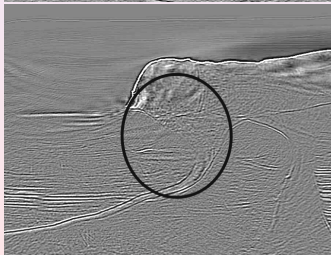
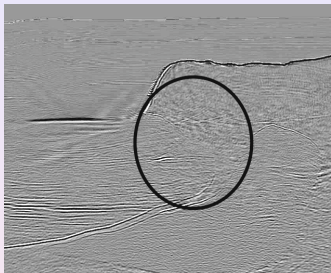


Tilted elliptic cylindrical conical-wave migration & Ellipsoidal shot-profile migration

Jeff Shragge and Guojian Shan

SEP Meeting 2008
SEP-134: Pages 123-134, 135-146

Toward 3D generalized coordinate migration



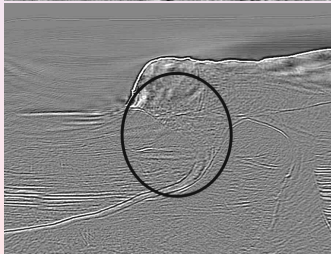
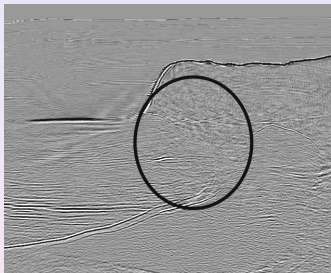
Imaging issues

- Imaging steep structure in complex geology remains problematic with wave-equation methods
 - Poor down-going illumination
 - Incorrect velocity models
- Image turning wavefield components
 - Inline and crossline azimuths
- Hale et al. (1992), Zhang and McMechan (1997), Zhang and Xu (2006), Shan (2008)

Migration strategies

- Reverse-time migration
 - Accurate .. very expensive
- One-way wavefield extrapolation
 - Cheaper ... lower accuracy and high-angle limited

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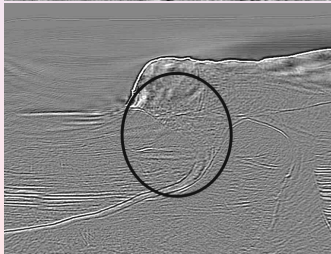
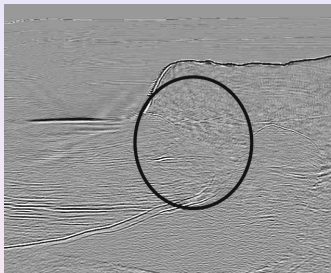
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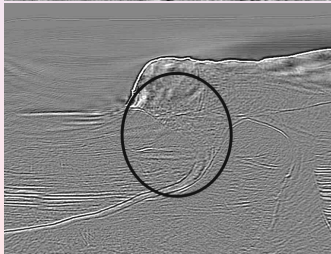
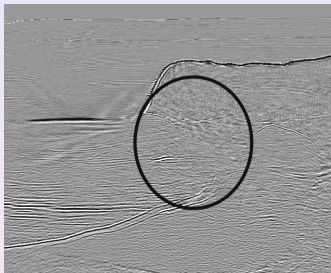
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Turning-wave imaging

Coordinate transformation strategy

- Change migration geometry to overcome steep-dip limitations while taking advantage of lower one-way extrapolation costs
- Analytic coordinate system examples:
 - 3D tilted Cartesian (Shan, 2008), 2D elliptic (Shragge and Shan, 2008)
- Generally requires reformulating the Helmholtz equation
 - Riemannian wavefield extrapolation (Sava and Fomel, 2006; Shragge, 2008)
- Method steps between Cartesian (\mathbf{x}) and Canonical (ξ) domains:
 - 1 Interpolate $v(\mathbf{x})$ velocity model to $v(\xi)$
 - 2 Propagate source and receiver wavefields on ξ domain
 - 3 Correlate source and receiver to form $I(\xi)$ image
 - 4 Interpolate $I(\xi)$ to $I(\mathbf{x})$

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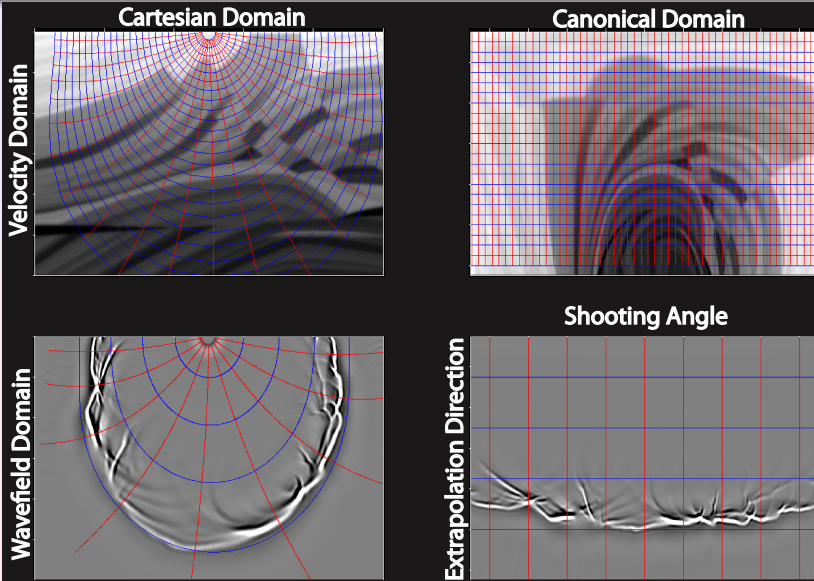
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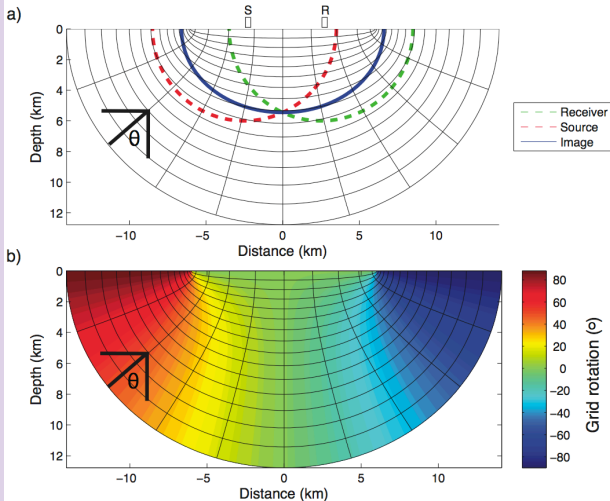
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Example I : 2D ray-coordinate Green's function



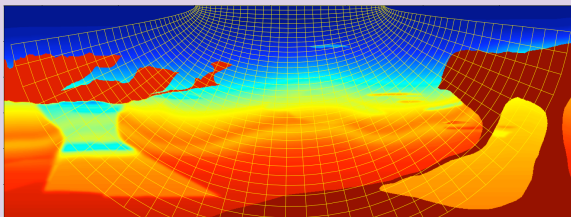
Example II: 2D elliptic coordinate migration

Elliptic meshes

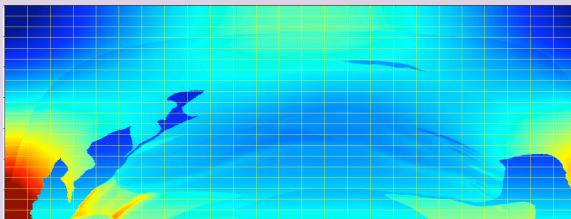


Example II: 2D elliptic coordinate migration

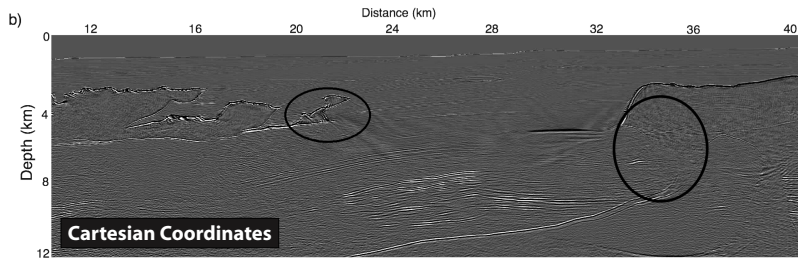
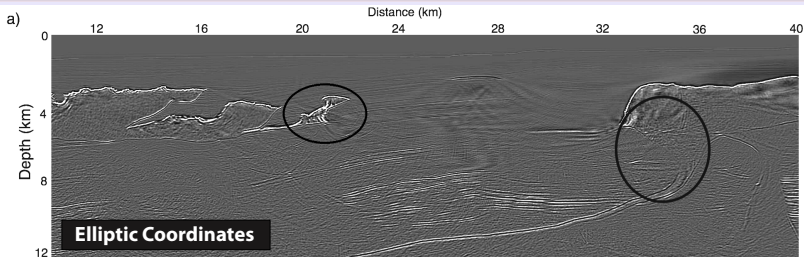
Cartesian velocity field



Elliptic velocity field



Example: Imaging in 2D elliptic coordinates



That's nice .. but what about 3D?

3D algorithm goals

- Analytic 3D coordinate systems \leftrightarrow analytic 3D extrapolation wavenumbers
- Simple extrapolation wavenumber that introduces minimal errors into the extrapolation operator implementation
- Geometry should not introduce significant computational/memory overhead

Candidate 3D meshes and techniques

- Tilting elliptic cylindrical coordinates
 - Conical-wave migration of narrow/wide azimuth sail line data
- Ellipsoidal coordinates
 - Shot-profile or plane-wave migration of narrow/wide azimuth data

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 - Plane-wave migration review
 - Tilted elliptic cylindrical coordinates
 - Migration examples

- 2 **Ellipsoidal coordinate migration**
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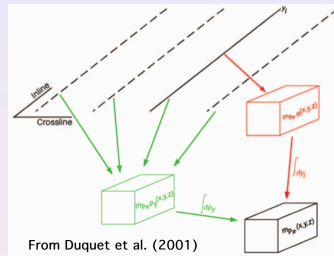
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3D plane- and conical-wave migration

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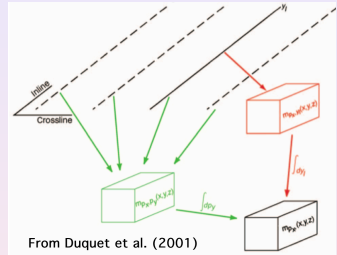
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 - Synthesize fewer composite receiver wavefields than in original volume
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 - Inline decomposition of individual sail-lines
 - Source wavefields \rightarrow conic sections
- Shot-profile migration (Liu et al., 2006)
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 - CWM equivalent when using a 'large' number of inline conical waves



3D plane- and conical-wave migration

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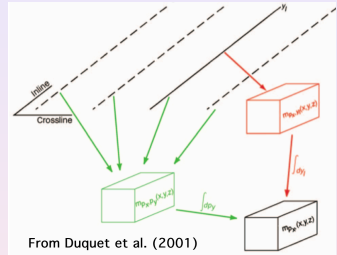
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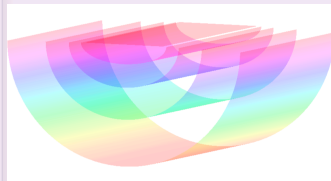
Plane-wave migration geometry

Question

For a given composite wavefield bulk propagation direction, how do we optimally propagate turning waves with one-way wavefield extrapolation?

Alternate 3D geometries

- Fully 3D plane-wave migration
 - Tilted Cartesian meshes (Shan, 2008)
 - Accurate, more efficient than shot-profile migration
 - Large memory/computational footprint from global wavefield/image volumes
- Conical-wave migration of individual sail lines
 - Inline propagation handled by tilted Cartesian grid
 - Crossline propagation handled by elliptic-like mesh
 - Tilted elliptic cylindrical coordinates



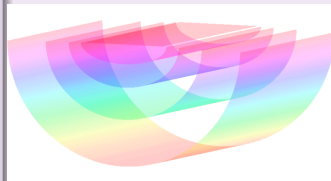
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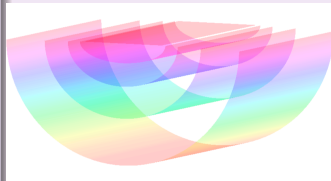
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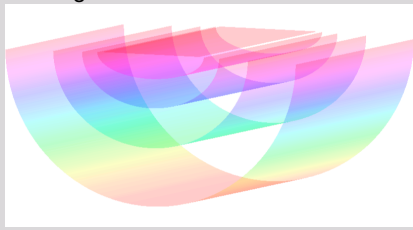
Tilted elliptic cylindrical coordinates

ξ -coordinate axes

- Extrapolation surfaces are confocal elliptic cylinders
- Initial surface: Tight box around acquired data
- Outward expanding crossline aperture
 - Opportunity for regridding as mesh expands outward

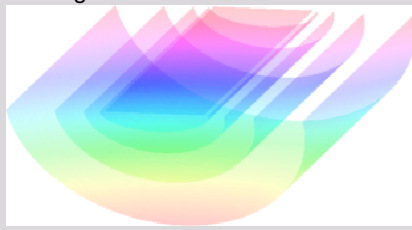
Example

Tilt Angle $\theta = 0^\circ$



Example

Tilt Angle $\theta = 25^\circ$



Cylindrical elliptic coordinates

Extrapolation wavenumber - no tilt

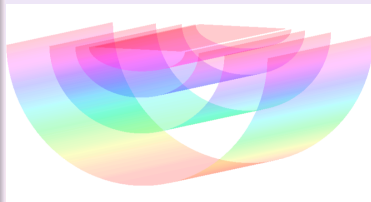
- Coordinate system definition:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \xi_1 \\ a \cosh \xi_3 \cos \xi_2 \\ a \sinh \xi_3 \sin \xi_2 \end{bmatrix}$$

- where a is scaling parameter
- Resulting extrapolation wavenumber

$$k_{\xi_3} = \pm \sqrt{A^2 s^2 \omega^2 - k_{\xi_1}^2 - A^2 k_{\xi_2}^2}$$

- $A = A(\xi)$ is a geometric scaling parameter
- k_{ξ_j} is a wavenumber in the ξ_j direction
- Anisotropy in k_{ξ_2} wavenumber



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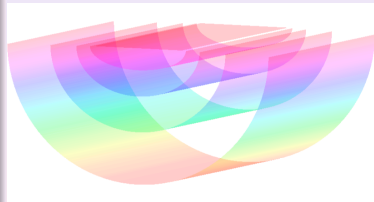
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Cylindrical elliptic coordinates

Extrapolation wavenumber - with tilt

- Coordinate system definition:

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \xi_1 \cos \theta - a \cosh \xi_3 \cos \xi_2 \sin \theta \\ a \cosh \xi_3 \cos \xi_2 \\ \xi_1 \sin \theta + a \sinh \xi_3 \sin \xi_2 \cos \theta \end{bmatrix}$$

- where a is scaling parameter and θ is tilt angle
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FFD Implementation - I

Question

Can the extrapolation wavenumber be implemented in FD schemes?

Dispersion relationship comparisons

- Exploit wavefield extrapolation schemes for elliptic anisotropy

$$\text{Elliptic cylindrical : } \frac{k_{\xi_3}}{A\omega S} = \sqrt{1 - \frac{k_{\xi_1}^2}{A^2\omega^2 S^2} - A^2 \frac{k_{\xi_2}^2}{A^2\omega^2 S^2}}$$

$$\text{Elliptic anisotropy : } \frac{k_{x_3}}{\omega S} \Big|_{\epsilon=\delta} = \sqrt{1 - (1 + 2\epsilon) \frac{k_{x_1}^2}{\omega^2 S^2} - (1 + 2\epsilon) \frac{k_{x_2}^2}{\omega^2 S^2}}$$

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FFD Implementation - II

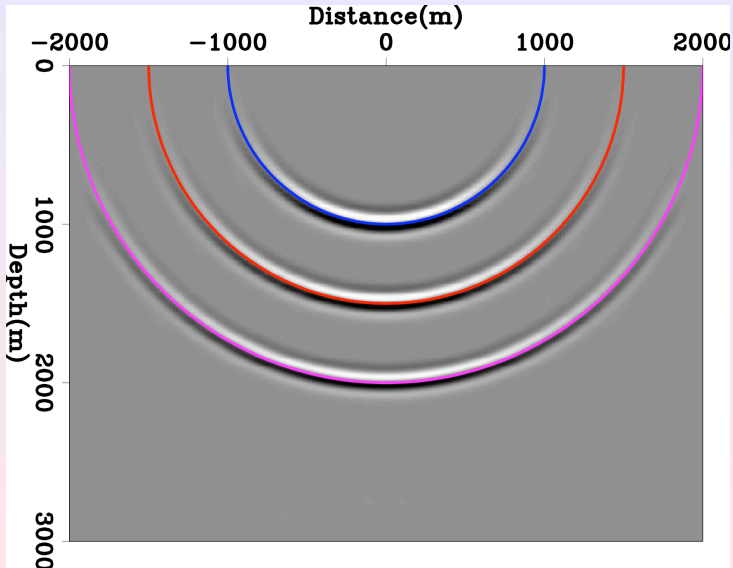
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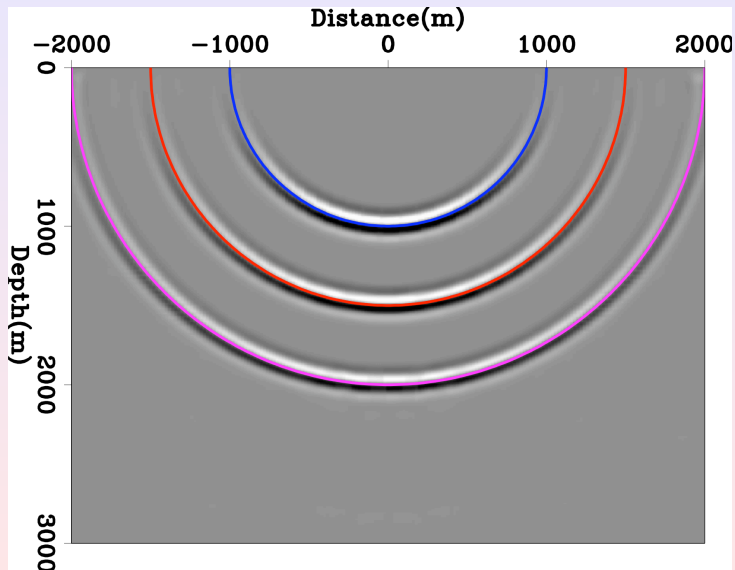
Dispersion relationship comparisons

- Optimized 3D implicit FD extrapolation algorithm (Shan, 2008)
 - Spatial phase-shift term
 - Lower-order tridiagonal solver - split in ξ_1 and ξ_2
 - Higher-order tridiagonal solver - split in ξ_1 and ξ_2
- Anisotropy handled by different scaling factors in ξ_1 and ξ_2
- Fourier-domain phase-correction filter (Li, 1991)
- Additional costs: 2 extra vector multiplications in tridiagonal solver

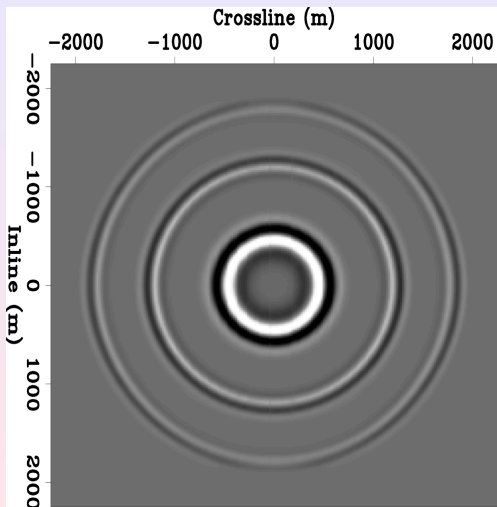
Impulse response: Inline



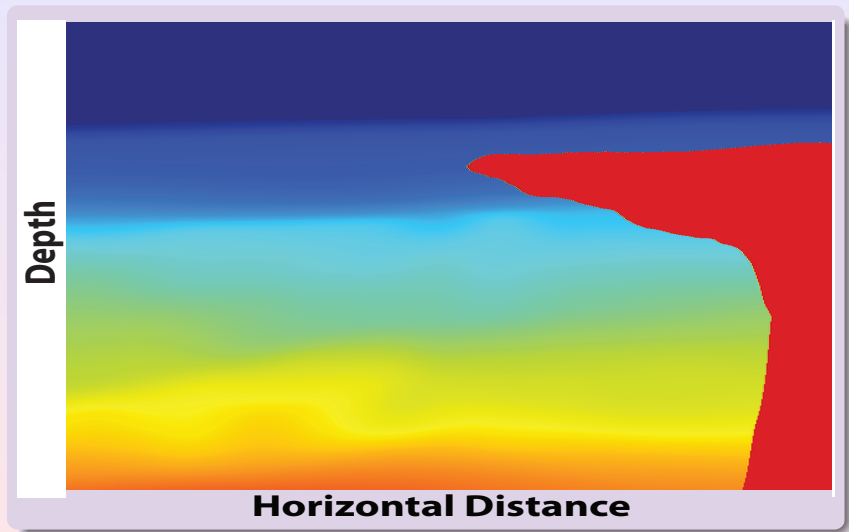
Impulse response: Crossline



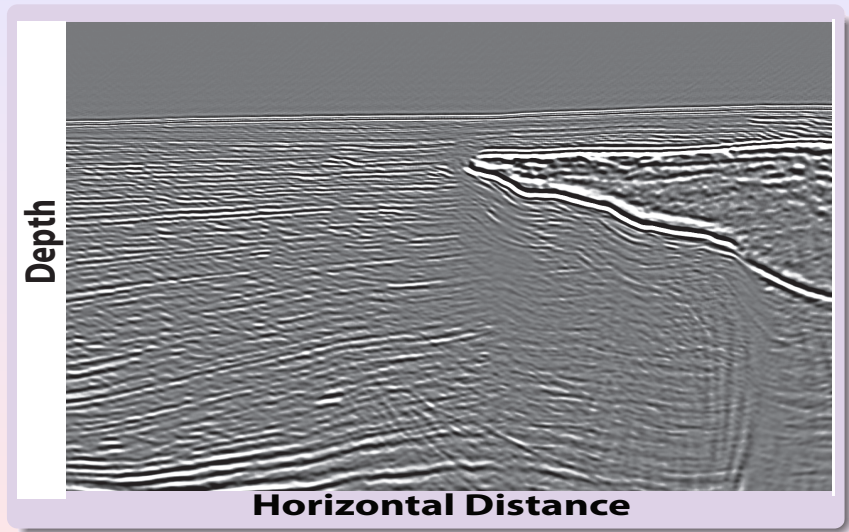
Impulse response: Depth slice



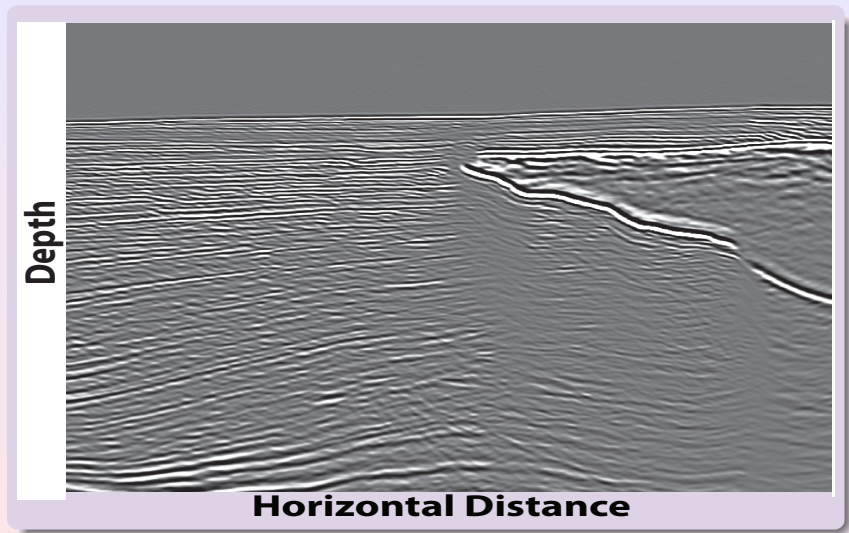
2D Gulf of Mexico: Velocity model



2D Gulf of Mexico: 61 plane waves, 61 coordinate system tilts



2D Gulf of Mexico: 61 plane waves, no coordinate system tilt



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 - Plane-wave migration review
 - Tilted elliptic cylindrical coordinates
 - Migration examples

- 2 **Ellipsoidal coordinate migration**
 - Elliptic coordinate design

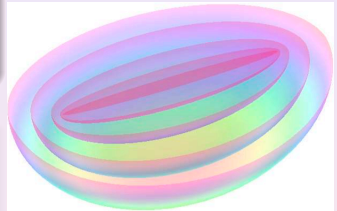
3D ellipsoidal coordinate design

Turning-wave migration requirements

- Analytic coordinate system representation
- Propagate turning waves to all azimuths
- Geometry should not introduce significant computational/memory overhead

Coordinate system design constraints

- Conform to acquisition surface
- Both narrow/wide azimuth geometries
- Generate fairly simple extrapolation wavenumber
- Both shot-profile and 'patched' plane-wave migration



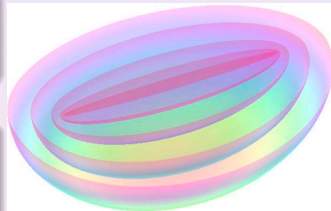
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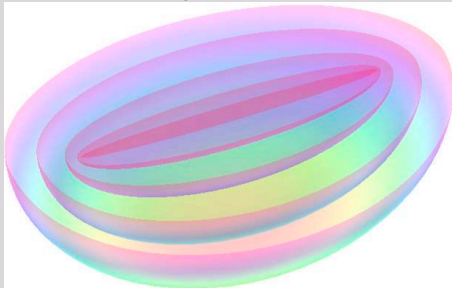
Ellipsoidal coordinate examples

ξ -coordinate system axes

- Extrapolation direction ξ_3 surfaces are confocal ellipsoids
- Extrapolation wavenumber similar to elliptic anisotropy
- Implement same FD kernel (with different ξ_1 and ξ_2 scaling factors)

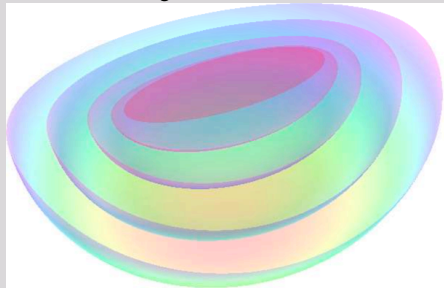
Example

Narrow-azimuth geometries



Example

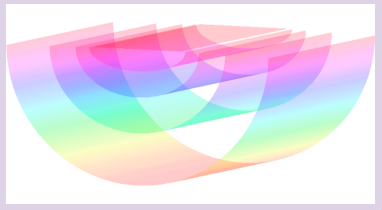
Wide-azimuth geometries



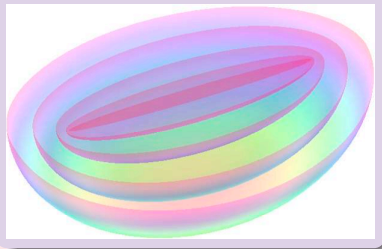
Summary

- Wave-equation migration in 3D generalized coordinates is feasible
- No more difficult to implement than elliptic anisotropy propagation
- Potential imaging benefits:
 - Migrate steeply dipping and turning wave events to all azimuths
 - Outward expanding aperture \rightarrow start with small aperture and play regridding games
- Future work: 3D field test on narrow or wide-azimuth data
- **Looking for a data set in complex geology with crossline turning waves**

Elliptic cylindrical



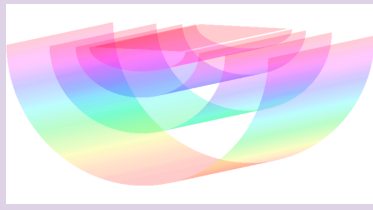
Ellipsoidal



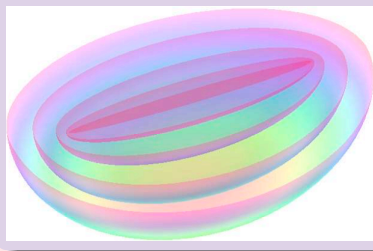
Acknowledgments

- BP and ExxonMobil for Gulf of Mexico field data
- Guojian Shan and Bob Clapp for helpful discussions

Elliptic cylindrical



Ellipsoidal



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SEP Meeting 2008
SEP-134: Pages 123-134, 135-146