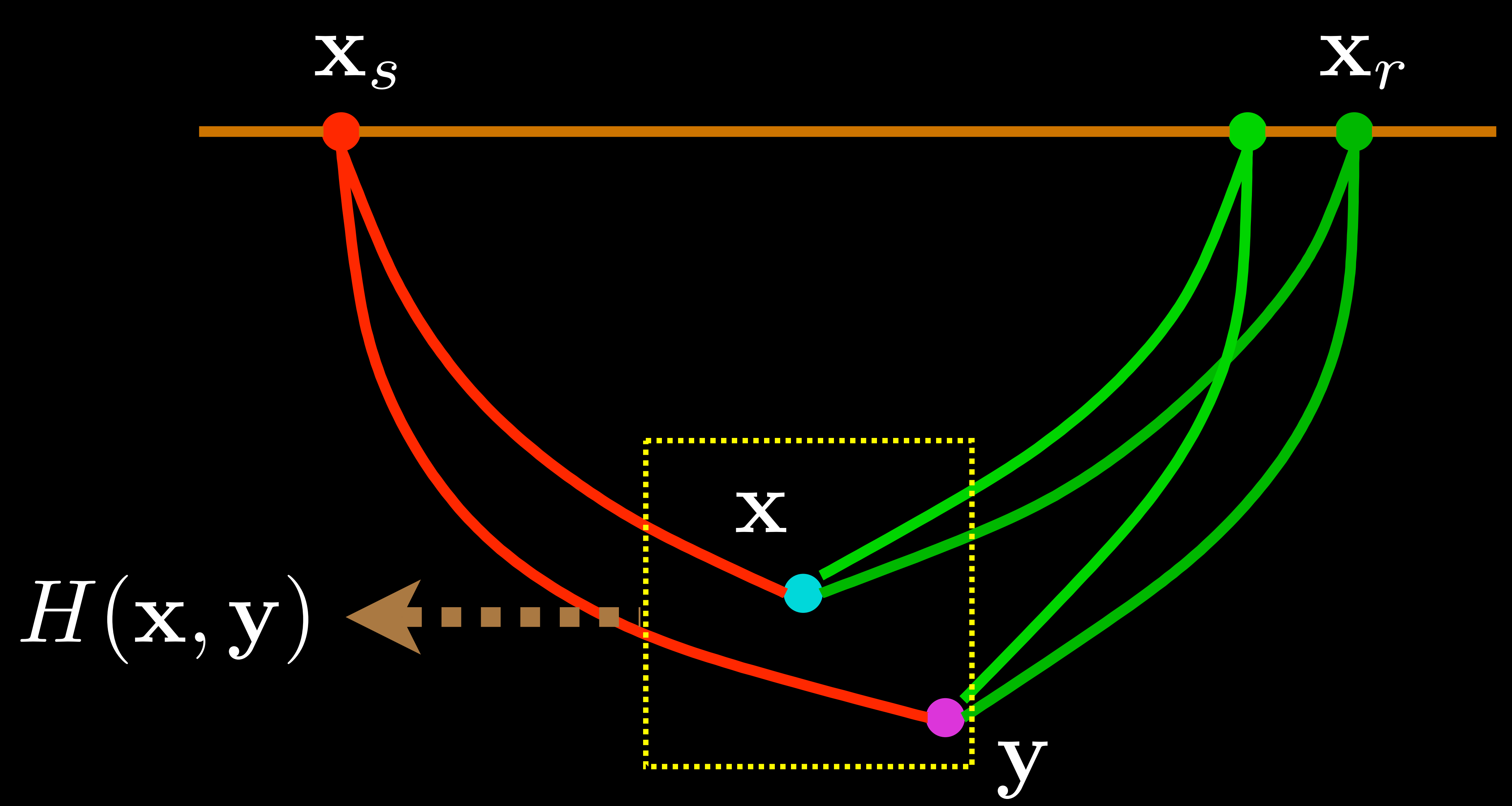

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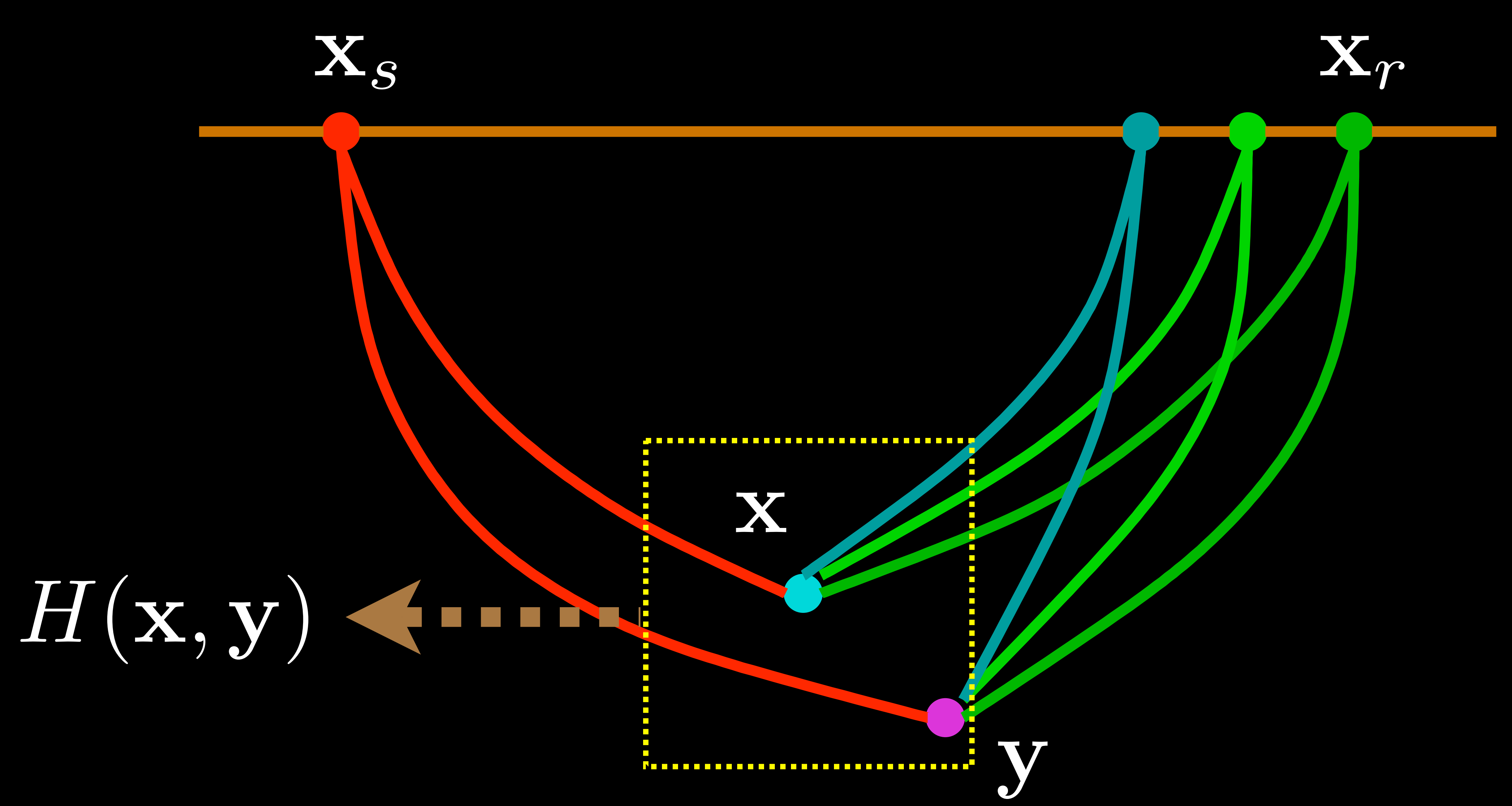

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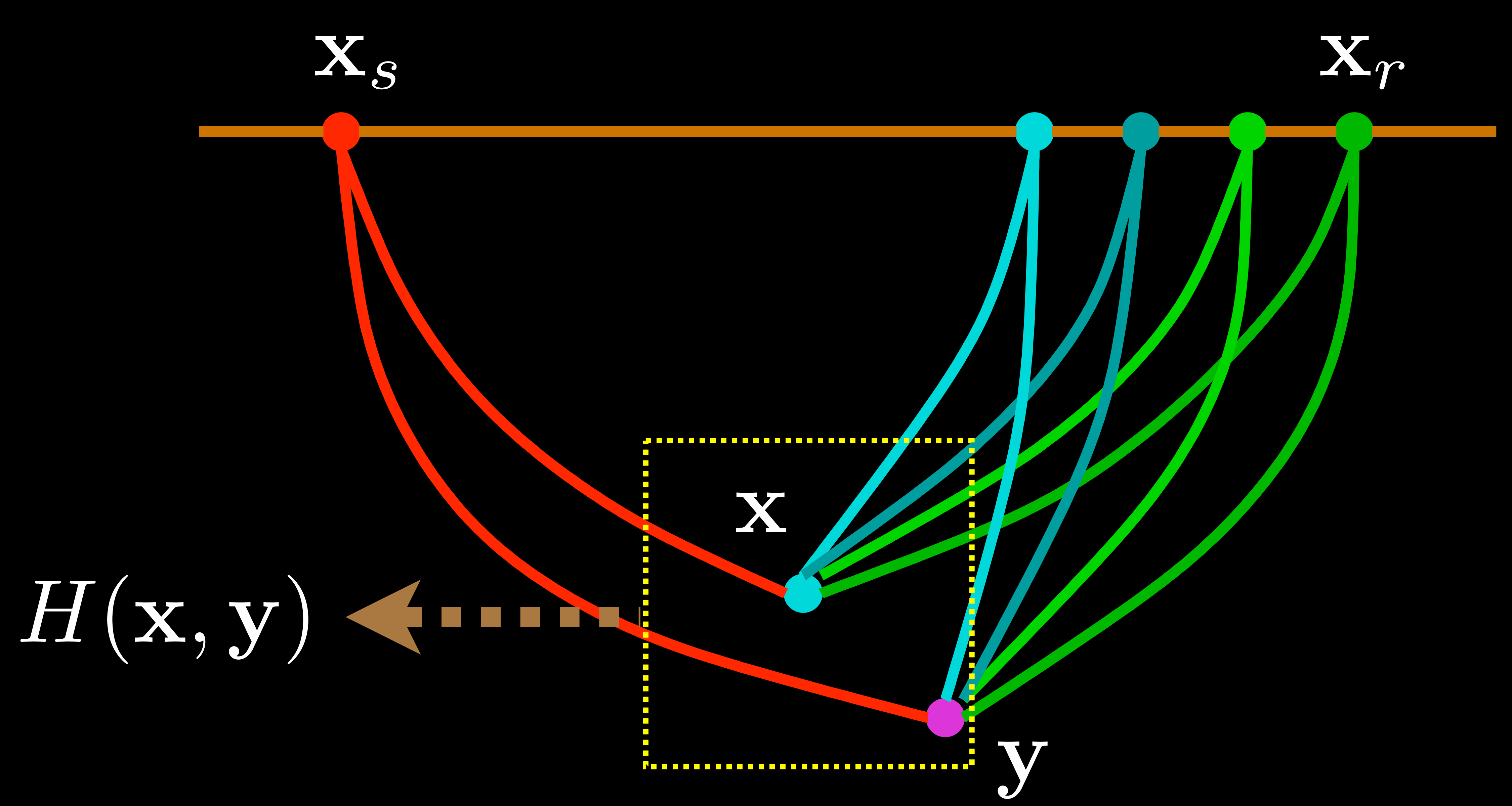

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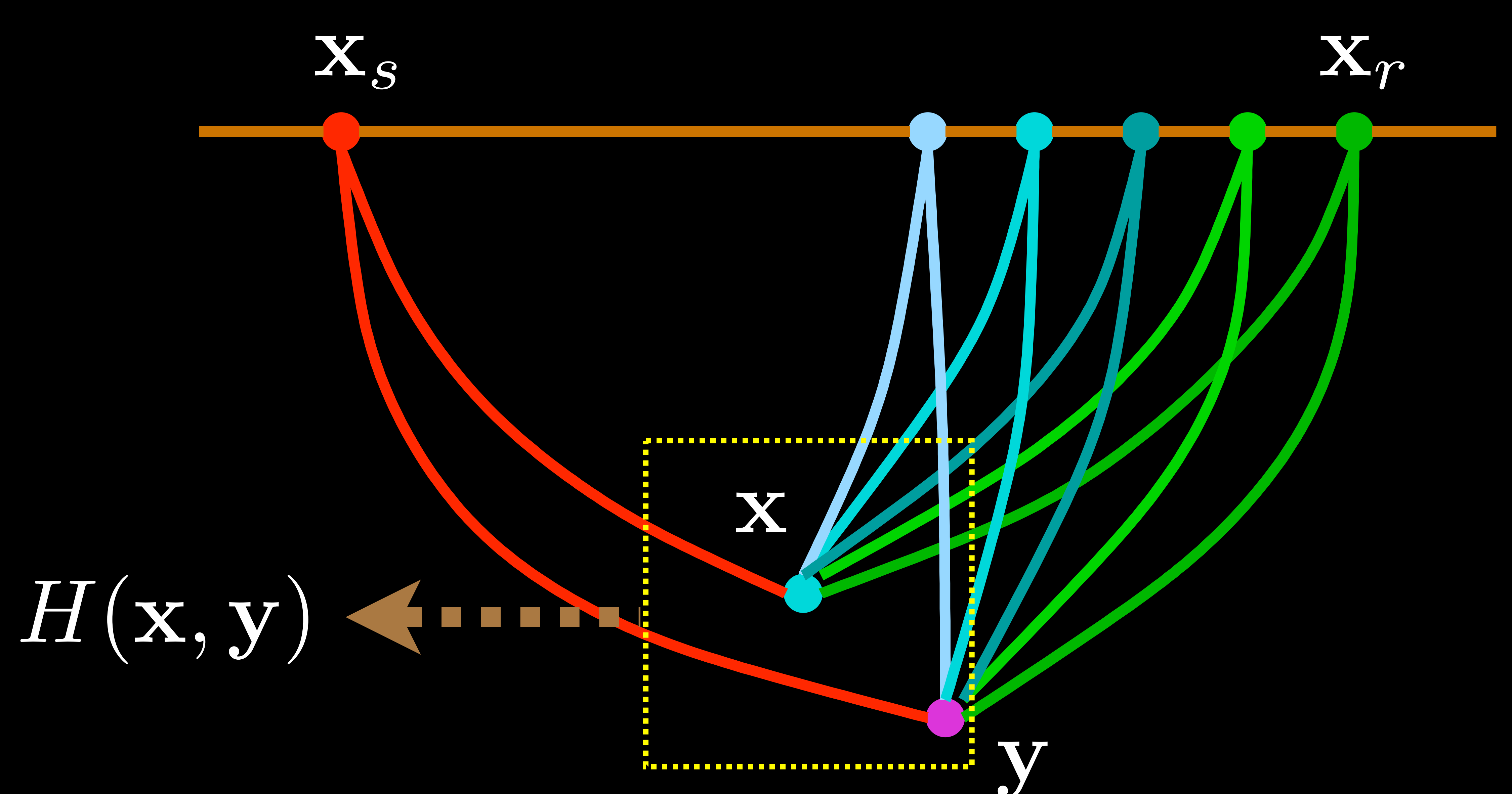


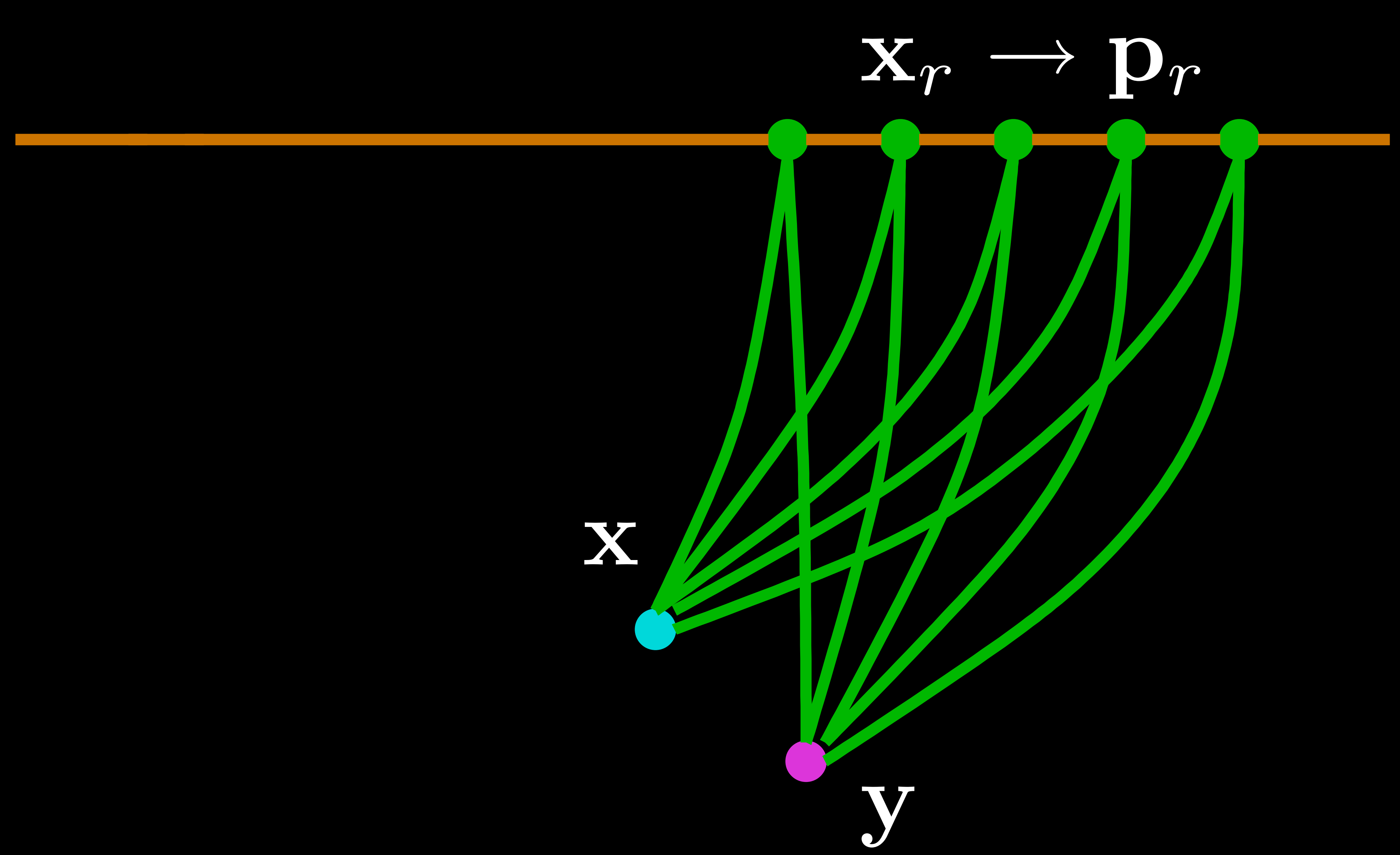
Encoding of the receiver Green's functions

Receiver-side encoded Green's function:

$$R(\mathbf{x}, \mathbf{p}_r, \omega; \mathbf{x}_s) = \sum_{\mathbf{x}_r} w(\mathbf{x}_r, \mathbf{x}_s) G(\mathbf{x}, \mathbf{x}_r, \omega) \beta(\mathbf{x}_r, \mathbf{p}_r, \omega)$$

Receiver-side phase-encoded Hessian

$$\widetilde{H}(\mathbf{x},\mathbf{y},\mathbf{p}_r) = \sum_{\omega}\sum_{\mathbf{x}_s} G(\mathbf{x},\mathbf{x}_s,\omega)G'(\mathbf{y},\mathbf{x}_s,\omega)$$
$$\times R(\mathbf{x},\mathbf{p}_r,\omega;\mathbf{x}_s)R'(\mathbf{y},\mathbf{p}_r,\omega;\mathbf{x}_s)$$



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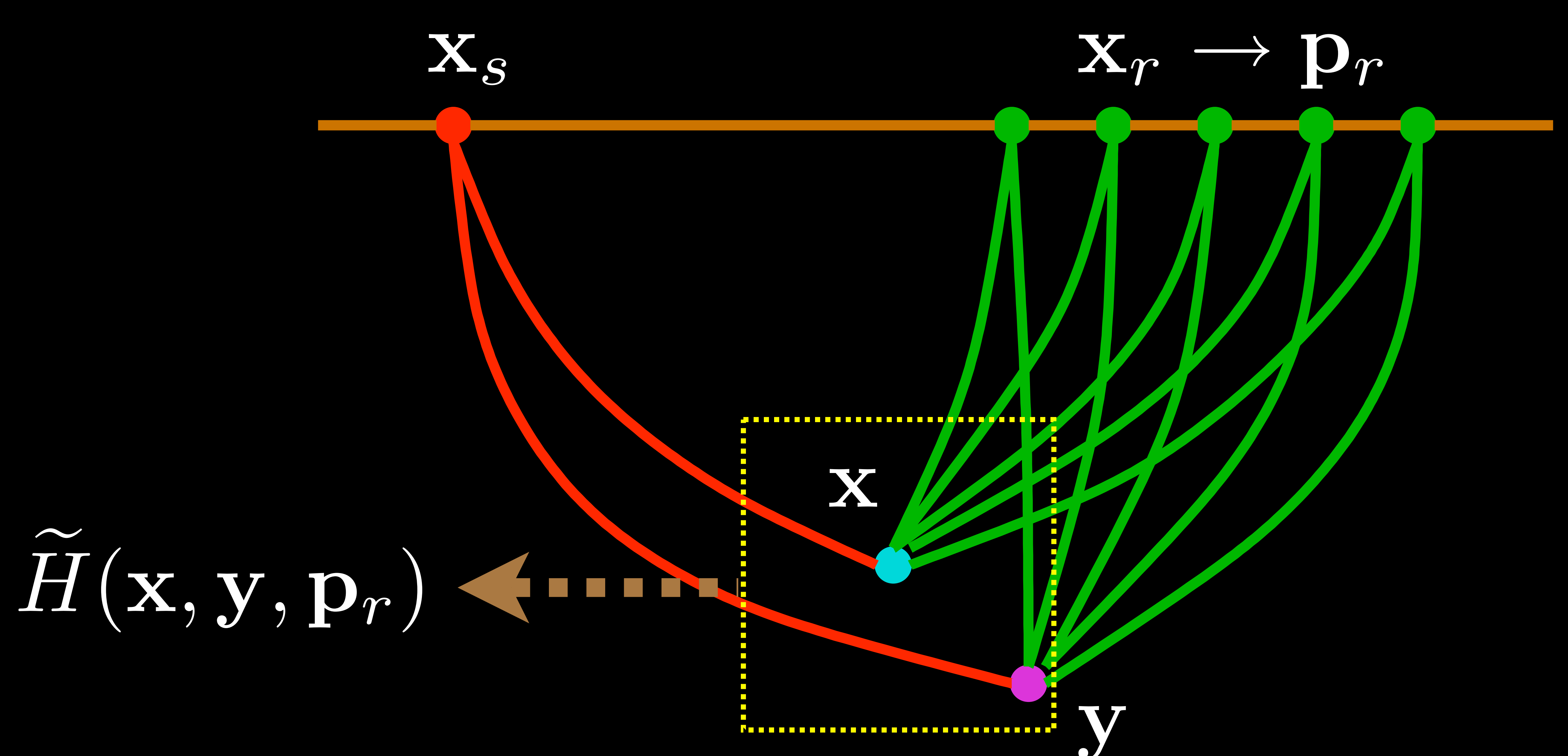


Receiver-side randomly encoded Hessian

Random-phase function:

$$\beta(\mathbf{x}_r, \mathbf{p}_r, \omega) = e^{i\gamma(\mathbf{x}_r, \mathbf{p}_r, \omega)}$$

If there are many sources, one realization is sufficient to attenuate most of the crosstalk:

$$\widetilde{H}(\mathbf{x}, \mathbf{y}, \mathbf{p}_r) \approx H(\mathbf{x}, \mathbf{y})$$

Cost = One shot-profile migration

Encoding of source and receiver Green's functions

For OBS or land acquisition geometry: $w(\mathbf{x}_r, \mathbf{x}_s) = w_r(\mathbf{x}_r) w_s(\mathbf{x}_s)$

Source-side encoded Green's function:

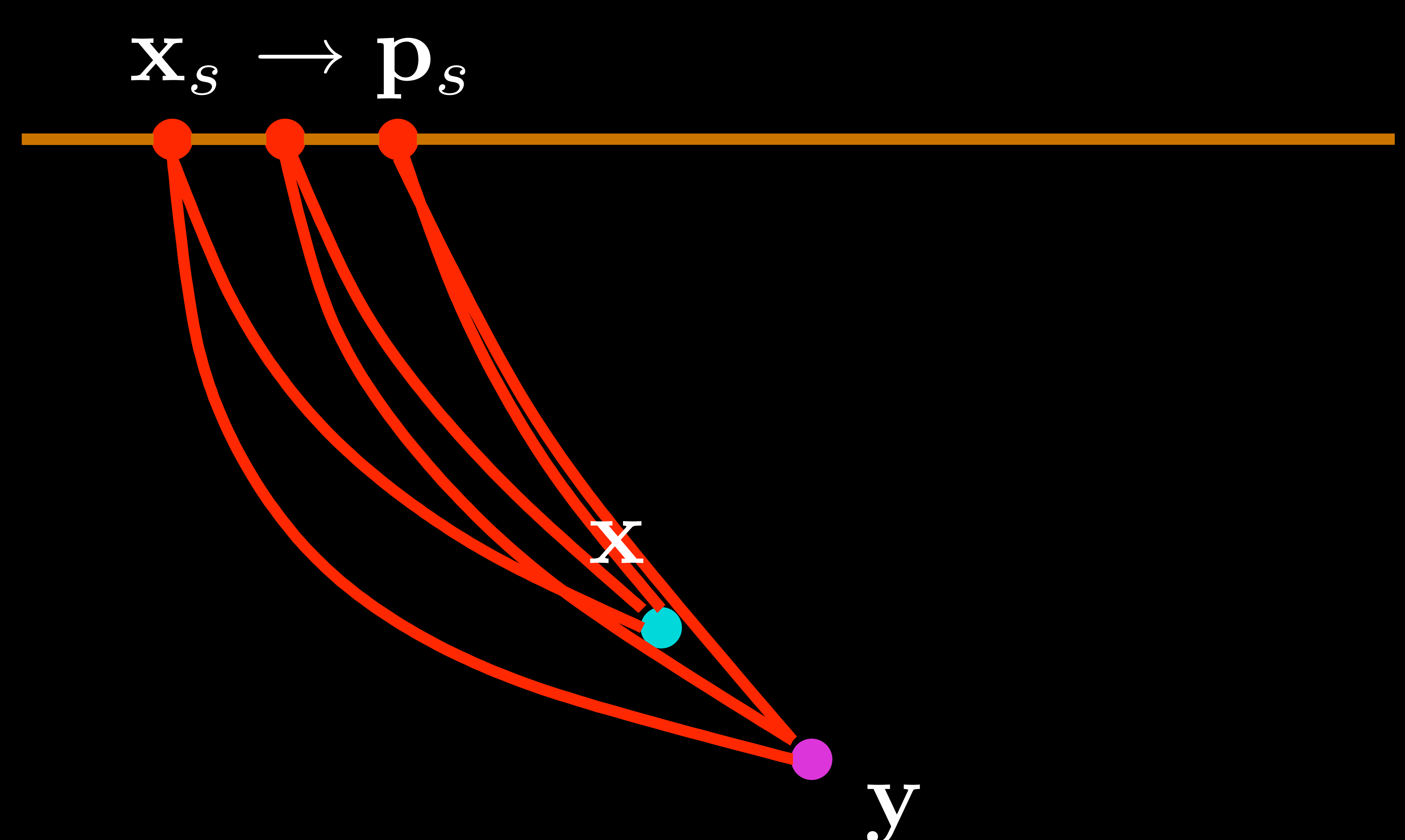
$$S(\mathbf{x}, \mathbf{p}_s, \omega) = \sum_{\mathbf{x}_s} w_s(\mathbf{x}_s) G(\mathbf{x}, \mathbf{x}_s, \omega) \alpha(\mathbf{x}_s, \mathbf{p}_s, \omega)$$

Receiver-side encoded Green's function:

$$R(\mathbf{x}, \mathbf{p}_r, \omega) = \sum_{\mathbf{x}_r} w_r(\mathbf{x}_r) G(\mathbf{x}, \mathbf{x}_r, \omega) \beta(\mathbf{x}_r, \mathbf{p}_r, \omega)$$

Simultaneously phase-encoded Hessian

$$\widetilde{H}(\mathbf{x},\mathbf{y},\mathbf{p}_s,\mathbf{p}_r) = \sum_{\omega} S(\mathbf{x},\mathbf{p}_s,\omega)S'(\mathbf{y},\mathbf{p}_s,\omega) \times R(\mathbf{x},\mathbf{p}_r,\omega)R'(\mathbf{y},\mathbf{p}_r,\omega)$$



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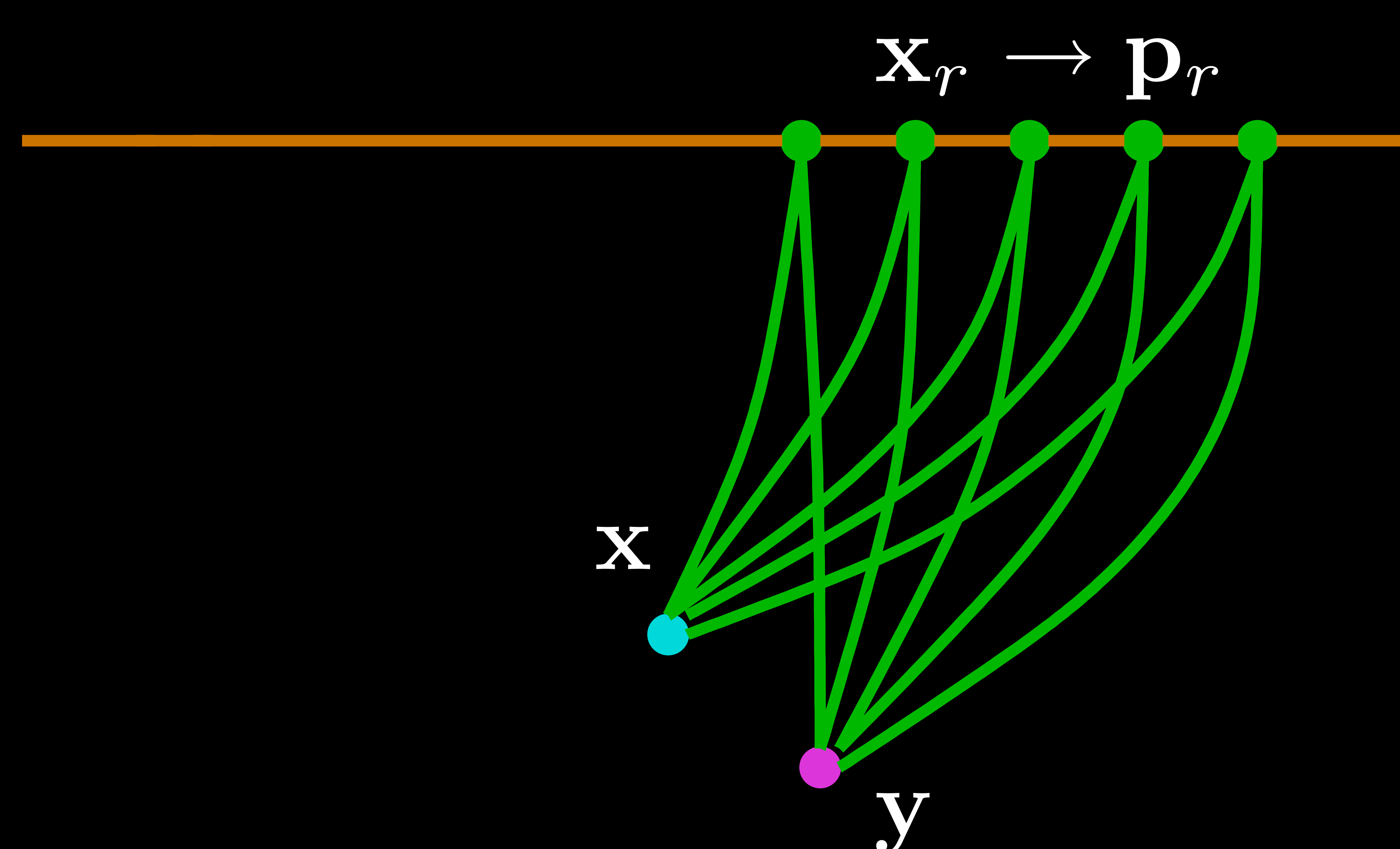
$$\widetilde{H}(\mathbf{x},\mathbf{y},\mathbf{p}_s,\mathbf{p}_r) = \sum_{\omega} S(\mathbf{x},\mathbf{p}_s,\omega)S'(\mathbf{y},\mathbf{p}_s,\omega) \times R(\mathbf{x},\mathbf{p}_r,\omega)R'(\mathbf{y},\mathbf{p}_r,\omega)$$



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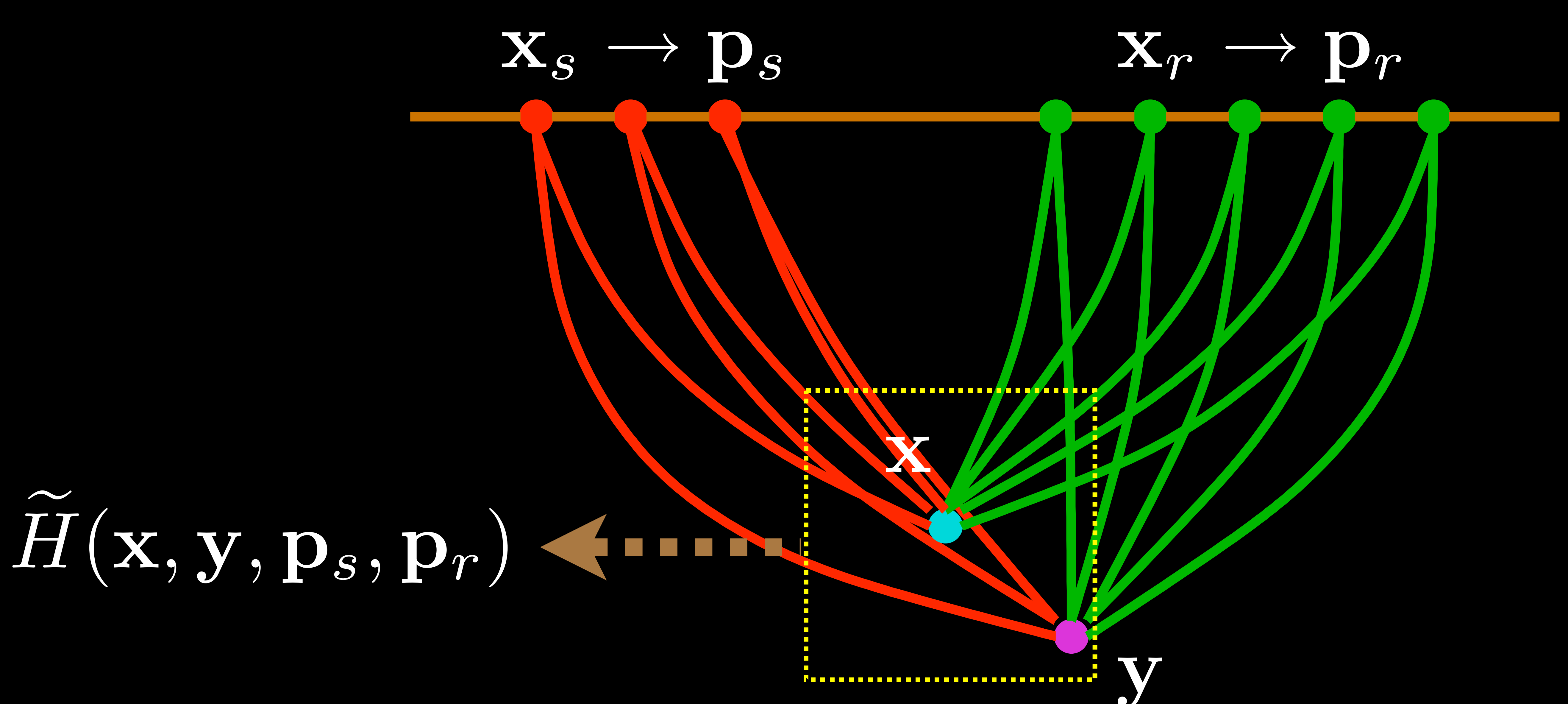


A mixed phase-encoding scheme

Linear phase function for sources: $\alpha(\mathbf{x}_s, \mathbf{p}_s, \omega) = e^{i\omega \mathbf{p}_s \cdot \mathbf{x}_s}$

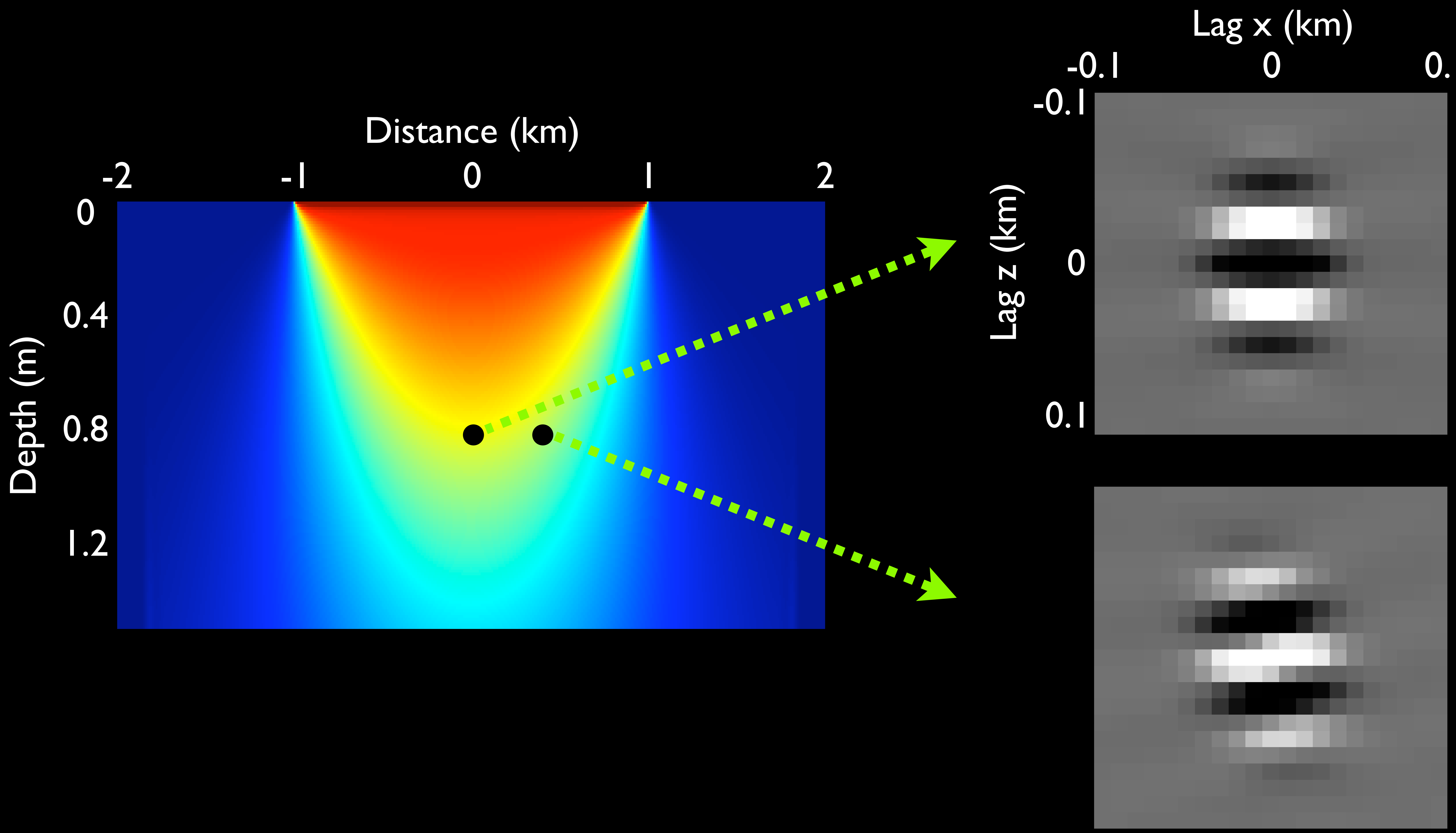
Random phase function for receivers: $\beta(\mathbf{x}_r, \mathbf{p}_r, \omega) = e^{i\gamma(\mathbf{x}_r, \mathbf{p}_r, \omega)}$

Stacking over source-ray parameters

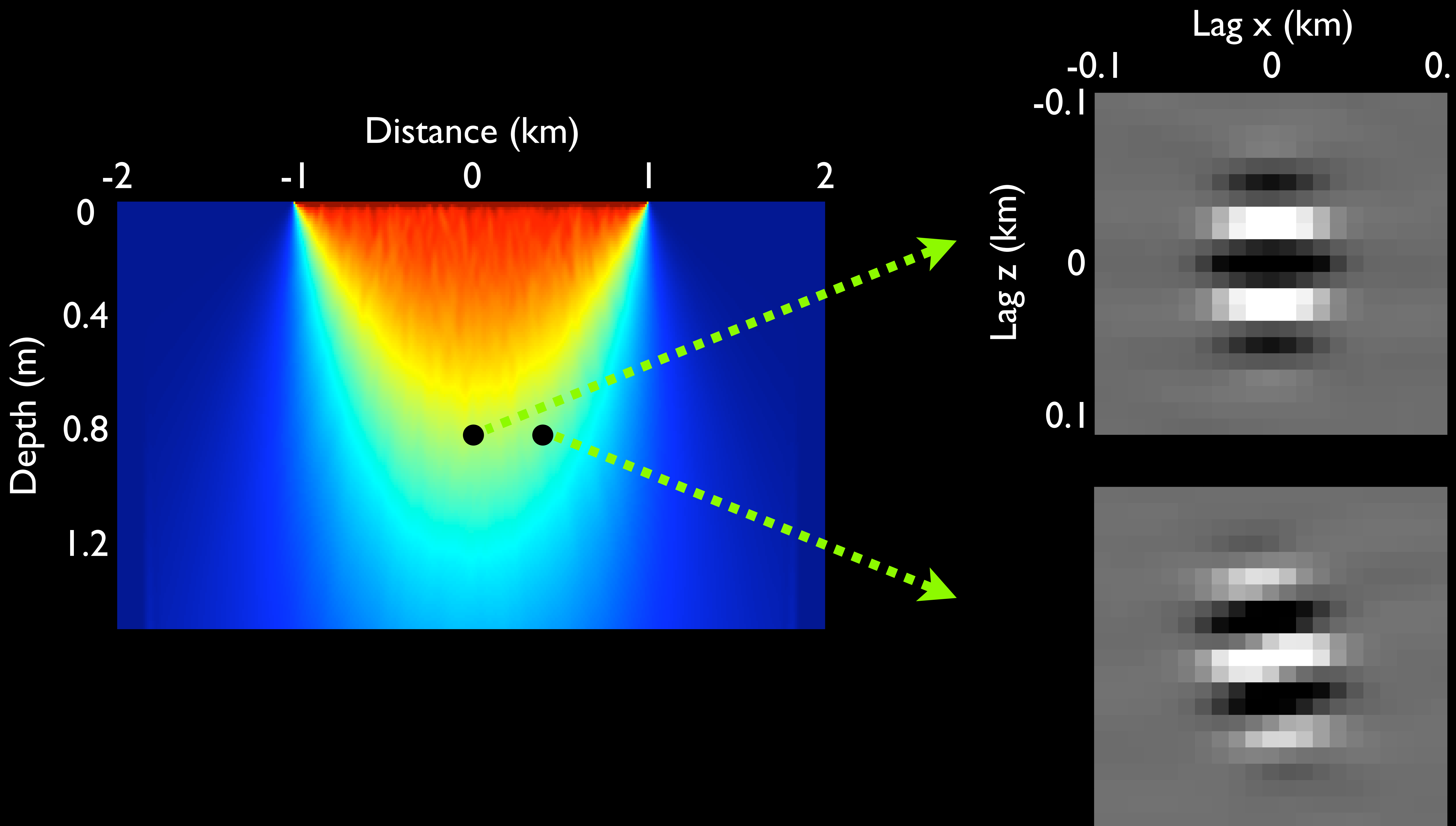
$$\sum_{\mathbf{p}_s} \widetilde{H}(\mathbf{x}, \mathbf{y}, \mathbf{p}_s, \mathbf{p}_r) \approx H(\mathbf{x}, \mathbf{y})$$

Cost = One plane-wave source migration

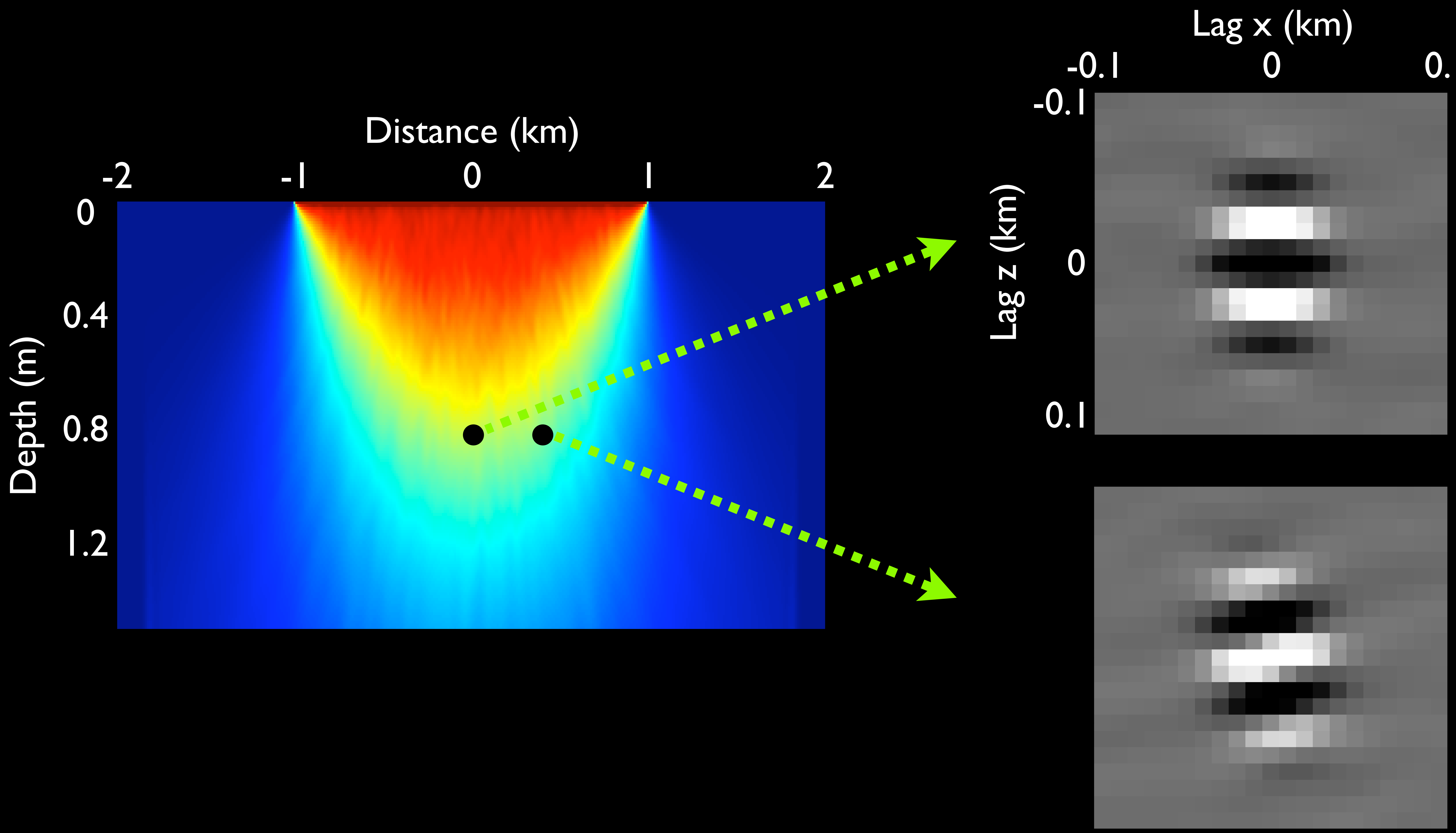
Exact Hessian



Receiver-side random-phase encoding



Mixed phase encoding



Issues with least-squares migration/inversion

$$F(\mathbf{m}) = ||\mathbf{L}\mathbf{m} - \mathbf{d}_{\mathrm{obs}}||_2^2$$

$$J(\mathbf{m}) = ||\mathbf{H}\mathbf{m} - \mathbf{m}_{\mathrm{mig}}||_2^2$$

- Computational cost
- Operator mismatch
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➡ Target-oriented inversion with phase-encoded Hessian

➡ Regularization that accurately estimate the inverse of the model covariance

Constraint that promotes sparsity

Regularized inversion

$$J(\mathbf{m}) = ||\mathbf{H}\mathbf{m} - \mathbf{m}_{\mathrm{mig}}||_2^2 + \epsilon S(\mathbf{m})$$

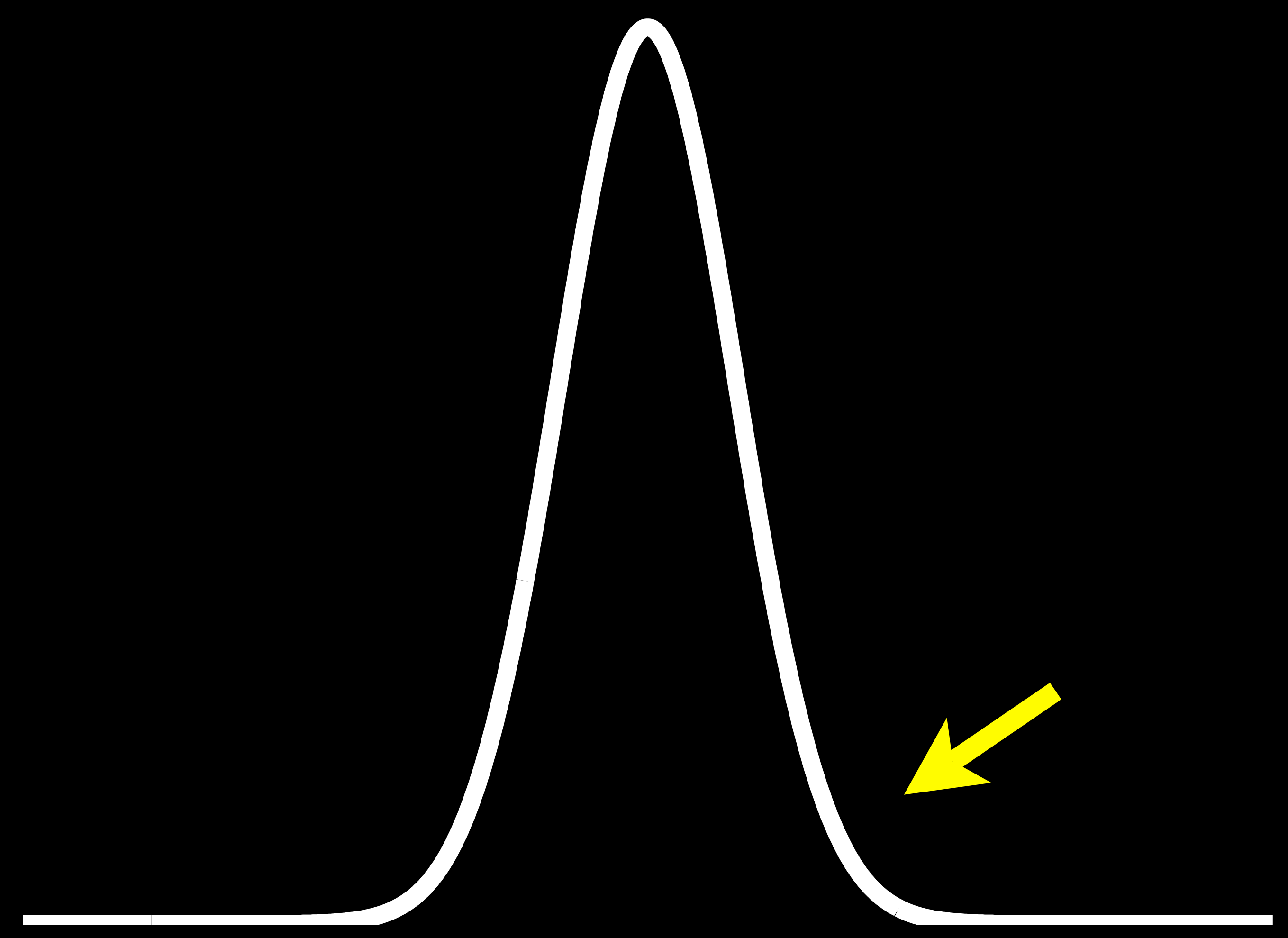
L2-norm damping: $$S(\mathbf{m}) = ||\mathbf{m}||_2^2$$

Sparseness constraint:
(Cauchy norm)

$$S(\mathbf{m}) = \sum_{\mathbf{x}} \log(1 + m^2(\mathbf{x})/\sigma^2)$$

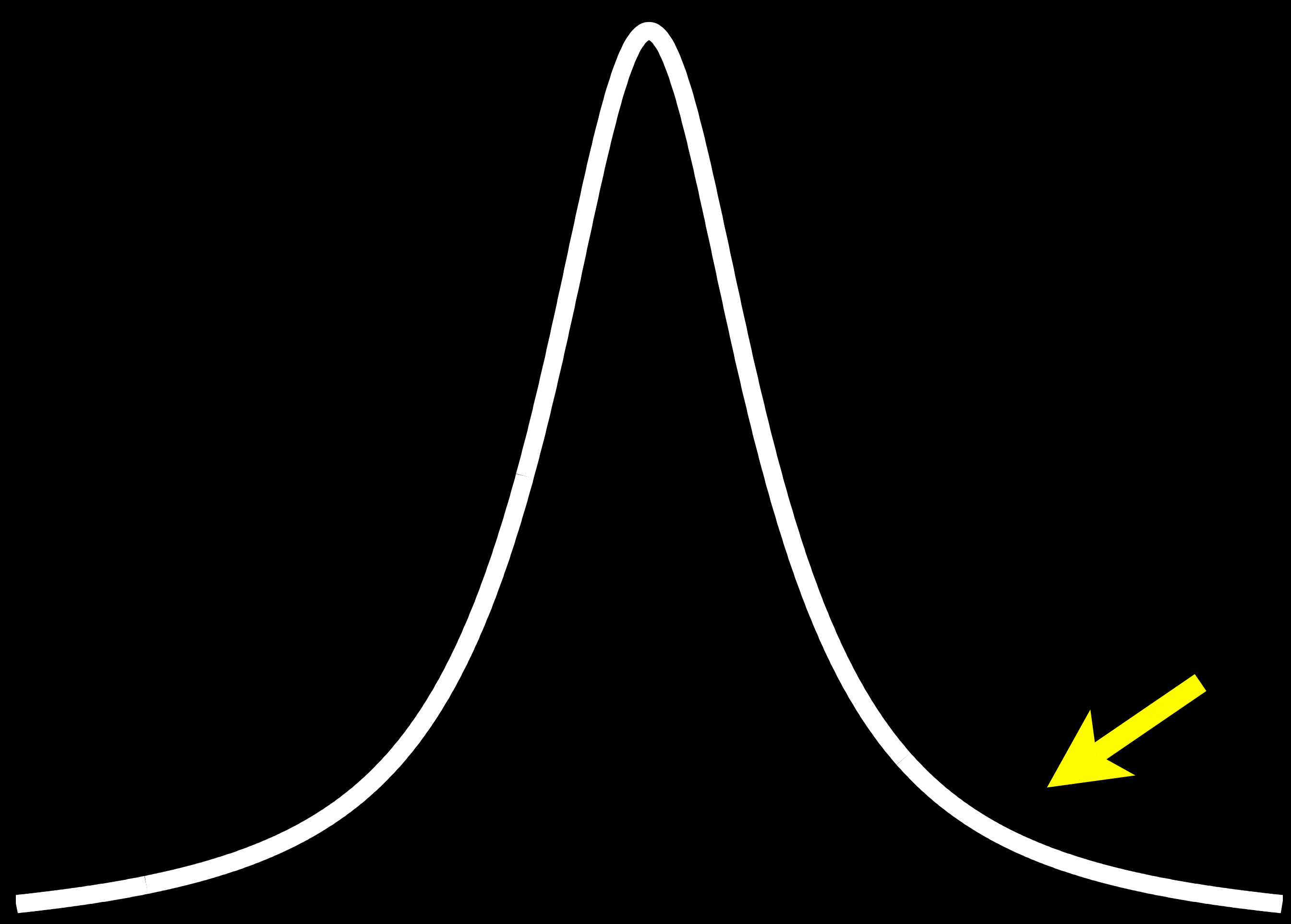
Cauchy distribution has longer tails

Gaussian distribution



$$S(\mathbf{m}) = ||\mathbf{m}||_2^2$$

Cauchy distribution



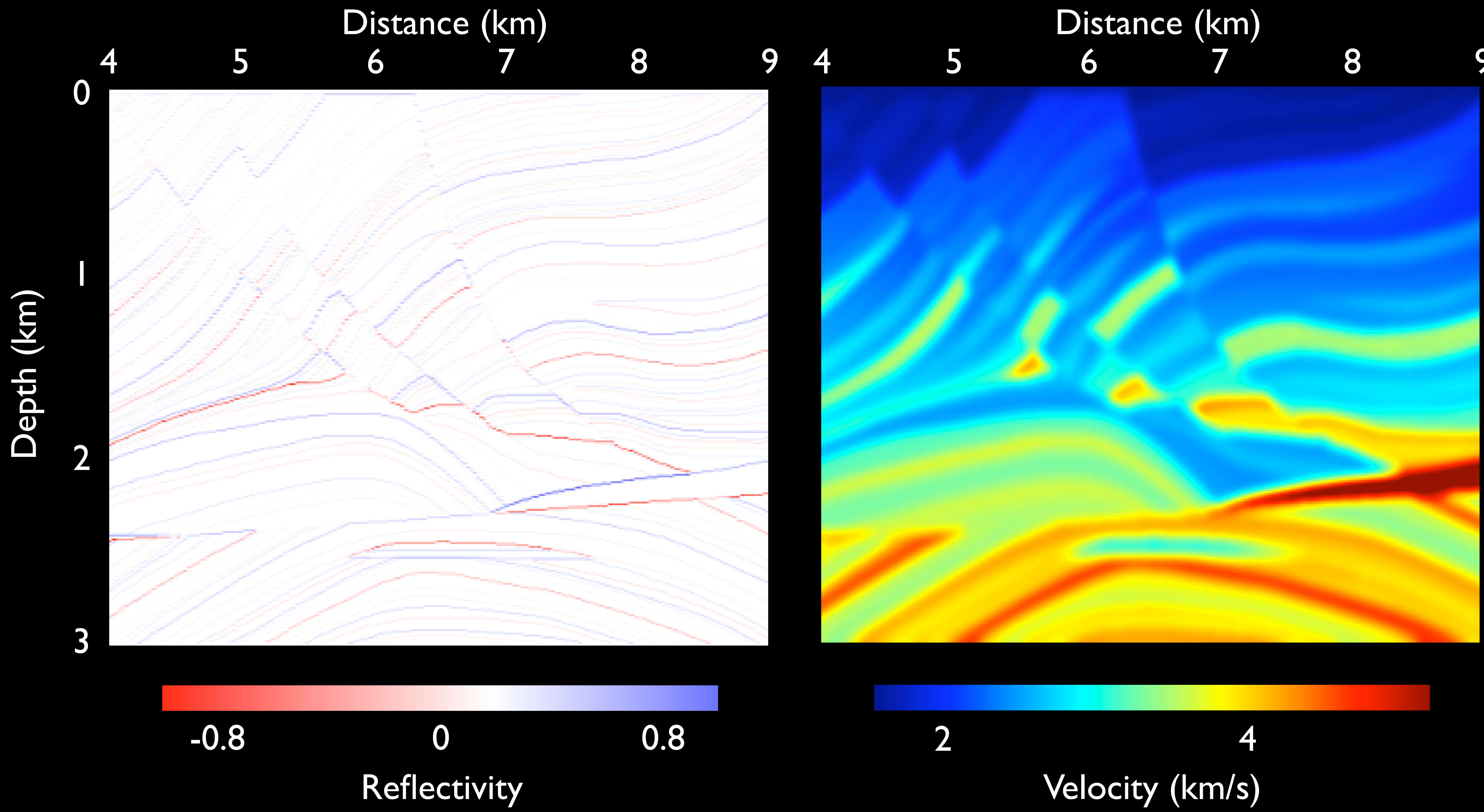
$$S(\mathbf{m}) = \sum_{\mathbf{x}} \log(1 + m^2(\mathbf{x})/\sigma^2)$$

Darche, 1989; Sacchi and Ulrych, 1995; Guitton, 2000; Tang, 2006

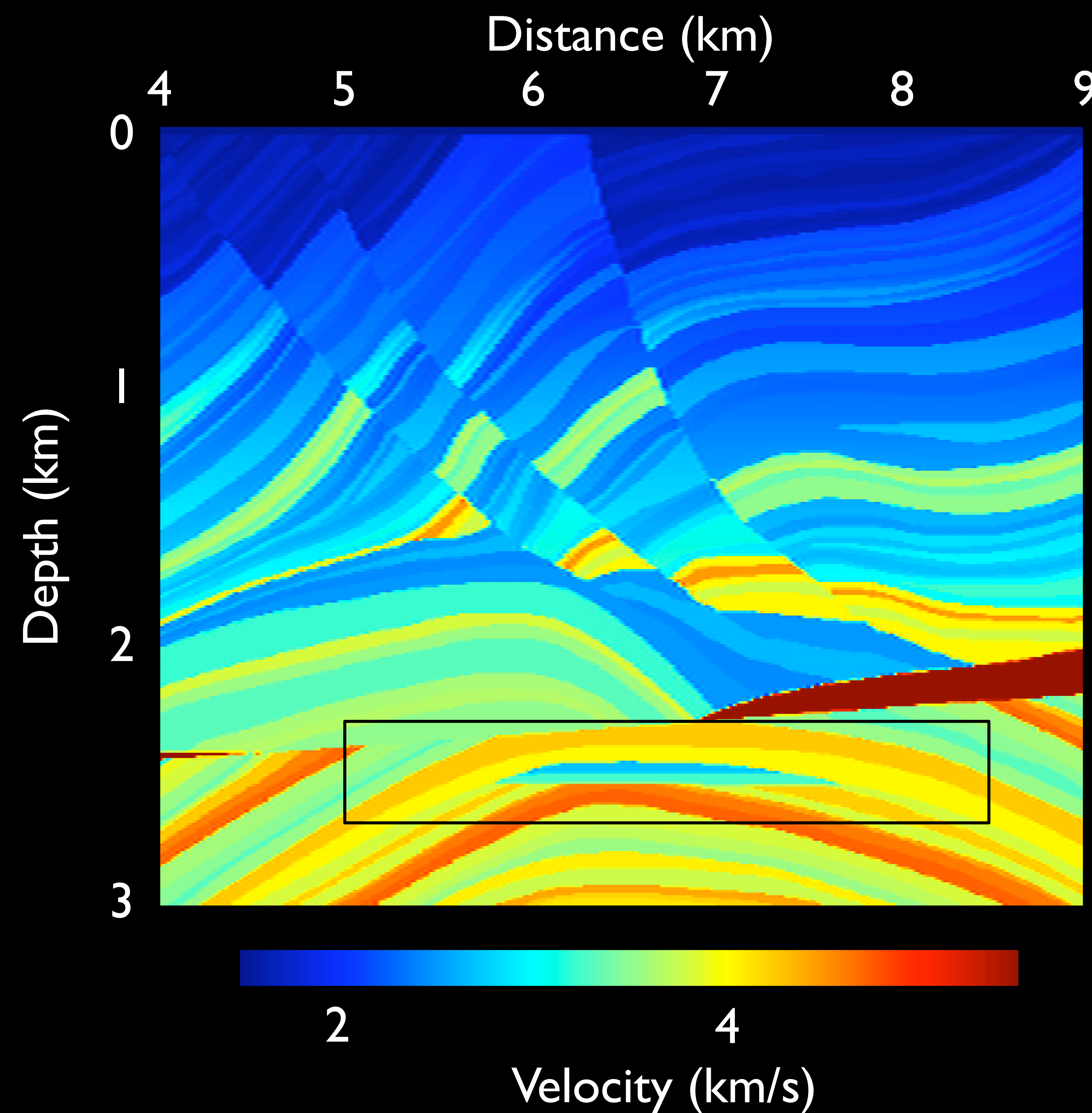
Synthetic data examples

- **One-way inversion of one-way Born data**
- **One-way inversion of two-way acoustic data**

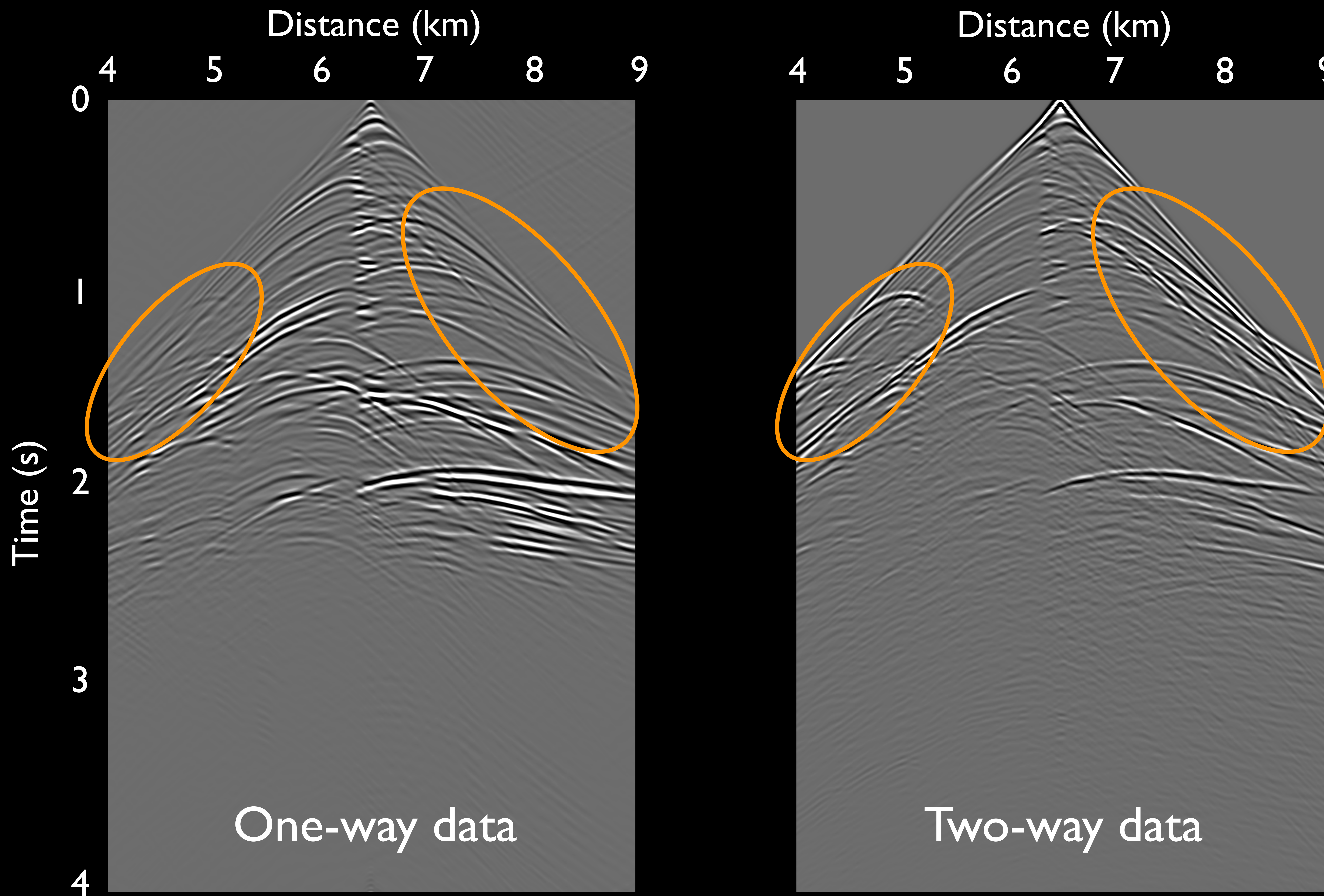
Reflectivity and background velocity



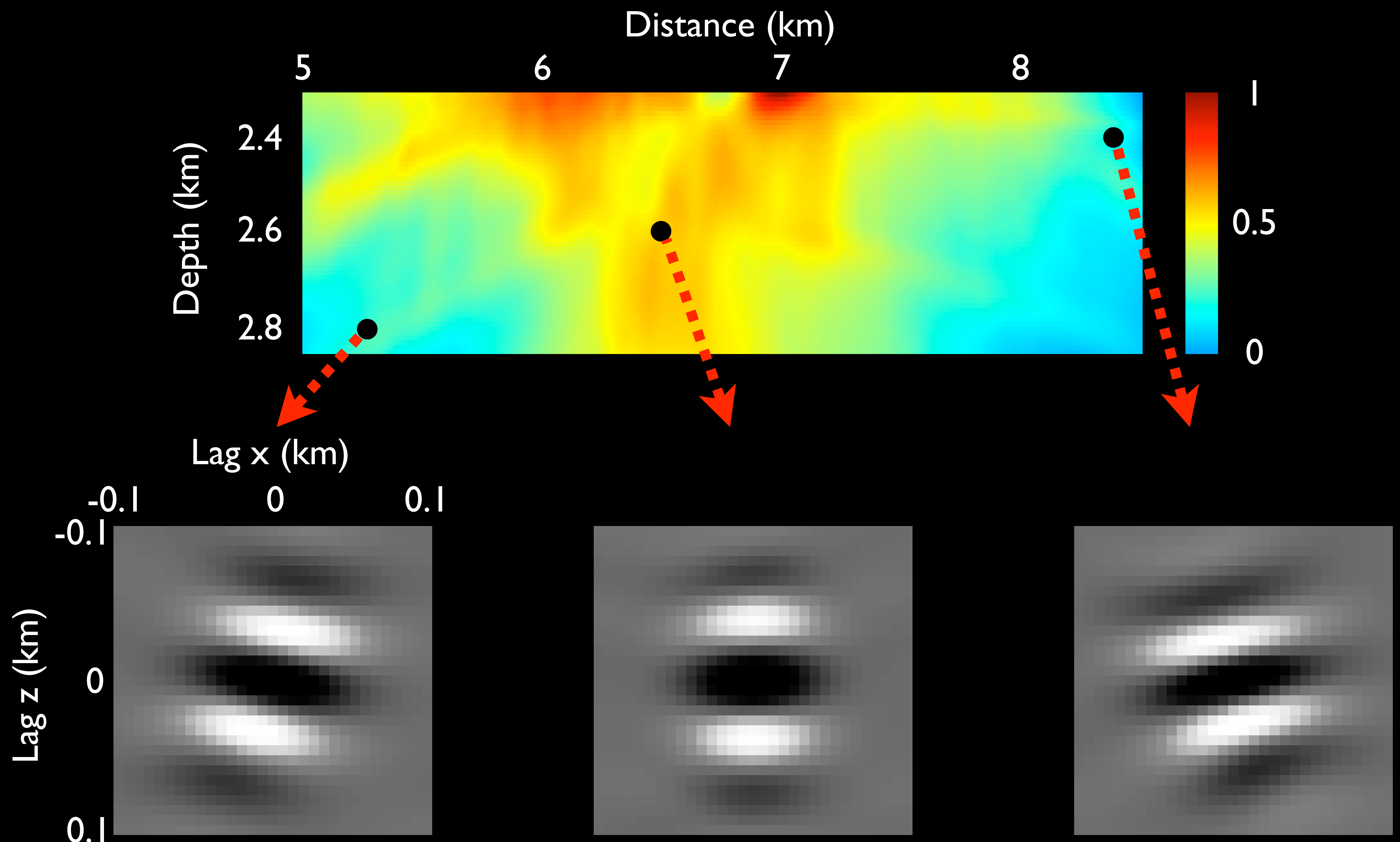
Stratigraphic velocity model



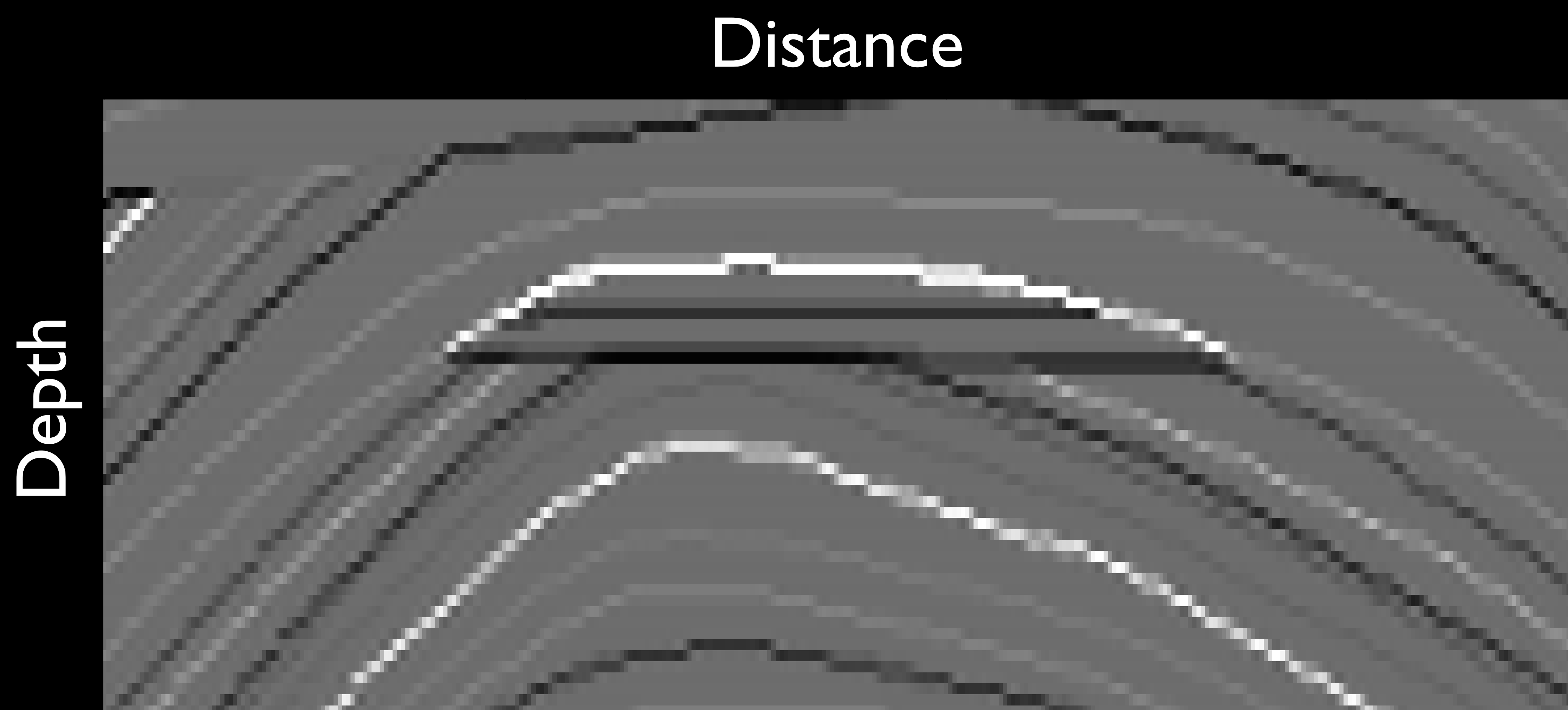
One-way Born data vs. two-way acoustic data



The receiver-side randomly encoded Hessian



One-way inversion of one-way Born data



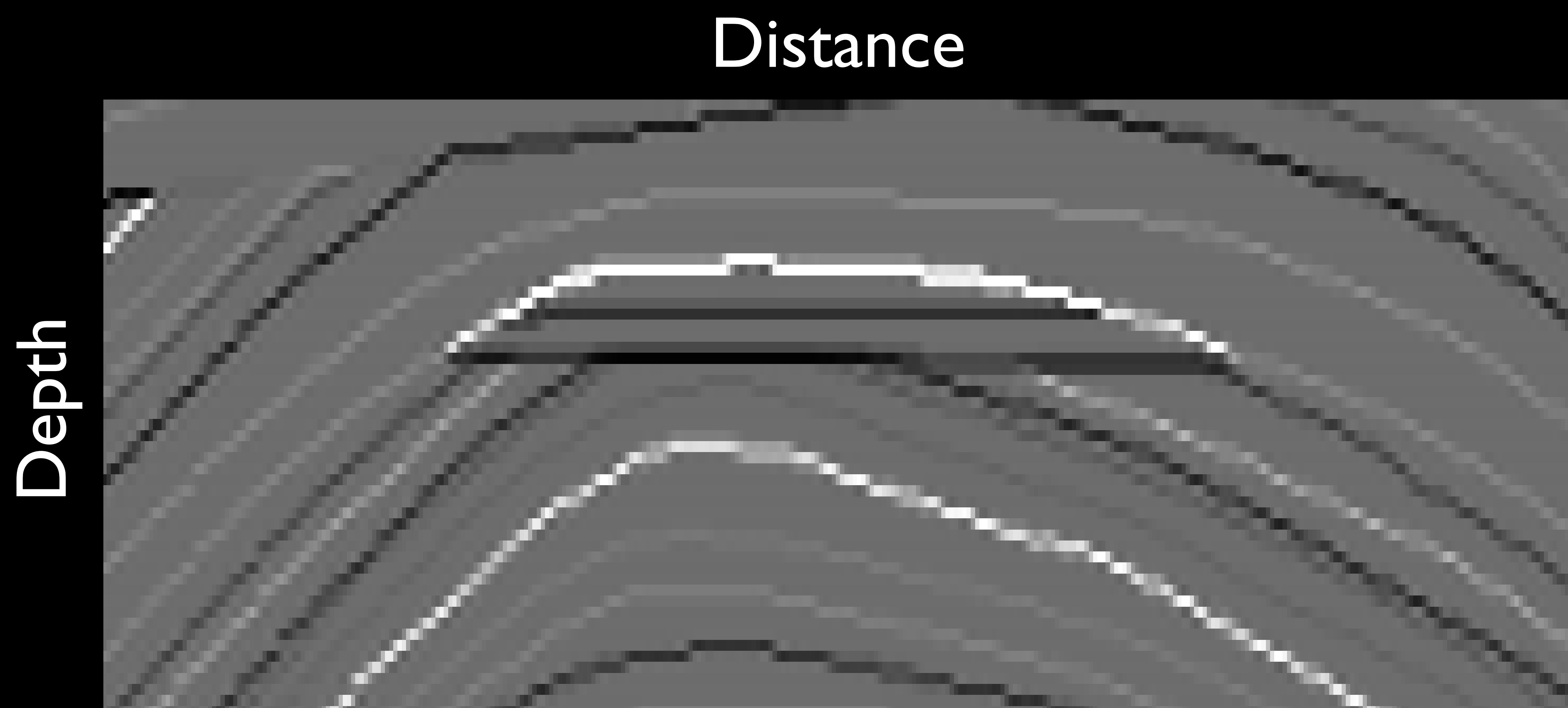
True reflectivity

Migration

Inversion (damping)

Inversion (sparseness)

One-way inversion of two-way acoustic data



True reflectivity

Migration

Inversion (damping)

Inversion (sparseness)

Conclusions

- **Phase encoding provides a cost effective way for computing the explicit Hessian**

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- **Inversion helps to correct the effects of uneven illumination**

Conclusions

- **Phase encoding provides a cost effective way for computing the explicit Hessian**
- **Cost for target-oriented inversion is reduced to two migrations**
- **Inversion helps to correct the effects of uneven illumination**
- **Inversion with sparseness constraints further enhances the resolution, but it may also over-penalize weak refections**

Thanks

Phase-encoded Hessian

Hessian in the original domain:

$$F(\mathbf{m}) = ||\mathbf{L}\mathbf{m} - \mathbf{d}_{\mathrm{obs}}||_2^2 \qquad \mathbf{H} = \mathbf{L}'\mathbf{L}$$

Hessian in the encoding domain:

$$\widetilde{F}(\mathbf{m}) = ||\mathbf{B}\mathbf{L}\mathbf{m} - \mathbf{B}\mathbf{d}_{\mathrm{obs}}||_2^2 \qquad \widetilde{\mathbf{H}} = \mathbf{L}'\mathbf{B}'\mathbf{B}\mathbf{L}$$

Choose encoding function(s), such that

$$\mathbf{B}'\mathbf{B} \approx \mathbf{I} \qquad \widetilde{\mathbf{H}} \approx \mathbf{L}'\mathbf{L} = \mathbf{H}$$

Encoding of the receiver Green's functions

Receiver-side encoded Green's function:

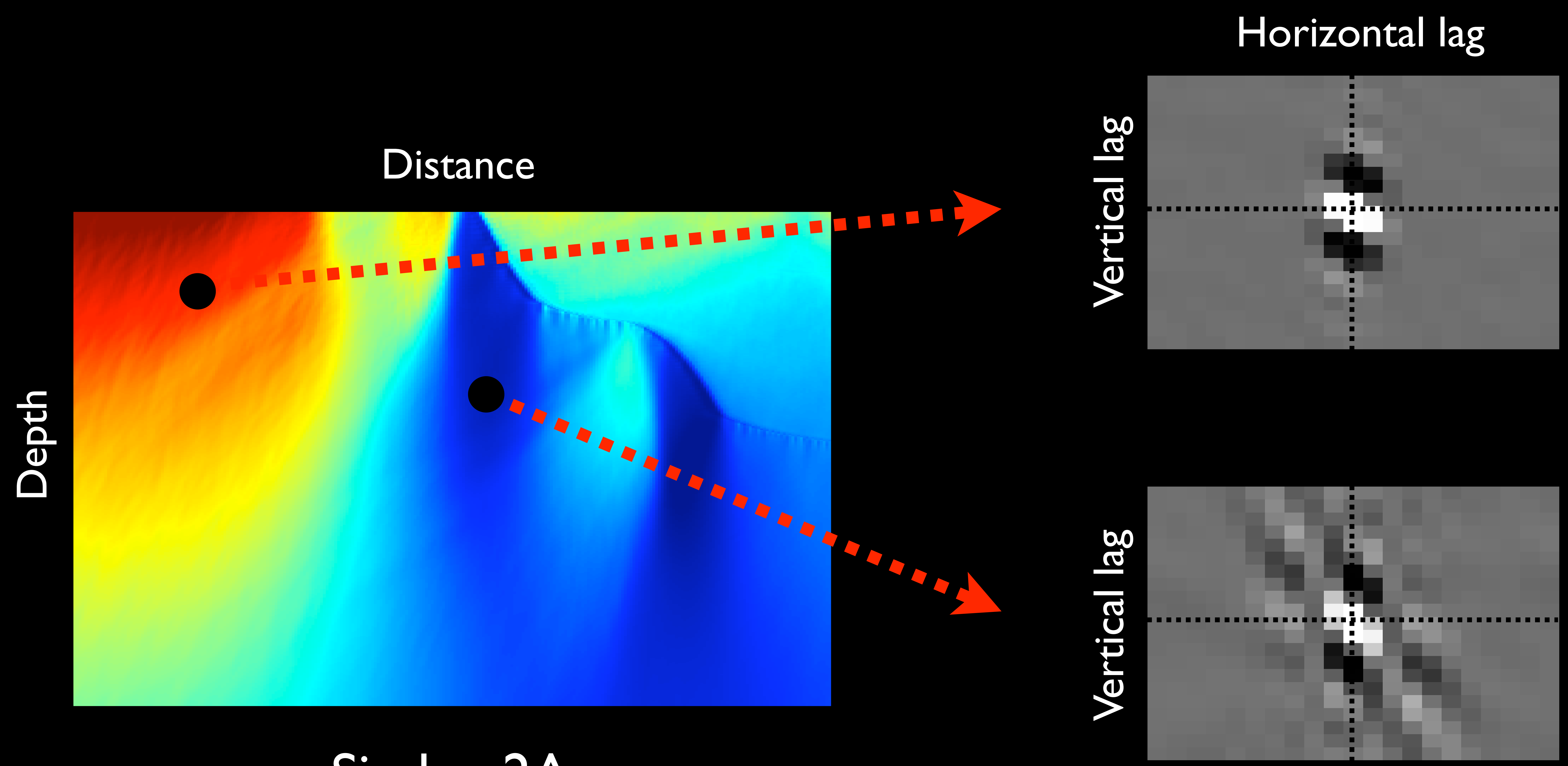
$$R(\mathbf{x}, \mathbf{p}_r, \omega; \mathbf{x}_s) = \sum_{\mathbf{x}_r} w(\mathbf{x}_r, \mathbf{x}_s) G(\mathbf{x}, \mathbf{x}_r, \omega) \beta(\mathbf{x}_r, \mathbf{p}_r, \omega)$$

Plane-wave phase encoding: $\beta(\mathbf{x}_r, \mathbf{p}_r, \omega) = e^{i\omega \mathbf{p}_r \cdot \mathbf{x}_r}$

Random phase encoding: $\beta(\mathbf{x}_r, \mathbf{p}_r, \omega) = e^{i\gamma(\mathbf{x}_r, \mathbf{p}_r, \omega)}$

Born modeling operator is non-unitary

The normal operator(Hessian): $\mathbf{H} = \mathbf{L}'\mathbf{L}$



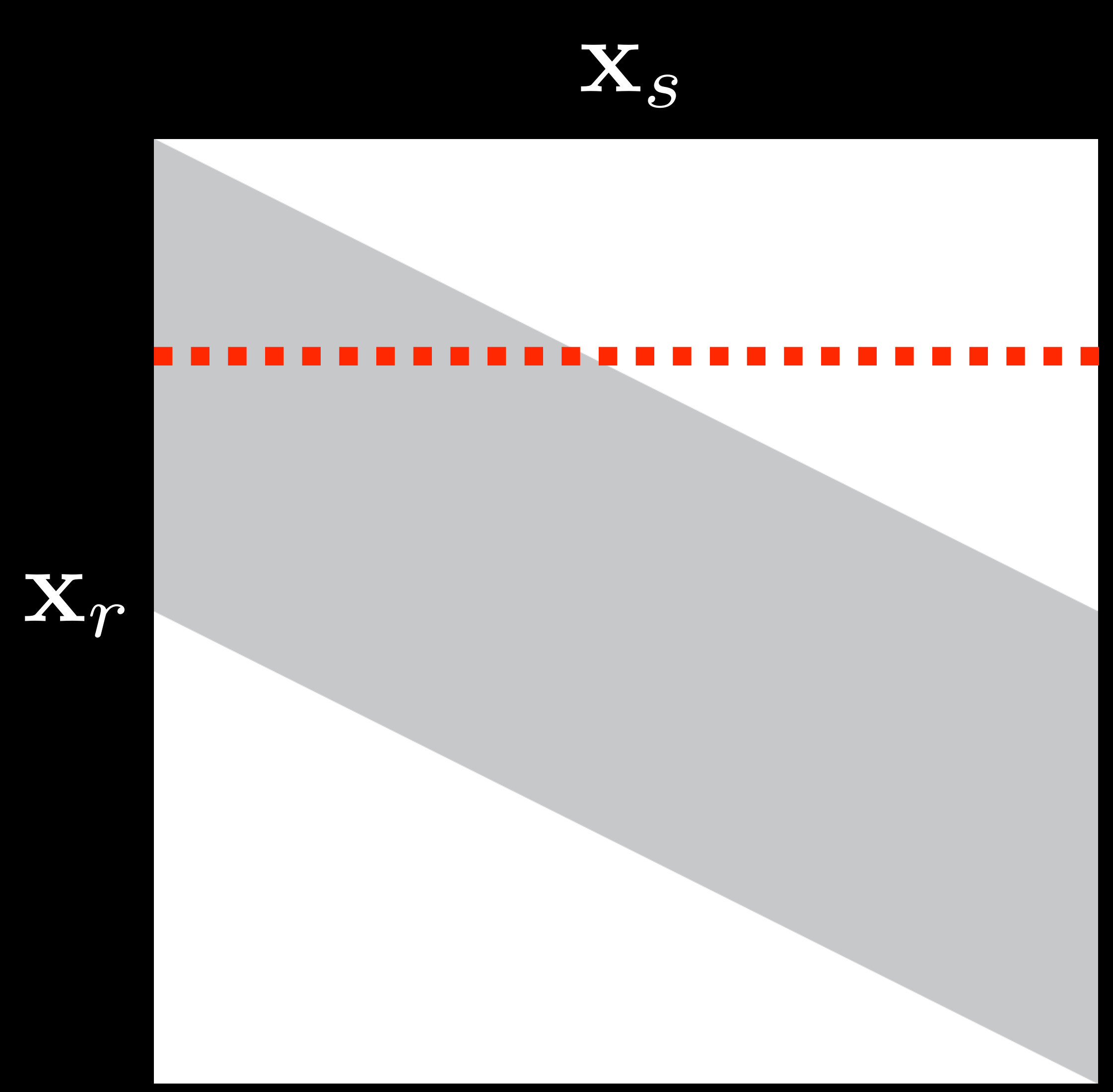
Sigsbee2A

Encoding-domain objective function

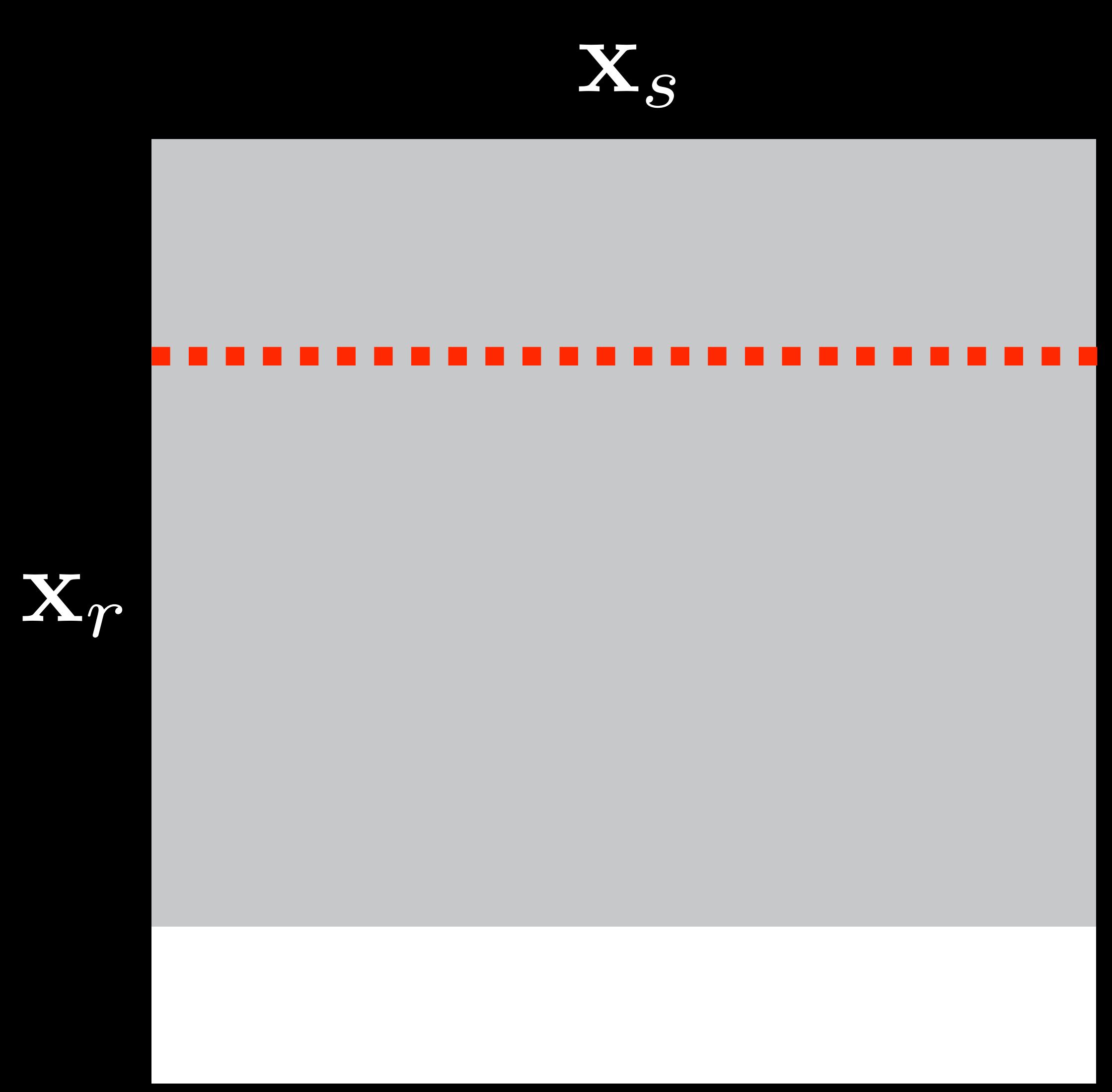
$$\widetilde{F}(\mathbf{m}) = ||\mathbf{BLm} - \mathbf{Bd}_{\mathrm{obs}}||_2^2$$

Encoding of the sources:

$$d(\mathbf{x}_r, \mathbf{p}_s, \omega) = \sum_{\mathbf{x}_s} w(\mathbf{x}_r, \mathbf{x}_s) d(\mathbf{x}_r, \mathbf{x}_s, \omega) \alpha(\mathbf{x}_s, \mathbf{p}_s, \omega)$$



2-D marine



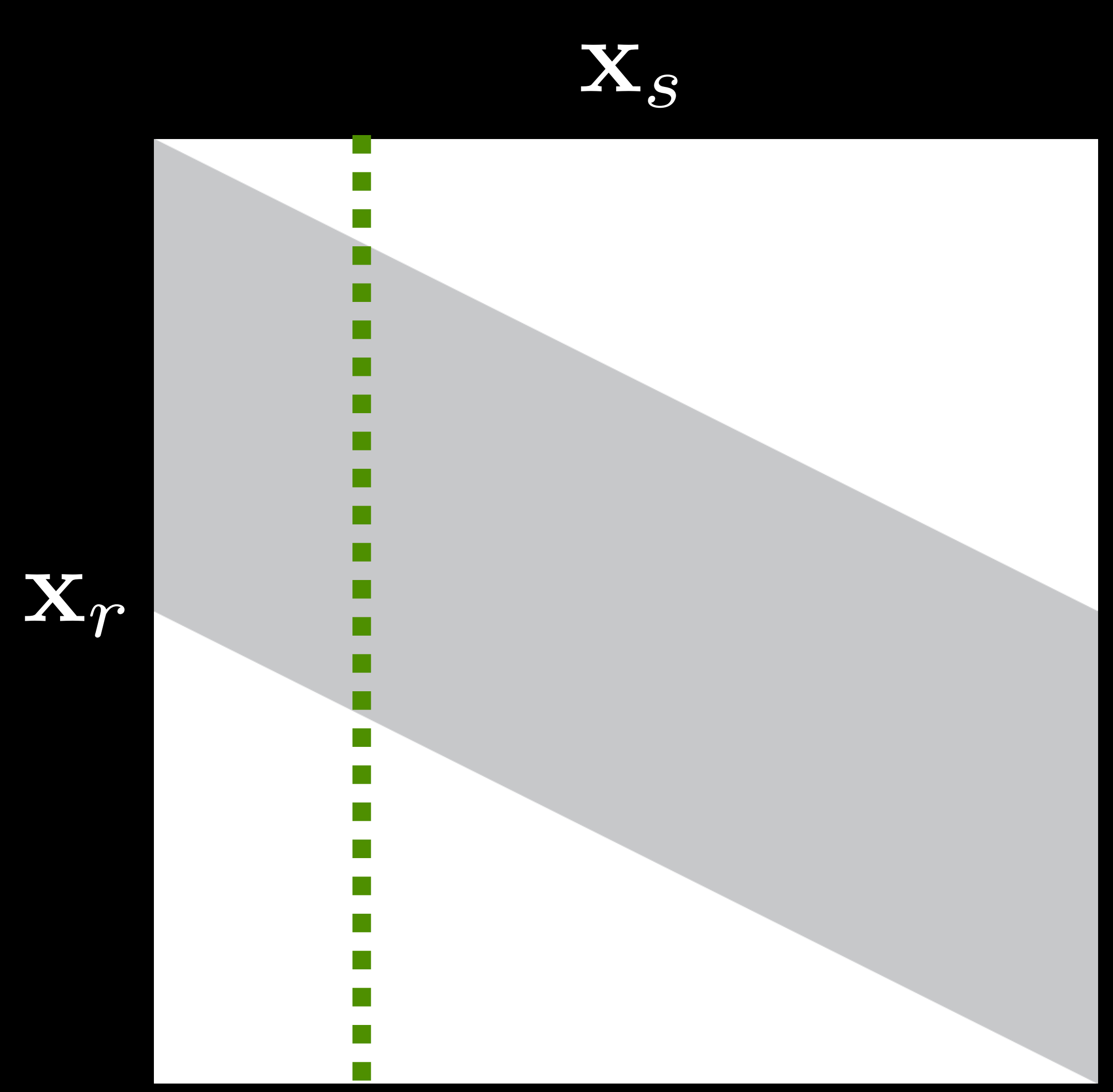
2-D OBS

Encoding-domain objective function

$$\widetilde{F}(\mathbf{m}) = ||\mathbf{BLm} - \mathbf{Bd}_{\text{obs}}||_2^2$$

Encoding of the receivers:

$$d(\mathbf{p}_r, \mathbf{x}_s, \omega) = \sum_{\mathbf{x}_r} w(\mathbf{x}_r, \mathbf{x}_s) d(\mathbf{x}_r, \mathbf{x}_s, \omega) \beta(\mathbf{x}_r, \mathbf{p}_r, \omega)$$



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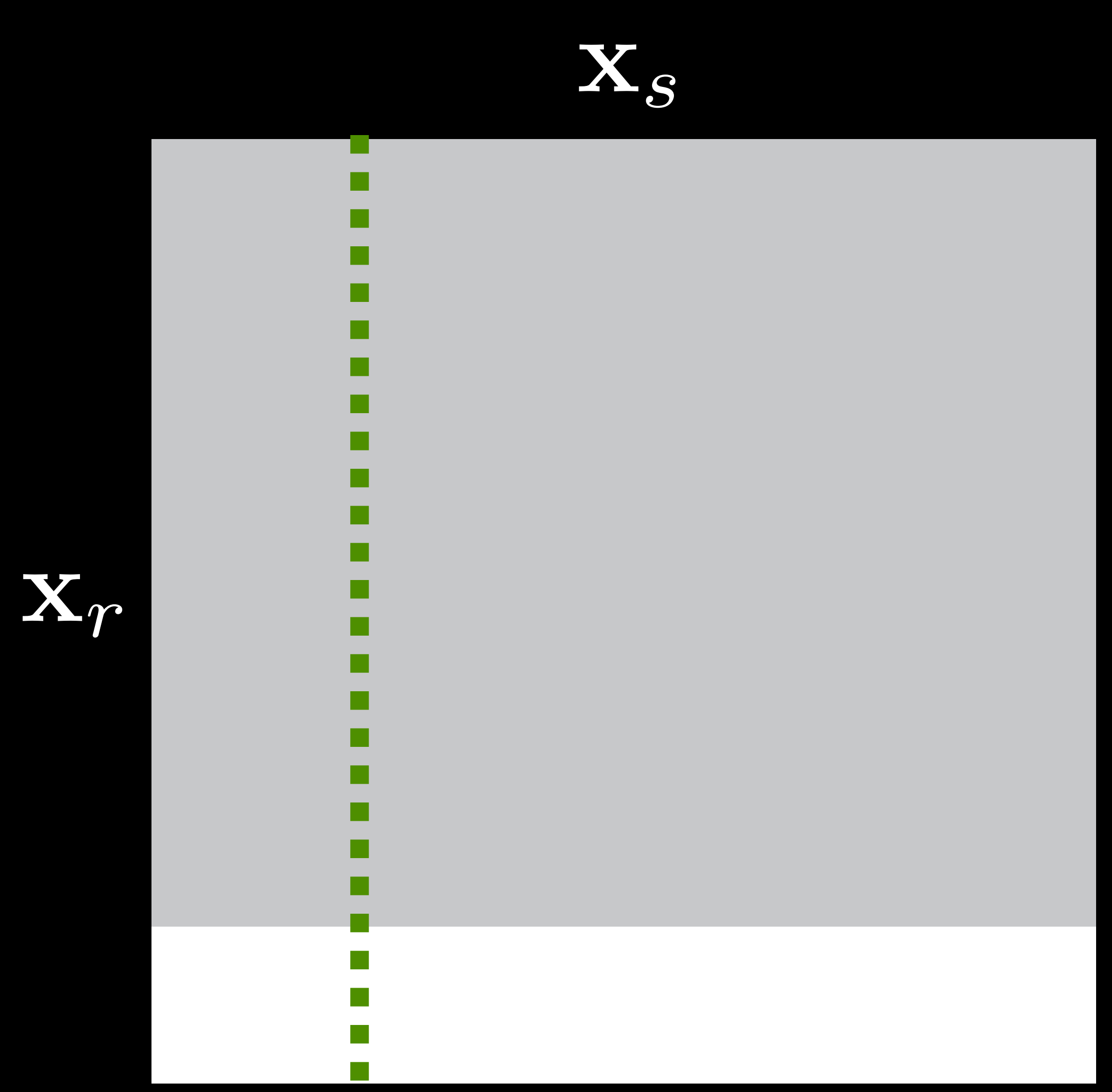

2-D OBS

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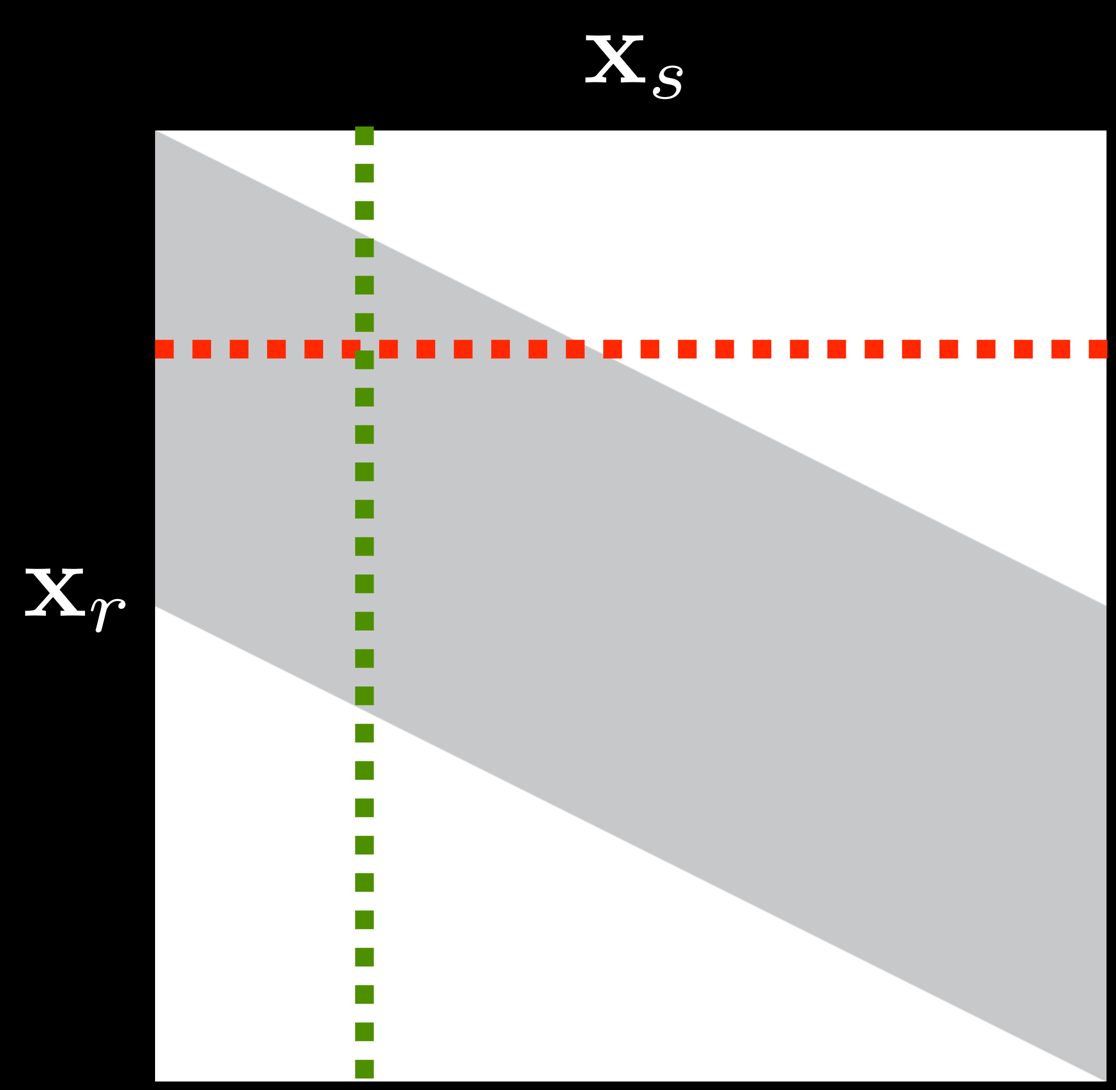
Encoding of the sources and receivers:

$$d(\mathbf{p}_r, \mathbf{p}_s, \omega) = \sum_{\mathbf{x}_r} \sum_{\mathbf{x}_s} w(\mathbf{x}_r, \mathbf{x}_s) d(\mathbf{x}_r, \mathbf{x}_s, \omega) \alpha(\mathbf{x}_s, \mathbf{p}_s, \omega) \beta(\mathbf{x}_r, \mathbf{p}_r, \omega)$$



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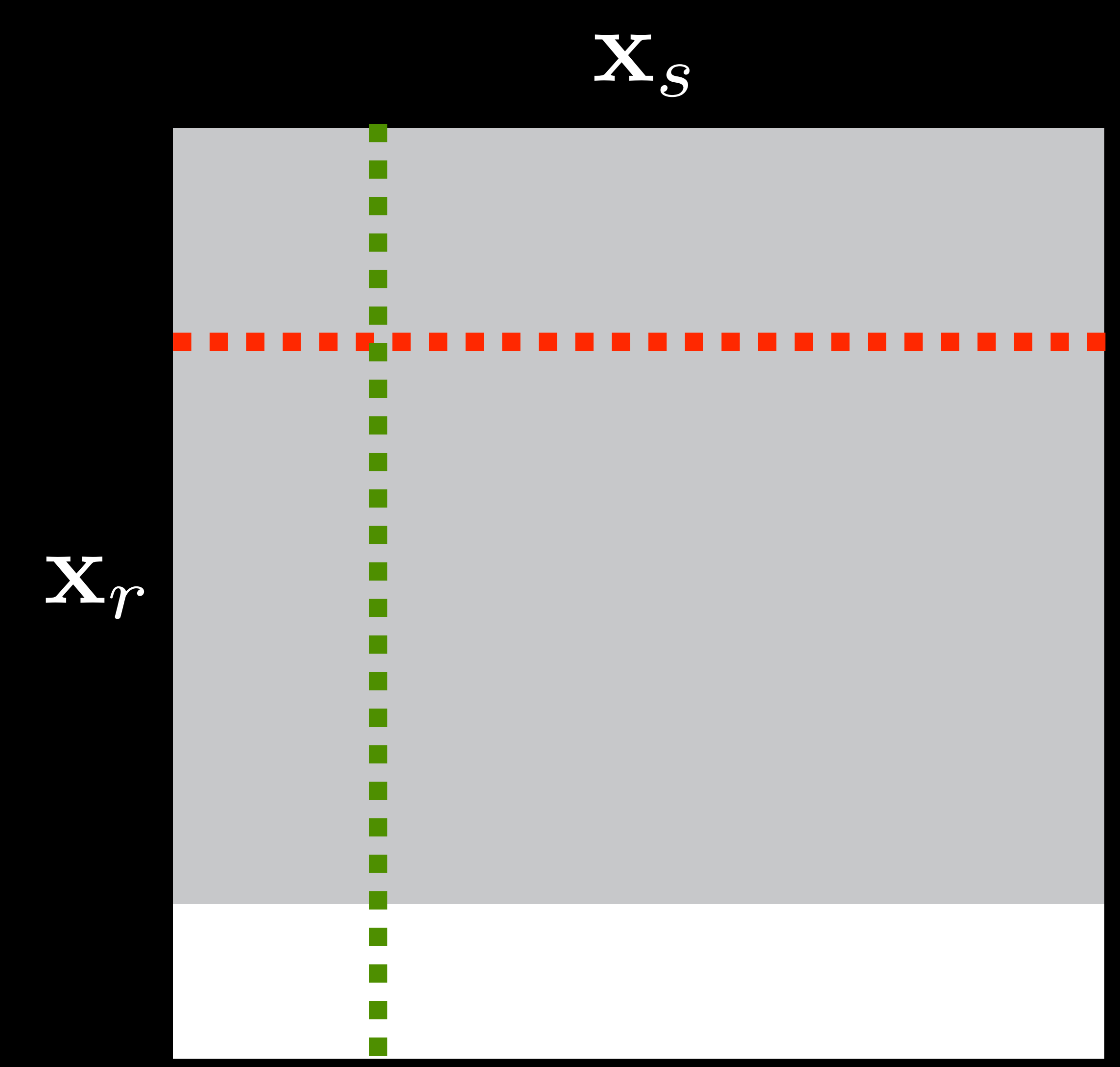

2-D OBS

Linear phase-encoding function

$$\alpha(\mathbf{x}_s, \mathbf{p}_s, \omega) = e^{i\omega \mathbf{p}_s \cdot \mathbf{x}_s}$$

$$\beta(\mathbf{x}_r, \mathbf{p}_r, \omega) = e^{i\omega \mathbf{p}_r \cdot \mathbf{x}_r}$$

$\mathbf{p}_s$: ray parameter for the source plane wave

$\mathbf{p}_r$: ray parameter for the receiver plane wave

Random phase-encoding function

$$\alpha(\mathbf{x}_s, \mathbf{p}_s, \omega) = e^{i\gamma(\mathbf{x}_s, \mathbf{p}_s, \omega)}$$

$$\beta(\mathbf{x}_r, \mathbf{p}_r, \omega) = e^{i\gamma(\mathbf{x}_r, \mathbf{p}_r, \omega)}$$

γ is defined to be a random sequence

$\mathbf{p}_r$: index of different realizations

$\mathbf{p}_s$: index of different realizations