Inversion theory of up- and down-going signal for ocean bottom data

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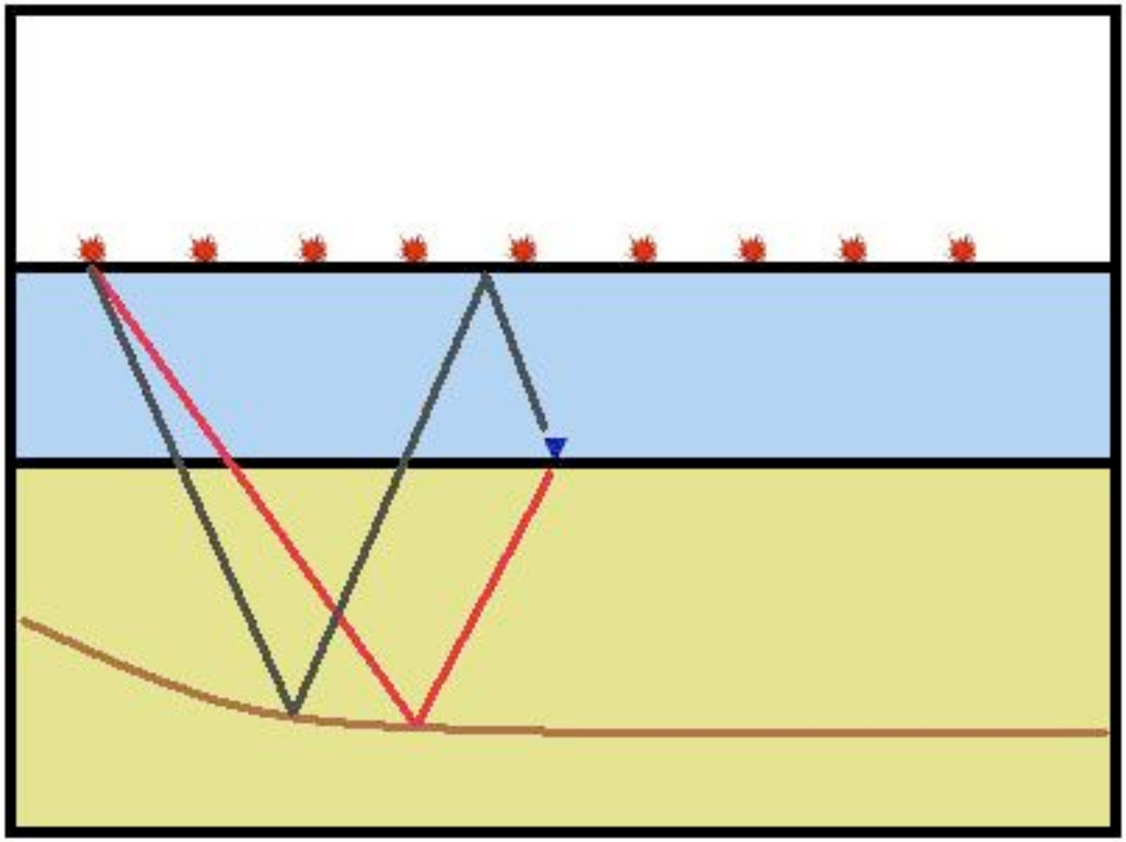
Overview

- Introduction
- Up-down separation
  - PZ summation
  - Over-Under Separation
- The Inverse Problem
  - Mirror Imaging operator
  - Complete operator
- Conclusion

Ocean Bottom Acquisition



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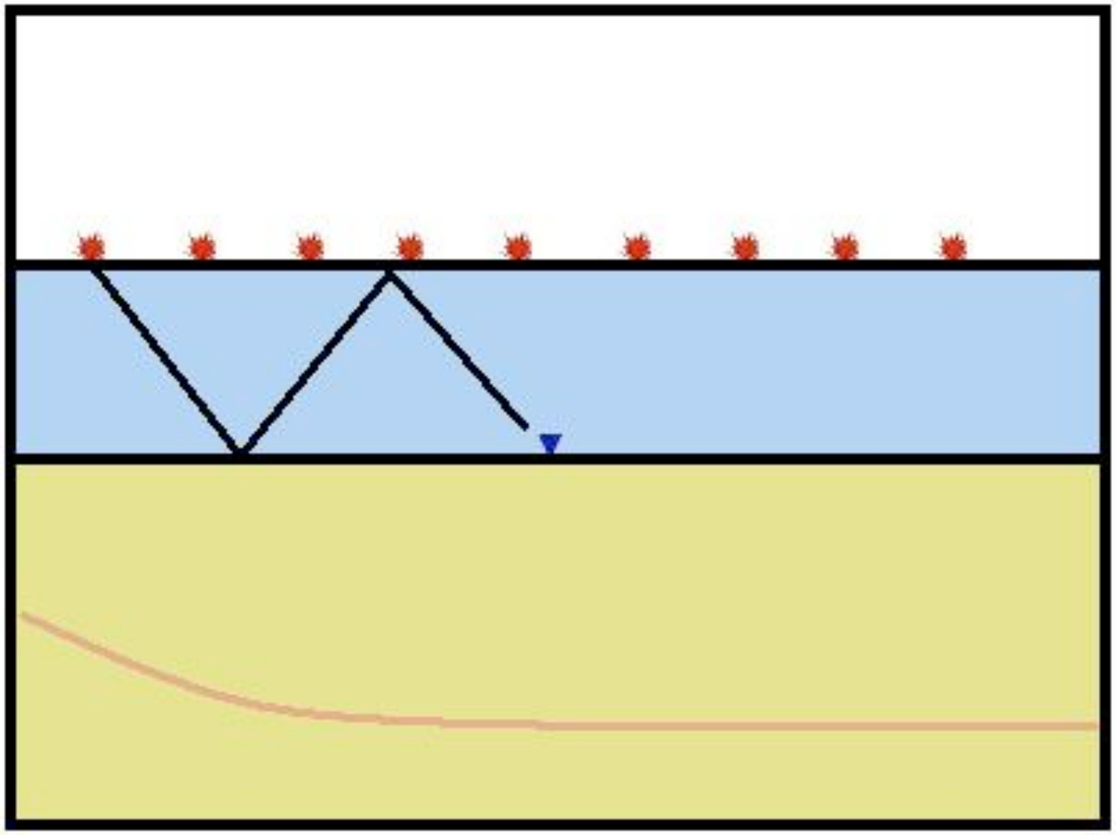
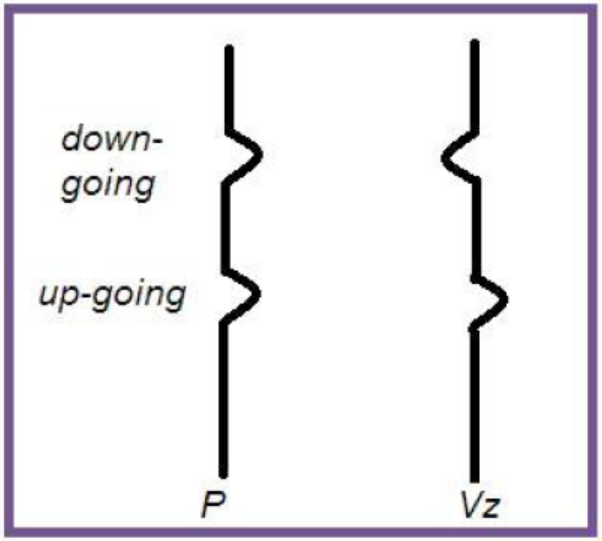


Separation of up and down-going wavefield

PZ summation



$$U(t,x) = \frac{1}{2}\left(P(t,x) + \text{scale}(x)Z(t,x)\right),$$
$$D(t,x) = \frac{1}{2}\left(P(t,x) - \text{scale}(x)Z(t,x)\right),$$

$U(t,x)$ is the up-going wavefield

$D(t,x)$ is the down-going wavefield

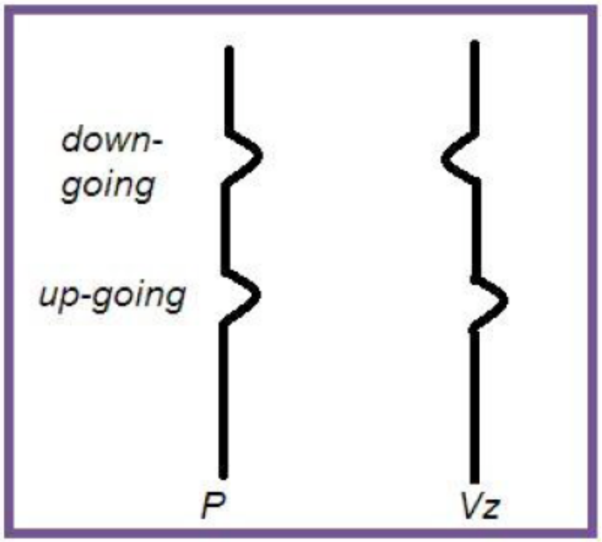
$P(t,x)$ is the pressure

$Z(t,x)$ is the vertical particle velocity

t is time and x is offset

Separation of up and down-going wavefield

PZ summation



$$U(k_x,\omega) = \frac{1}{2}\left[P(k_x,\omega) - \frac{\rho\omega}{k_z}Z(k_x,\omega)\right]$$

$$D(k_x,\omega) = \frac{1}{2}\left[P(k_x,\omega) + \frac{\rho\omega}{k_z}Z(k_x,\omega)\right]$$

ρ is water density

v is water velocity

k_x is wave-number in x

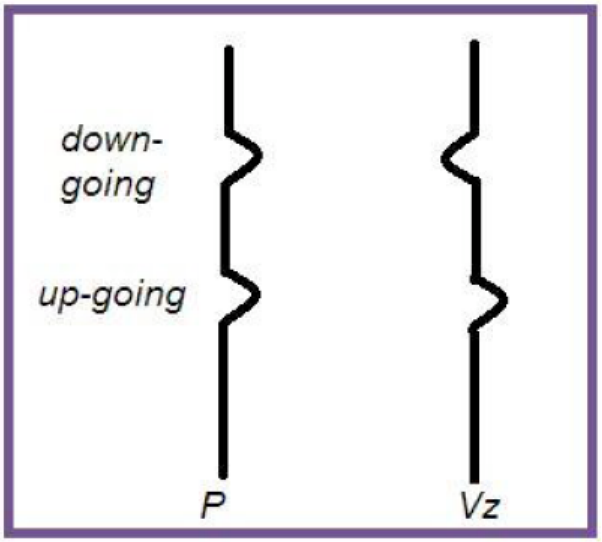
$k_z = \sqrt{\frac{\omega^2}{v^2} - k_x^2}$ is the vertical wave-number

ω is the frequency

Amundsen (1993)

Separation of up and down-going wavefield

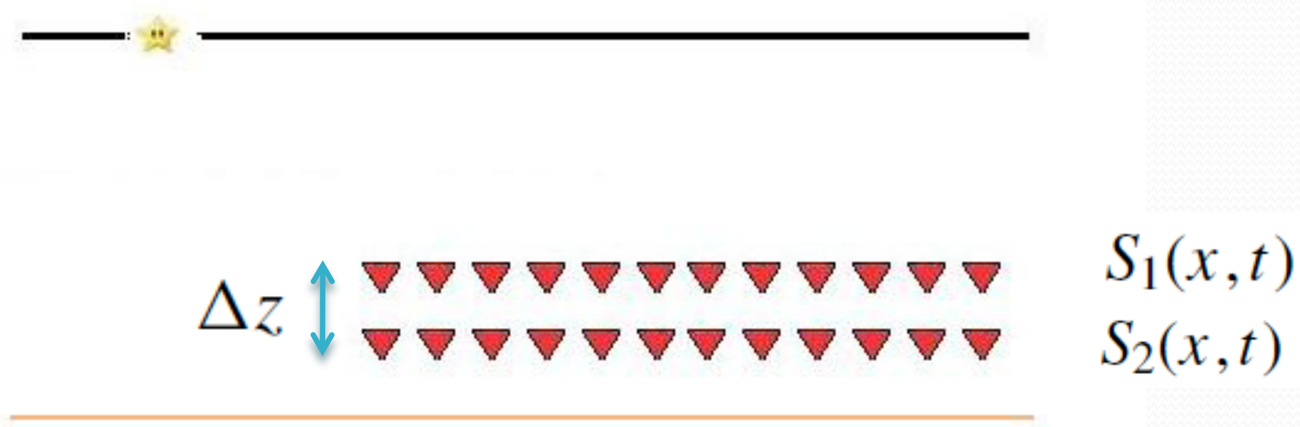
PZ summation



$$\begin{bmatrix} \mathbf{d}_{\uparrow} \\ \mathbf{d}_{\downarrow} \end{bmatrix} = \mathbf{S}_{pz}^{-1} \begin{bmatrix} \mathbf{d}_P \\ \mathbf{d}_Z \end{bmatrix}$$

Separation of up and down-going wavefield

Over-Under Separation



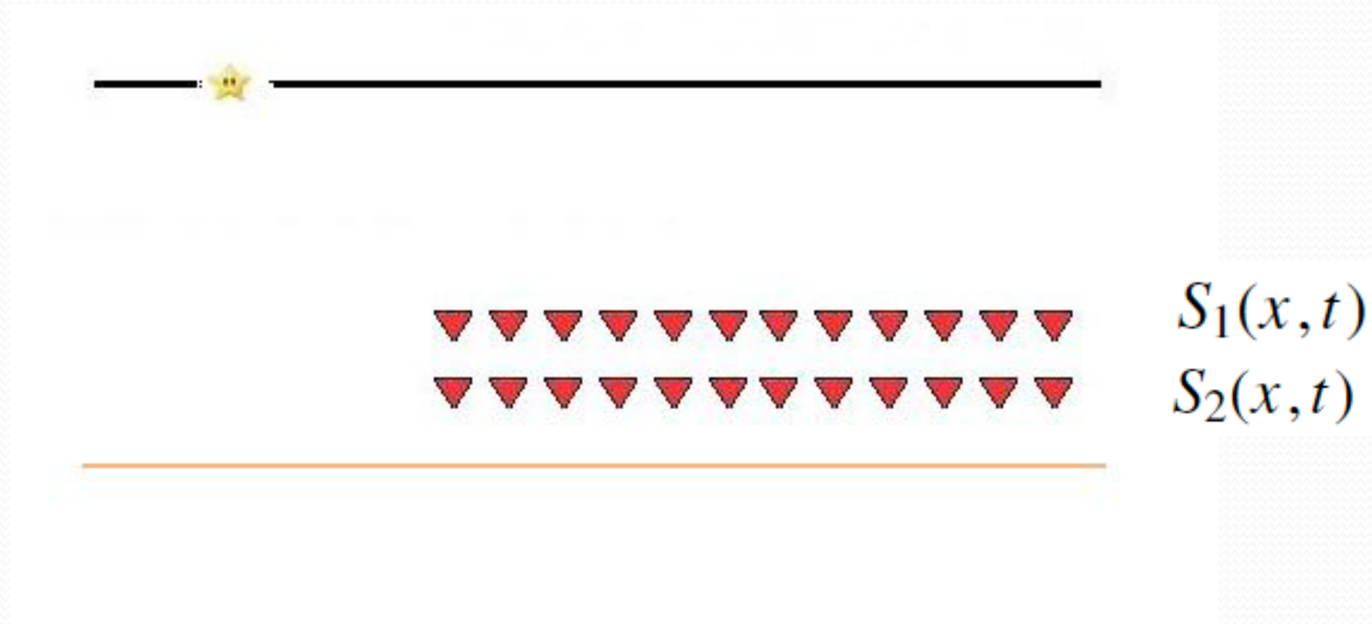
$S_1(x,t)$ is the recorded wavefield for over array

$S_2(x,t)$ is the recorded wavefield for under array

Δz is the spacing between over and under array

Separation of up and down-going wavefield

Over-Under Separation



$$S_1(\omega,k_x) = U_1(\omega,k_x) + D_1(\omega,k_x)$$

$$S_2(\omega,k_x) = U_2(\omega,k_x) + D_2(\omega,k_x)$$

Separation of up and down-going wavefield

Over-Under Separation

$$e^{ik_z \Delta z} D_1 = D_2$$

$$U_1 = U_2 e^{ik_z \Delta z}$$

Separation of up and down-going wavefield

Over-Under Separation

$$U_2 = \frac{S_2 - e^{ik_z \Delta z} S_1}{1 - e^{2ik_z \Delta z}},$$

$$D_2 = \frac{e^{ik_z \Delta z} S_1 - e^{2ik_z \Delta z} S_2}{1 - e^{2ik_z \Delta z}}.$$

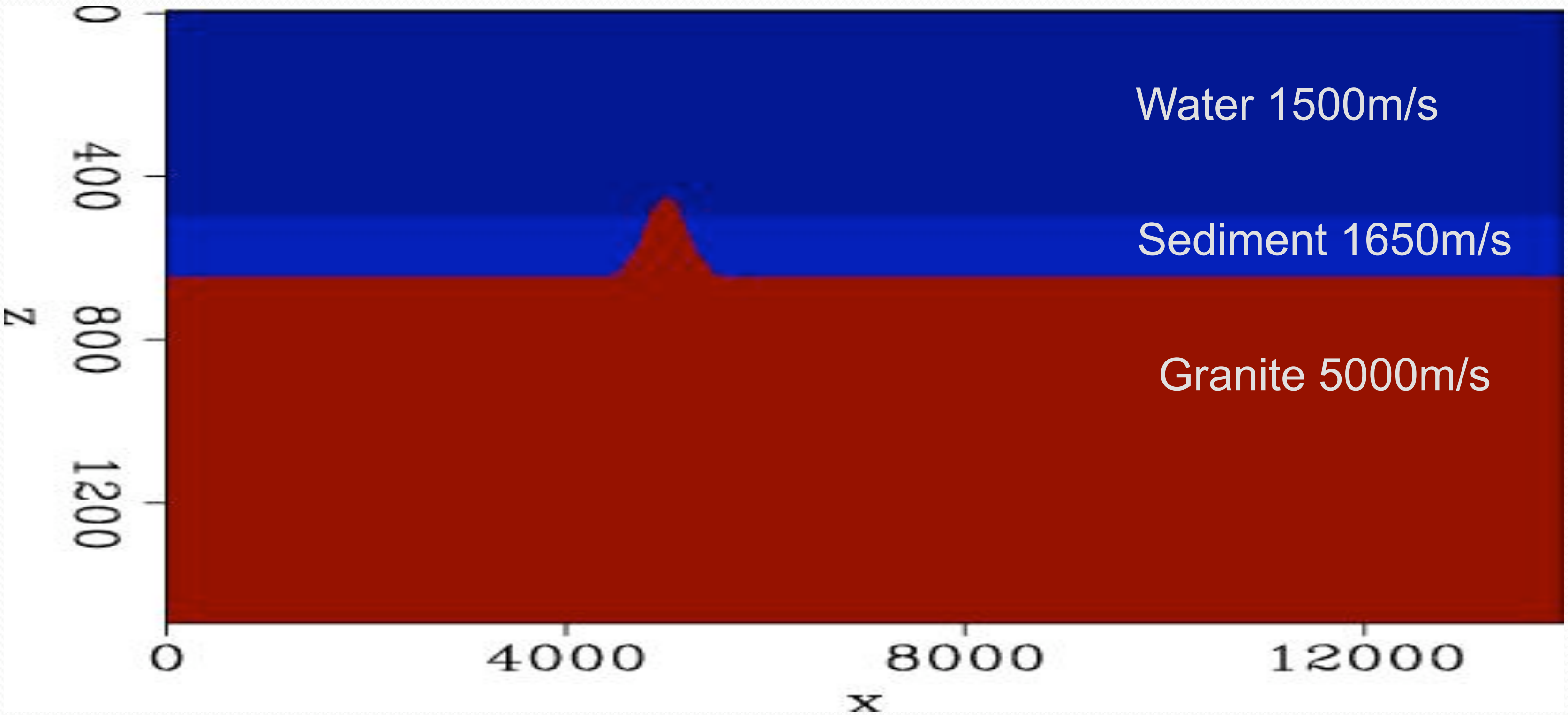
Separation of up and down-going wavefield

Over-Under Separation

$$\begin{bmatrix} \mathbf{d}_{\uparrow} \\ \mathbf{d}_{\downarrow} \end{bmatrix} = \mathbf{S}_{ou}^{-1} \begin{bmatrix} \mathbf{d}_{over} \\ \mathbf{d}_{under} \end{bmatrix}$$

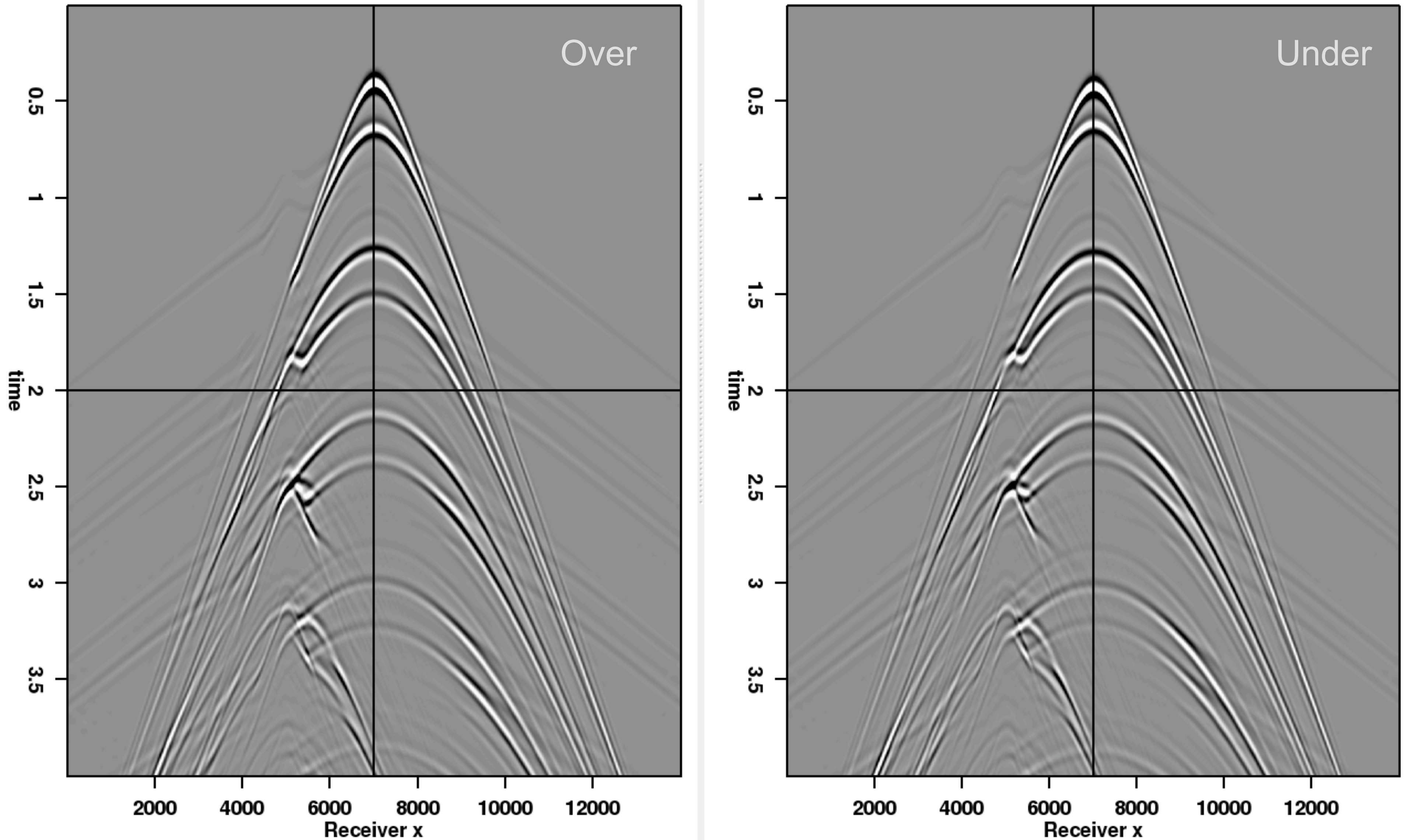
Separation of up and down-going wavefield

Over-Under Separation: Synthetic Example



Separation of up and down-going wavefield

Over-Under Separation: Synthetic Example



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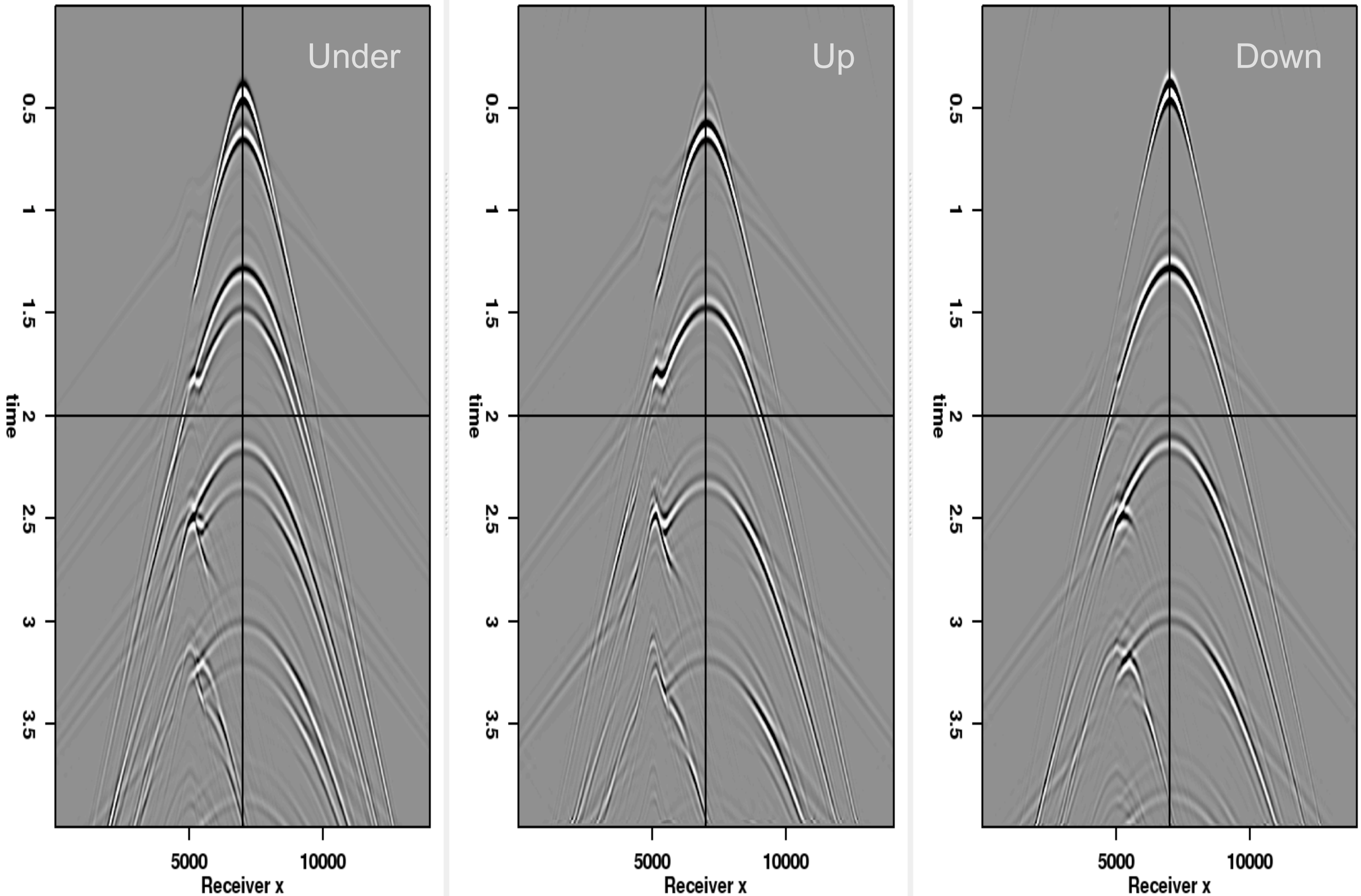


The Inverse Problem

Joint fitting of up- and down-going data.

$$min\left(\|\mathbf{L}_{\uparrow}\mathbf{m}-\mathbf{d}_{\uparrow}\|^2+\|\mathbf{L}_{\downarrow}\mathbf{m}-\mathbf{d}_{\downarrow}\|^2\right)$$

$\mathbf{m}$ is the model that describe the image (ex. velocity model)

$\mathbf{d}_{\uparrow}$ is the up-going wavefield at the receiver

$\mathbf{d}_{\downarrow}$ is the down-going wavefield at the receiver

$\mathbf{L}_{\uparrow}$ is an operator that models up-going wavefield at the receiver

$\mathbf{L}_{\downarrow}$ is an operator that models down-going wavefield at the receiver

$\|\cdot\|$ is the L_2 norm

Why the inverse problem?

Joint fitting of up- and down-going data.

$$min\left(\|\mathbf{L}_{\uparrow}\mathbf{m}-\mathbf{d}_{\uparrow}\|^{2}+\|\mathbf{L}_{\downarrow}\mathbf{m}-\mathbf{d}_{\downarrow}\|^{2}\right)$$

Traditional migration:

$$\tilde{\mathbf{m}}=\mathbf{L}'_{\uparrow}\mathbf{d}_{\uparrow}$$

Why the inverse problem?

Joint fitting of up- and down-going data.

$$min\left(\|\mathbf{L}_{\uparrow}\mathbf{m}-\mathbf{d}_{\uparrow}\|^{2}+\|\mathbf{L}_{\downarrow}\mathbf{m}-\mathbf{d}_{\downarrow}\|^{2}\right)$$

Mirror Imaging:

$$\tilde{\mathbf{m}}=\mathbf{L}_{\downarrow}'\mathbf{d}_{\downarrow}$$

Convert PZ into Up and Down

$$\begin{bmatrix} \mathbf{d}_{\uparrow} \\ \mathbf{d}_{\downarrow} \end{bmatrix} = \mathbf{S}_{pz}^{-1} \begin{bmatrix} \mathbf{d}_{P} \\ \mathbf{d}_{Z} \end{bmatrix}$$

$\mathbf{d}_p$ is the pressure data recorded

$\mathbf{d}_z$ is the vertical particle velocity data recorded

$\mathbf{d}_{\uparrow}$ is the up-going wavefield at the receiver

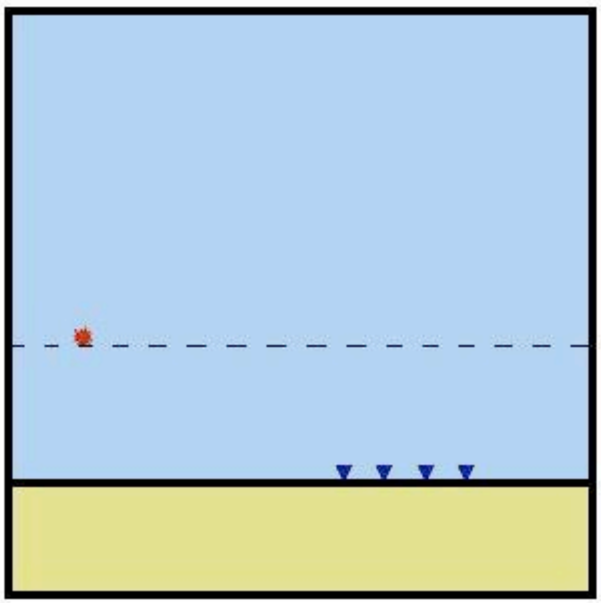
$\mathbf{d}_{\downarrow}$ is the down-going wavefield at the receiver

$\mathbf{S}_{pz}^{-1}$ separates PZ data into up-down data

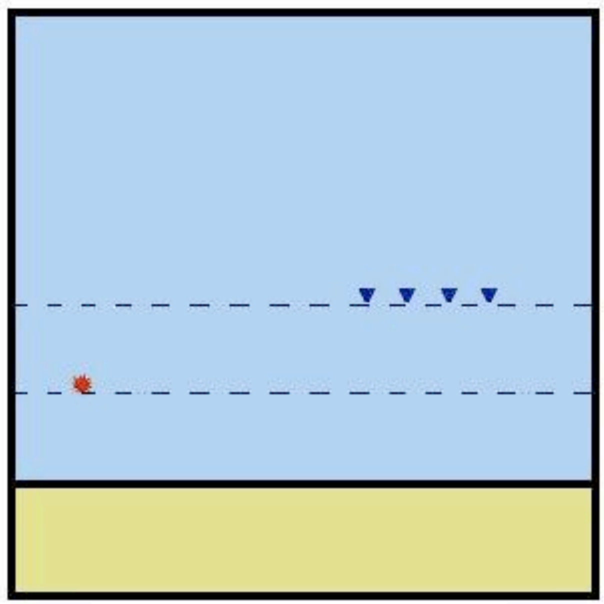
How to define …

$\mathbf{L}_{\uparrow}$ $\mathbf{L}_{\downarrow}$

The Inverse Problem: Mirror Imaging Operator



$\mathbf{L}_{\uparrow mirror}$

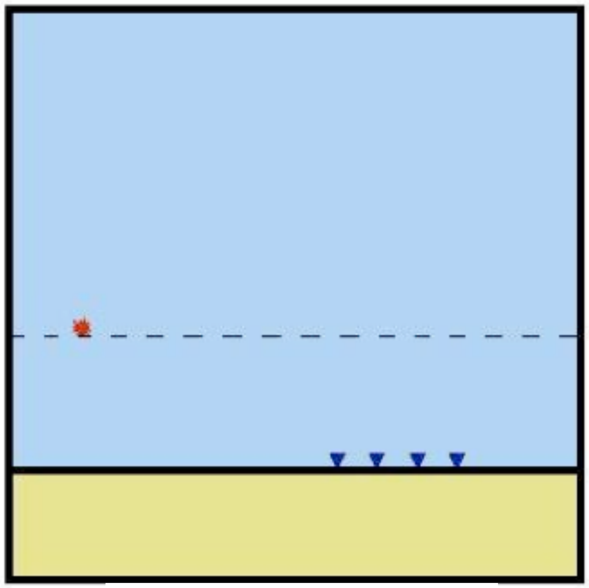


$\mathbf{L}_{\downarrow mirror}$

Stanford Exploration Project

The Inverse Problem: Mirror Imaging Operator

Accuracy of the up-going wavefield modeling operator



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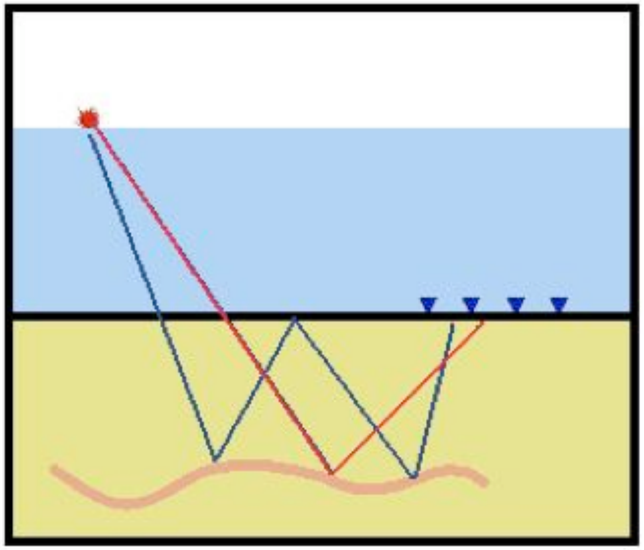

Included

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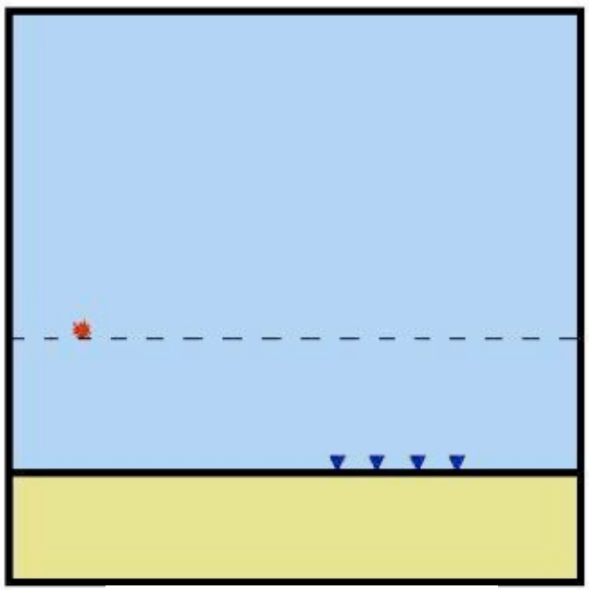

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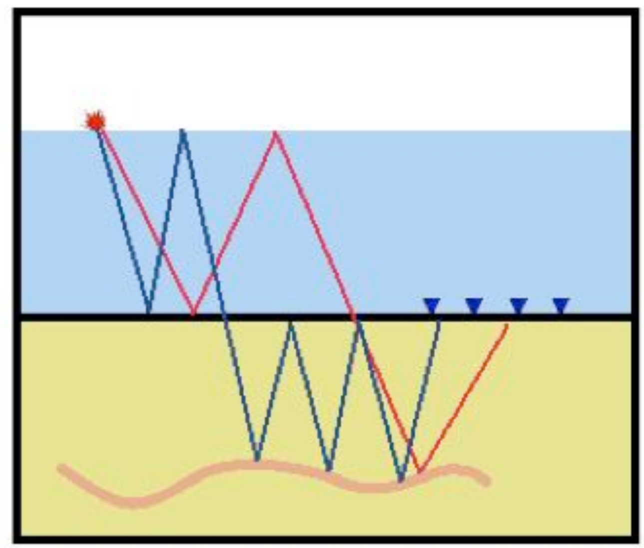
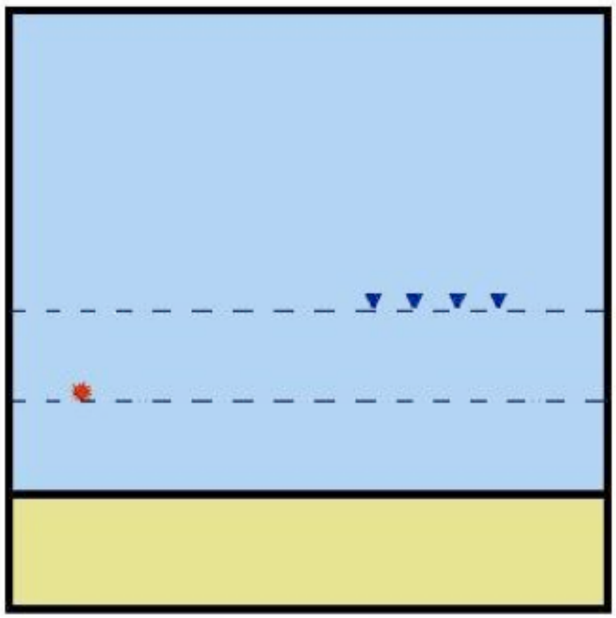


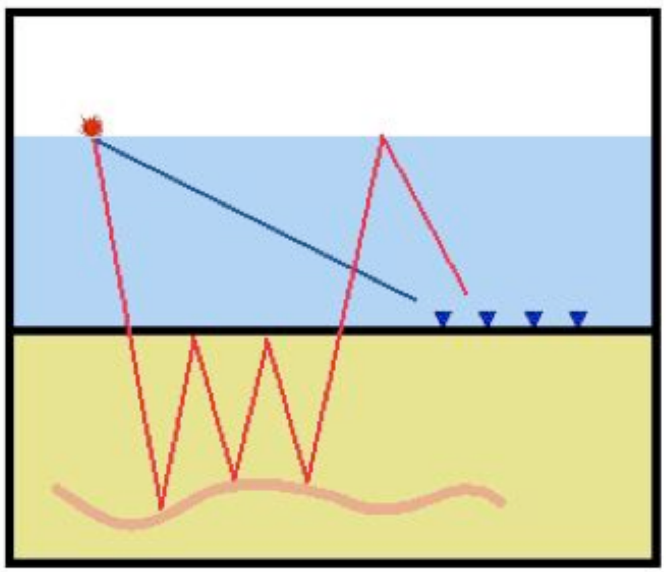
Excluded

The Inverse Problem: Mirror Imaging Operator

Accuracy of the down-going wavefield modeling operator



$\mathbf{L}_{\downarrow mirror}$



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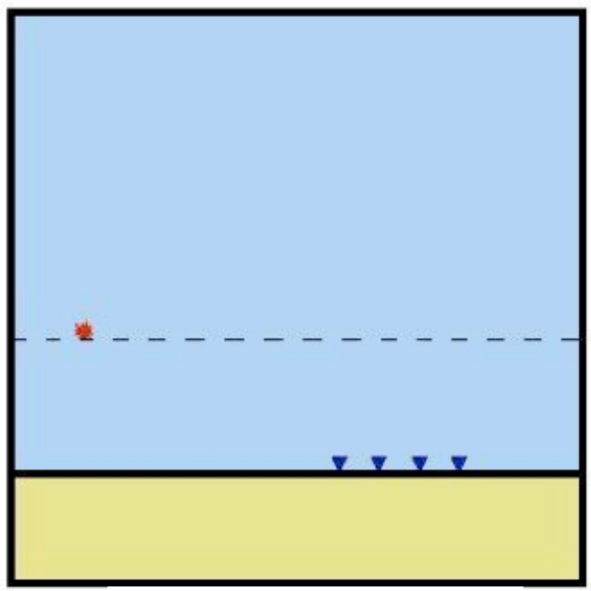
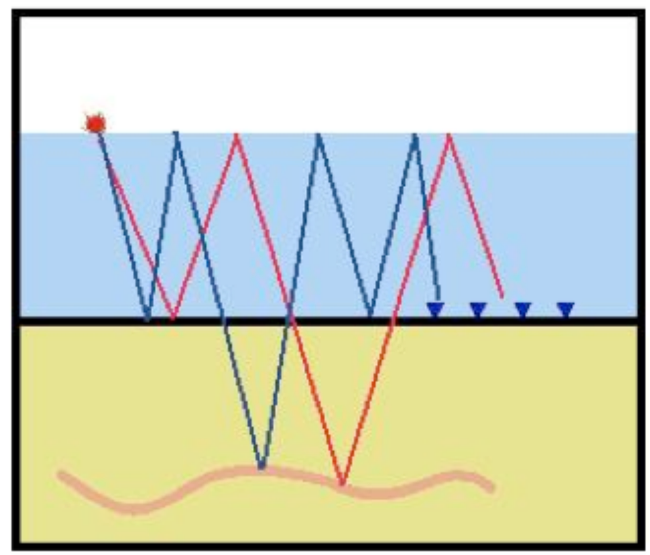
Stanford Exploration Project

Accuracy of the up-going wavefield modeling operator



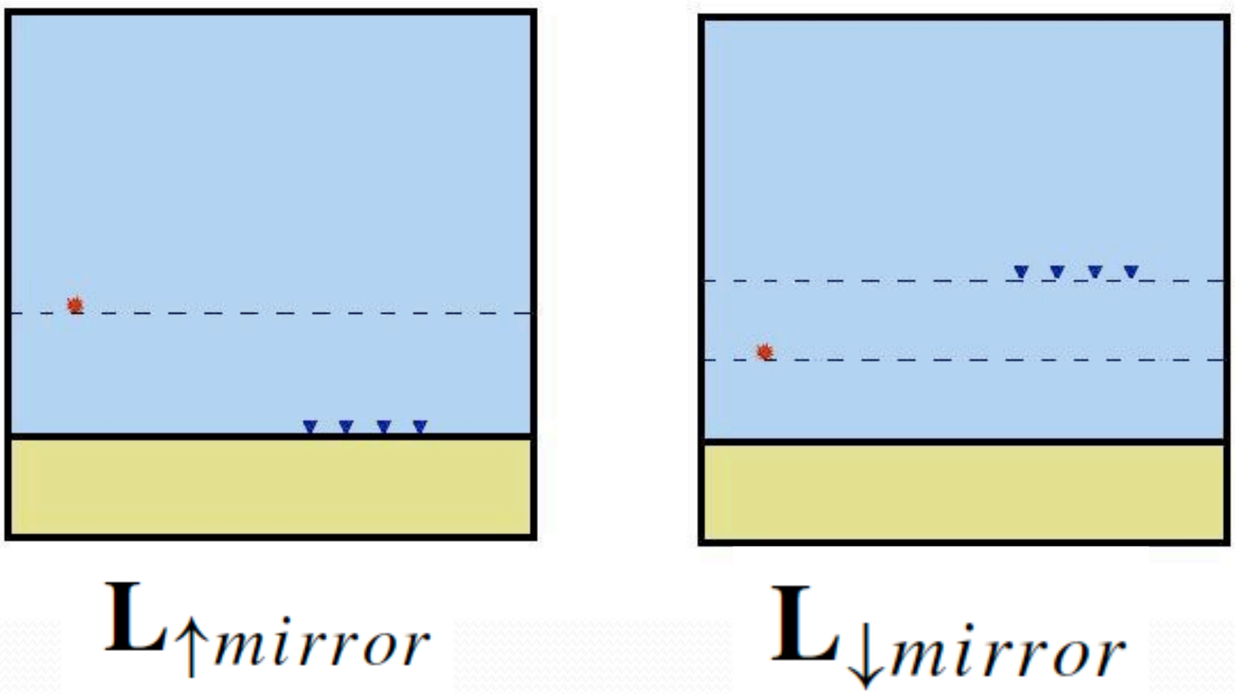
$\mathbf{L}_{\downarrow mirror}$



Excluded

Using mirror imaging operator in the inverse Problem

$$min\left(\|\mathbf{L}_{\uparrow}\mathbf{m}-\mathbf{d}_{\uparrow}\|^2+\|\mathbf{L}_{\downarrow}\mathbf{m}-\mathbf{d}_{\downarrow}\|^2\right)$$



The Inverse Problem: Complete Operator

$$\begin{bmatrix} \mathbf{L}_{\uparrow} \\ \mathbf{L}_{\downarrow} \end{bmatrix} = \mathbf{S}_{ou}^{-1}\mathbf{A}$$

$\mathbf{A}$ is a **two way acoustic** forward modeling operator.

$\mathbf{S}_{ou}^{-1}$ is separates over and under data

into up- and down-going wavefield

The Inverse Problem: Complete Operator

$$\begin{bmatrix} \mathbf{L}_{\uparrow} \\ \mathbf{L}_{\downarrow} \end{bmatrix} \mathbf{m} = \mathbf{S}_{ou}^{-1} \mathbf{A} \mathbf{m}$$

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The Inverse Problem: Complete Operator

Fitting goal

$$0 \approx \begin{bmatrix} \mathbf{L}_\uparrow \\ \mathbf{L}_\downarrow \end{bmatrix} \mathbf{m} - \begin{bmatrix} \mathbf{d}_\uparrow \\ \mathbf{d}_\downarrow \end{bmatrix} = \mathbf{S}_{ou}^{-1} \mathbf{A} \mathbf{m} - \mathbf{S}_{pz}^{-1} \begin{bmatrix} \mathbf{d}_p \\ \mathbf{d}_z \end{bmatrix}$$

The Inverse Problem: Complete Operator

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An alternate interpretation is fitting the P and Z data using only acoustic modeling

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Another alternate interpretation is fitting over/under data.

$$0 \approx \mathbf{A} \mathbf{m} - \mathbf{S}_{ou} \mathbf{S}_{pz}^{-1} \begin{bmatrix} \mathbf{d}_{p} \\ \mathbf{d}_{z} \end{bmatrix}$$

The Inverse Problem: Complete Operator

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❖ Formulated an inversion problem that make use of the up- and down-going signal of ocean bottom data

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❖ Will work on this problem next term.


Thank
You!