Angle-domain common-image gathers in generalized coordinates

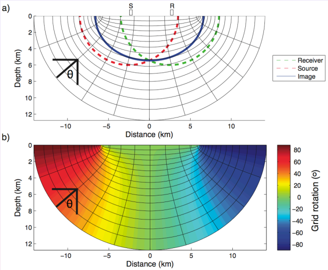
Jeff Shragge

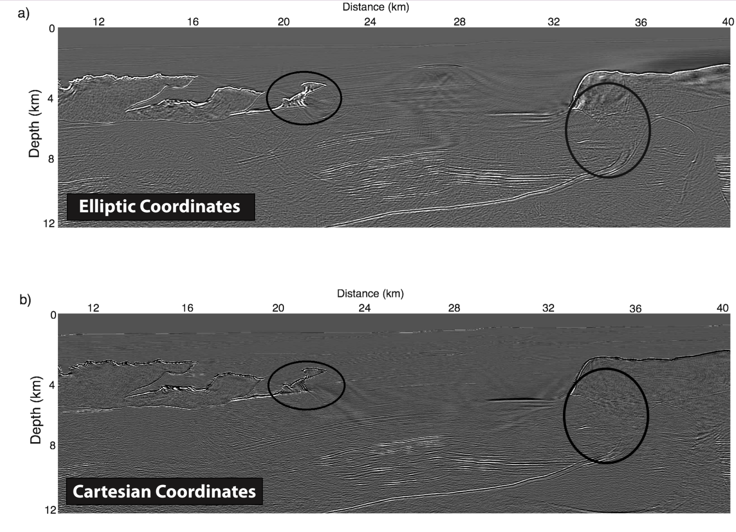
SEP Meeting 2008 - May 13, 2008
SEP 134 - Pages 111-122

Toward generalized ADCIGs

Review

- ADCIGs - diagnostic for velocity model accuracy
- Easily computed for Cartesian geometries
  - Shot-profile and shot-geophone migration
- What about in generalized coordinates?
  - SEP 2007 - 2D elliptic coordinates




a)
Distance (km)
12
16
20
24
28
32
36
40
0
4
8
12
Depth (km)
Elliptic Coordinates
b)
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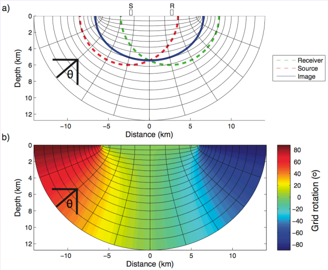
Cartesian Coordinates

Toward generalized ADCIGs

Review

- ADCIGs - diagnostic for velocity model accuracy
- Easily computed for Cartesian geometries
  - Shot-profile and shot-geophone migration
- What about in generalized coordinates?
  - SEP 2007 - 2D elliptic coordinates
- ADCIG theory currently not generally applicable
  - Extend ADCIG theory to general geometries



Outline

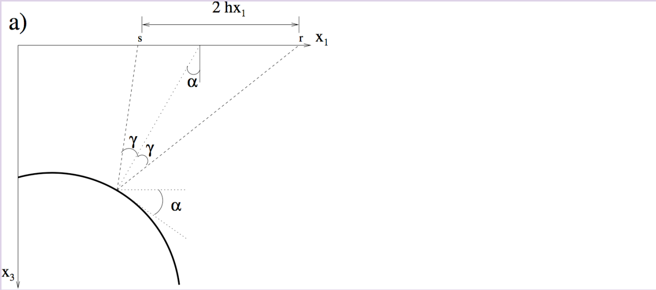
Outline

ADCIG theory

Cartesian coordinates

- Calculate reflection angle γ and reflector dip α
- Relate differential changes in travel time t to
  - Depth x_3 and subsurface offset h_{x_1}

Schematic



ADCIG theory

Cartesian ADCIG equations

- Relate fields t, x_3 and h_{x_1} to γ, α and slowness s (Sava and Fomel, 2003)

$$\left[\begin{array}{c}\frac{\partial t}{\partial h_{x_1}}\\ \frac{\partial t}{\partial x_3}\end{array}\right]\Bigg|_{t,x_1} = 2\,s\cos\alpha\left[\begin{array}{c}\sin\gamma\\ \cos\gamma\end{array}\right]$$

- Divide the two equations

$$\left.\frac{\partial x_3}{\partial h_{x_1}}\right|_{t,x_1} = -\frac{\partial t}{\partial h_{x_1}}\Big/\frac{\partial t}{\partial x_3} = -\tan\gamma$$

- Fourier-domain calculation

$$\tan\gamma = -\frac{k_{h_{x_1}}}{k_{x_3}}$$

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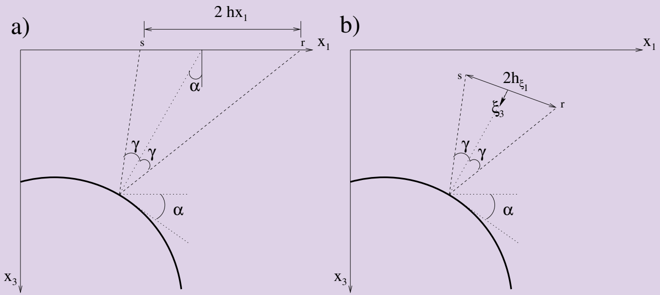
ADCIG theory

Generalized coordinates

- Calculate reflection angle γ and reflector dip α
- Relate differential changes in travel time *t* to
  - Extrapolation coordinate ξ_3 and subsurface shift coordinate h_{ξ_1}

Schematic

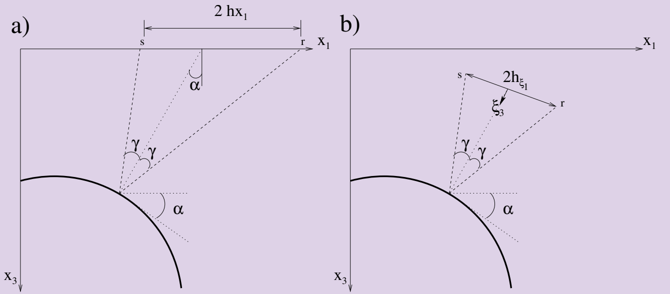


ADCIG theory

So what changes?

- Which ADCIG quantities are affected by this change of variables?
  - Constant reflection angle γ and reflector dip α
  - Different h_{ξ_1} and ξ_3 axis orientations

Schematic



Generalized coordinates

Key assumptions

- Coordinate systems related by one-to-one mappings:

$$\begin{aligned} x_1 &= f(\xi_1, \xi_3) \\ x_3 &= g(\xi_1, \xi_3) \end{aligned}$$

- Explicitly define two subsurface offset axes
- h_{x_1} - horizontal and linear shift along the x_1 axis
  - e.g. Cartesian coordinates - uniform shift along a horizontal line
- h_{ξ_1} - shift along the ξ_1 axis sharing the same geometry as the ξ_1 axis.
  - e.g. Elliptic coordinates - regular shift along an elliptic surface

ADCIG theory

Generalized coordinates

- Use partial derivative expansion

$$\left[\begin{array}{c} \frac{\partial t}{\partial h_{x_1}} \\ \frac{\partial t}{\partial x_3} \end{array}\right]\Bigg|_{\xi_1,t} = \left[\begin{array}{c} \frac{\partial t}{\partial h_{\xi_1}}\frac{\partial h_{\xi_1}}{\partial h_{x_1}} \\ \frac{\partial t}{\partial \xi_3}\frac{\partial \xi_3}{\partial x_3} \end{array}\right]\Bigg|_{\xi_1,t} = 2\,s\cos\alpha \left[\begin{array}{c} \sin\gamma \\ \cos\gamma \end{array}\right]$$

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- What does term $\frac{\partial h_{x_1}}{\partial h_{\xi_1}}$ represent?

ADCIG theory

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Defining subsurface stretch

What is subsurface stretch?

- Partial differential expansions yield

$$\frac{\partial h_{x_1}}{\partial h_{\xi_1}} = \frac{\partial h_{x_1}}{\partial x_1} \frac{\partial x_1}{\partial \xi_1} \Big/ \frac{\partial h_{\xi_1}}{\partial \xi_1}$$

  - $\frac{\partial h_{x_1}}{\partial x_1}$ - scaling between h_{x_1} and x_1 (e.g. $h_{x_1} = x_1$ such that $\frac{\partial h_{x_1}}{\partial x_1} = 1$)
  - $\frac{\partial x_1}{\partial \xi_1}$ - partial derivative mapping between coordinate systems
  - $\frac{\partial h_{\xi_1}}{\partial \xi_1}$ - scaling between h_{ξ_1} and ξ_1 (e.g. $h_{\xi_1} = \xi_1$ such that $\frac{\partial h_{\xi_1}}{\partial \xi_1} = 1$)
- General ADCIG equation for regular wavefield shifts

$$\tan\gamma = - \left.\frac{\partial \xi_3}{\partial h_{\xi_1}}\right|_{\xi_1,t} \left[\frac{\partial x_3}{\partial \xi_3} \Big/ \frac{\partial x_1}{\partial \xi_1} \right]$$

- Can calculate in Fourier domain if no geometric scaling introduced
  - Must calculate separately for each coordinate system used

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Outline

1. ADCIG Theory
   - Cartesian coordinates
   - Generalized coordinates

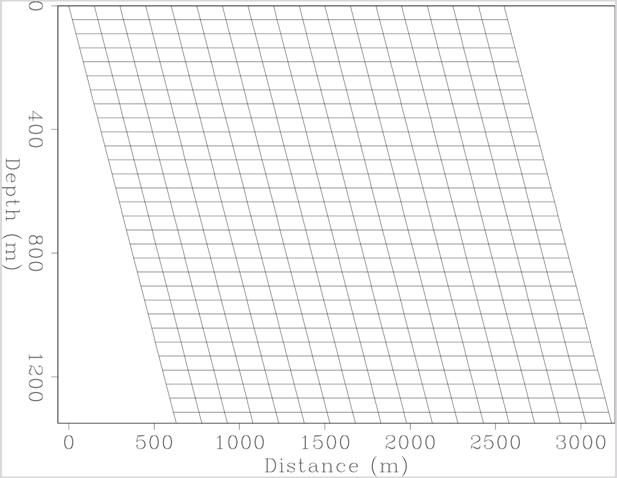
2. **Analytic examples**
   - Sheared Cartesian coordinates
   - Polar coordinates
   - Elliptic coordinates

3. Numerical example
   - Elliptic coordinates

Examples I - Sheared Cartesian

Example

Shear Angle 25°



Example I - Sheared Cartesian

Sheared Cartesian ADCIGs

- A Cartesian mesh sheared at angle θ is defined by

$$
\begin{aligned}
x_1 &= \xi_1 + \xi_3 \sin\theta \quad \longrightarrow \quad \frac{\partial x_1}{\partial \xi_1} = 1 \\
x_3 &= \xi_3 \cos\theta \quad \longrightarrow \quad \frac{\partial x_3}{\partial \xi_3} = \cos\theta
\end{aligned}
$$

- Sheared Cartesian coordinate ADCIGs are given by

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- Can calculate ADCIGs with Fourier methods

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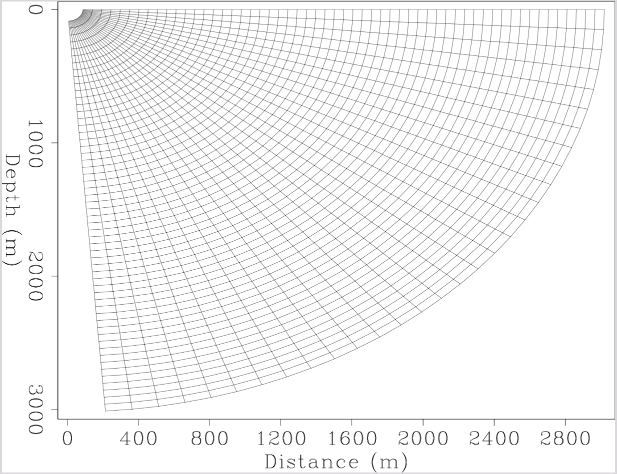
- Can calculate ADCIGs with Fourier methods

Example II - Polar coordinates

Example

Polar coordinate mesh



Example II - Polar coordinates

Polar coordinate ADCIG equations

- A polar coordinate mesh scaled by a is defined by

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x_1 &= \mathrm{a}\,\xi_1 \cos\xi_3 \quad \longrightarrow \quad \frac{\partial x_1}{\partial \xi_1} = \mathrm{a}\cos\xi_3 \\
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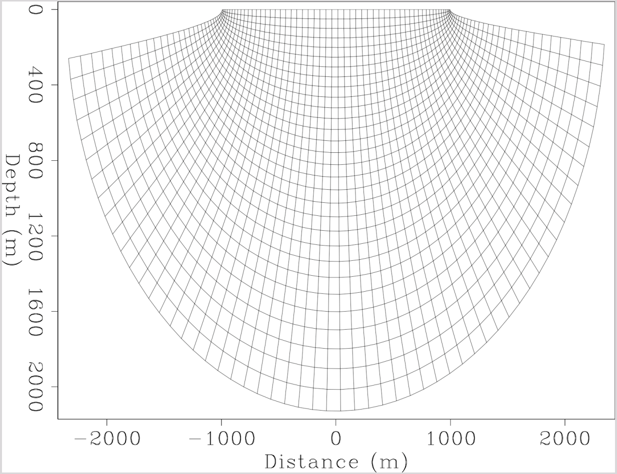
- Cannot calculate ADCIGs with Fourier methods

Example III - Elliptic coordinates

Example

Elliptic coordinate mesh



Example III - Elliptic coordinates

Elliptic coordinate ADCIG equations

- An elliptic coordinate mesh scaled by a is defined by

$$x_1 = \mathrm{a}\cosh\xi_3\cos\xi_1 \quad \rightarrow \quad \frac{\partial x_1}{\partial \xi_1} = \mathrm{a}\cosh\xi_3\sin\xi_1$$

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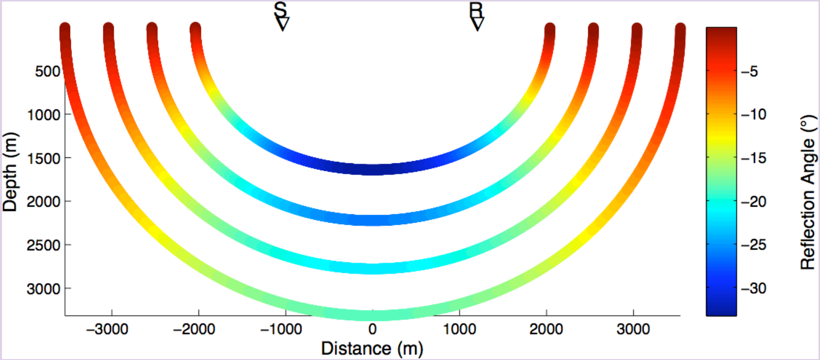
- Can calculate ADCIGs with Fourier methods

Outline

Expected reflection angles

Key questions:

1. Will we recover the correct reflection angles?
2. How well-resolved are the imaged ADCIGs?



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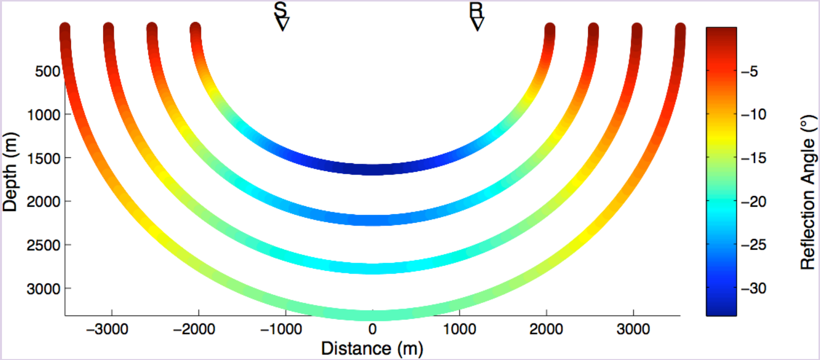
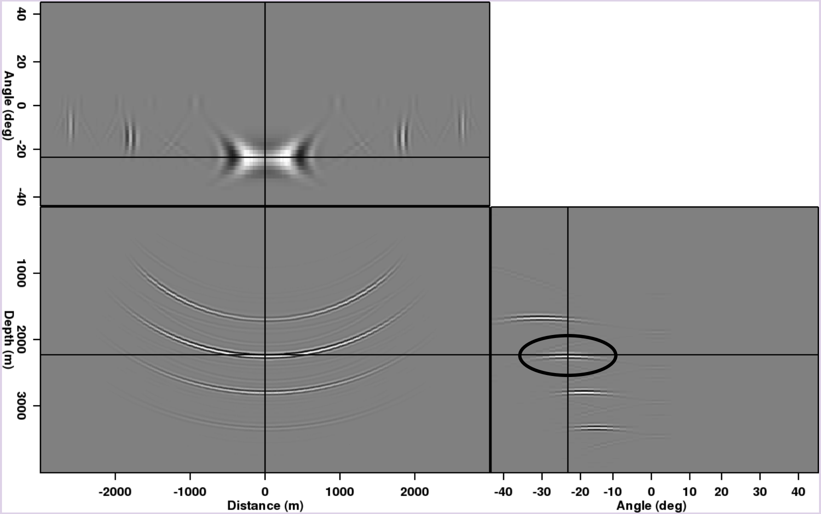
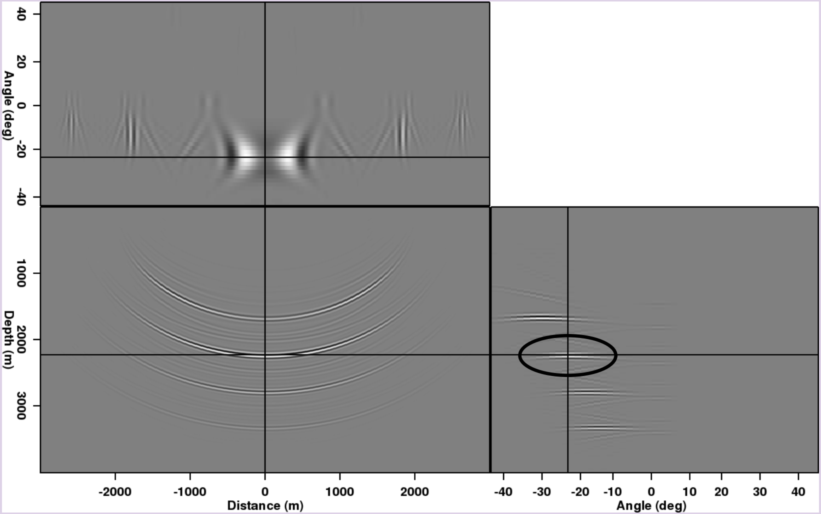


ADCIG Example: Elliptic coordinates



ADCIG Example: Cartesian coordinates



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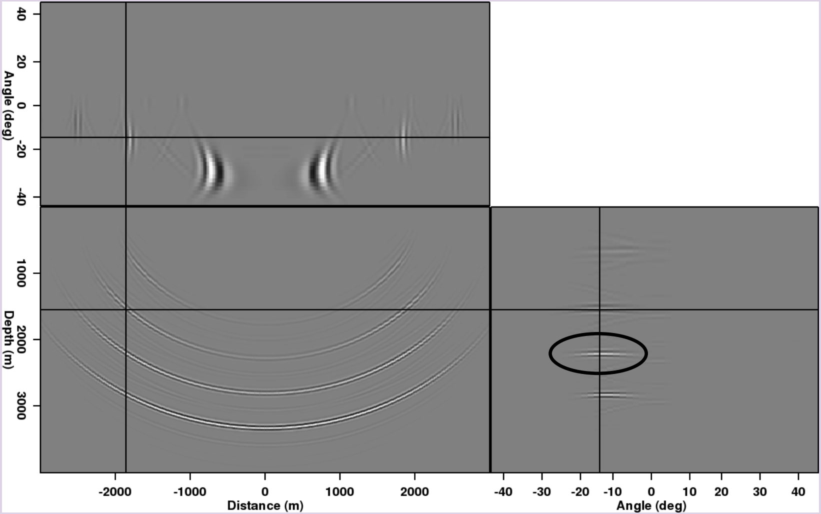

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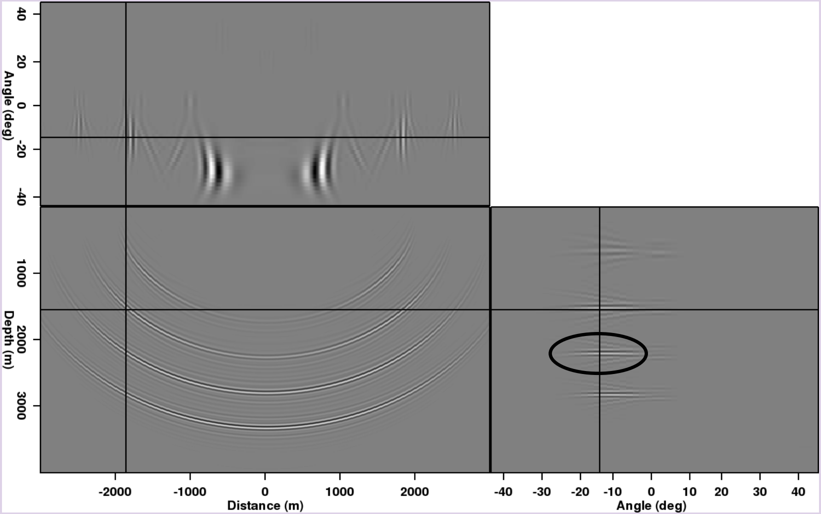
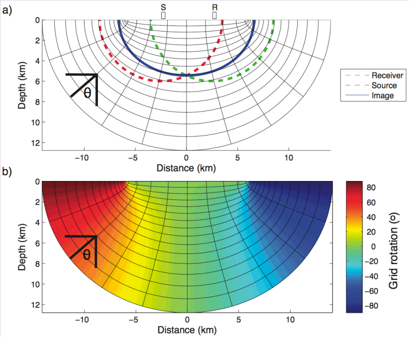


A comment on dip sensitivity

Varying sensitivity

- Computation of ADCIGs is sensitive to reflector dip
  - Horizontal ADCIGs most sensitive to flat structure
  - Vertical ADCIGS most sensitive to vertical structure
- Elliptic meshes have grid angles from $\theta = -80^\circ$ to 80°
  - Advantage: "Broadband" sensitivity



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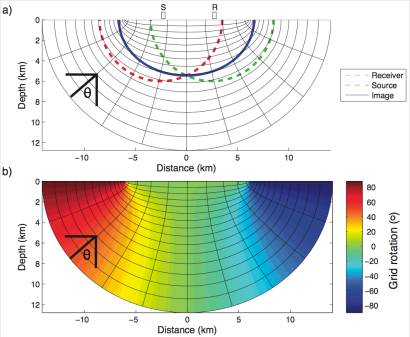
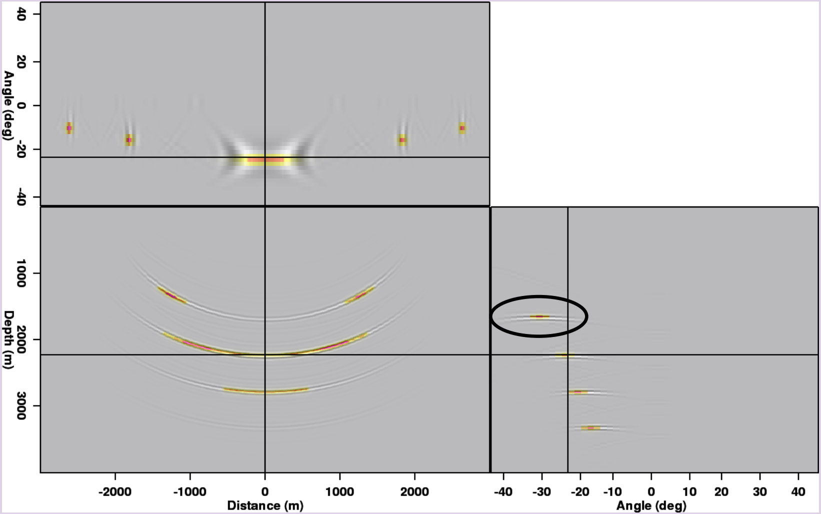
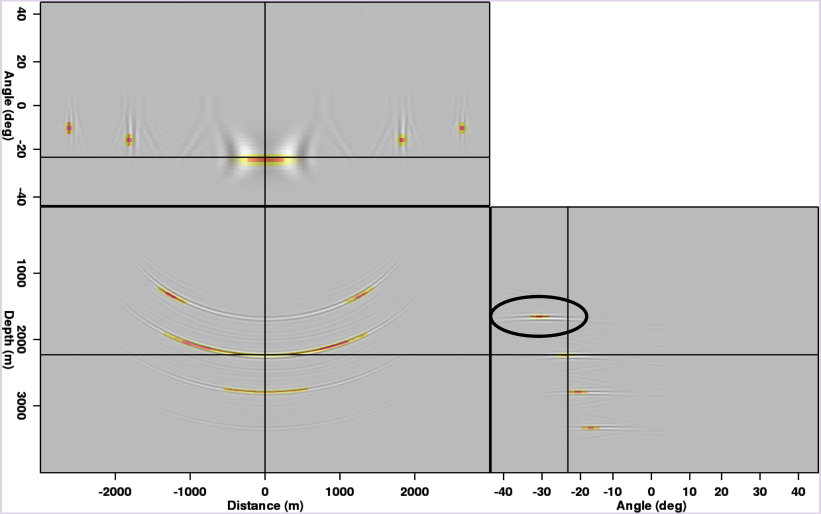


ADCIG example: Elliptic coordinates



ADCIG example: Cartesian coordinates



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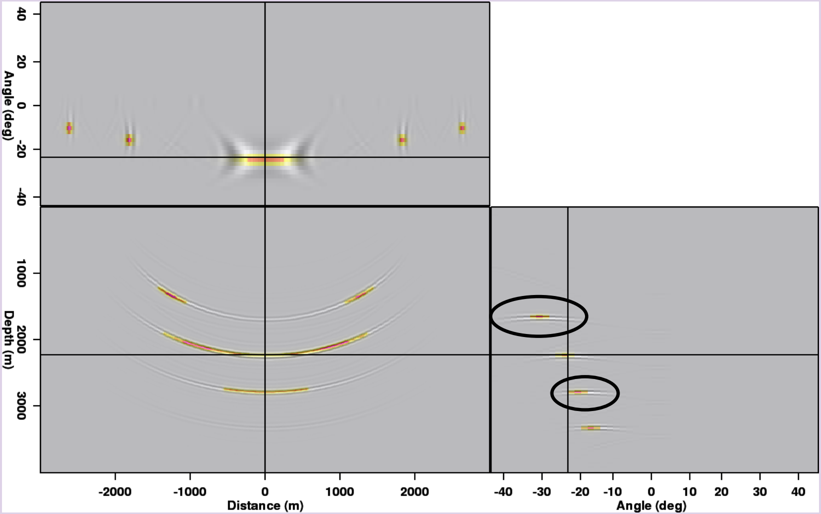
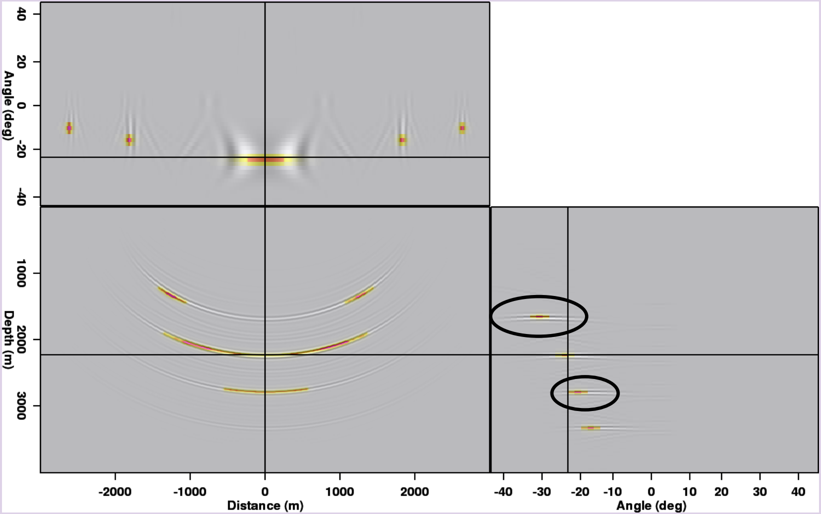


ADCIG example: Cartesian coordinates



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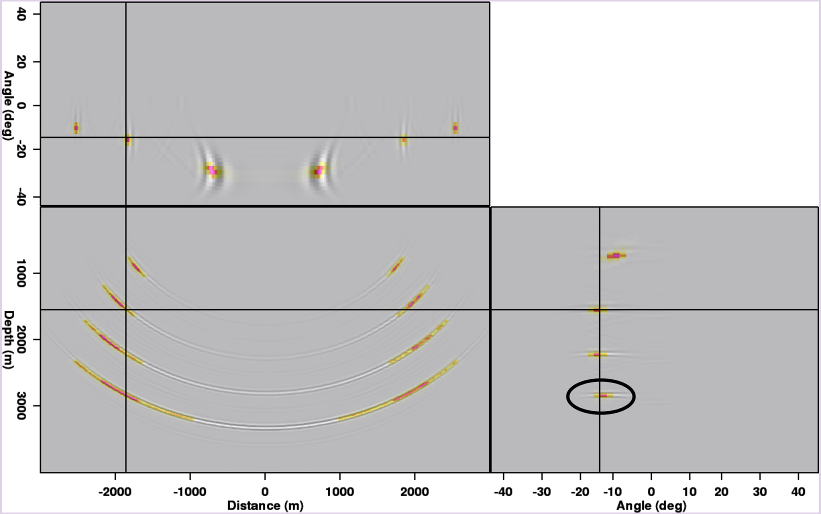
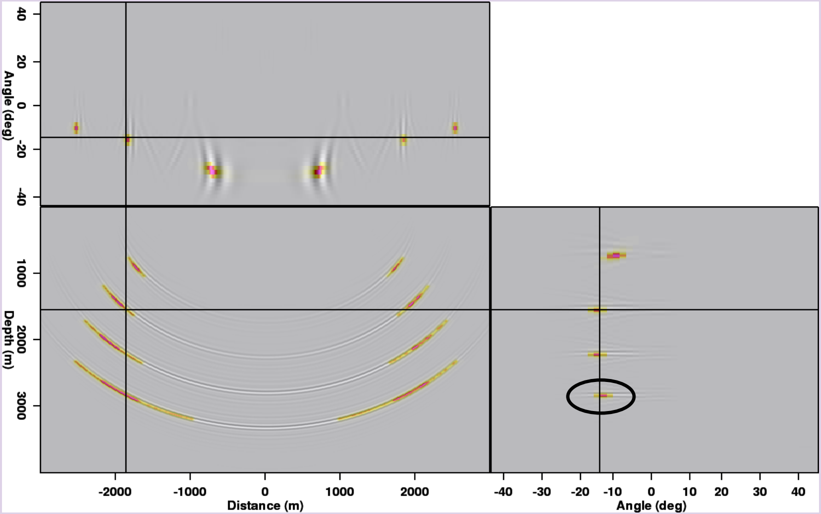


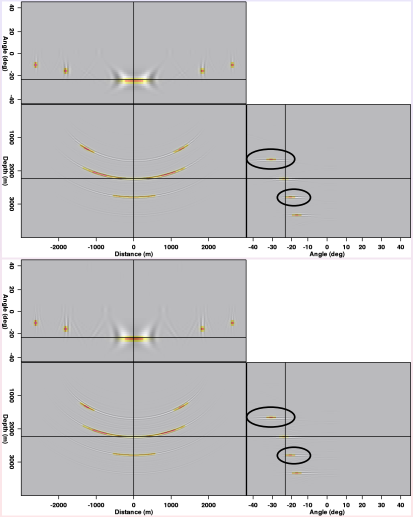
ADCIG example: Cartesian coordinates



Discussion

Offset source-receiver experiment

- Imaged ADCIG locations not always centered in correct location
  - Both Cartesian and Elliptic coordinates
- Center shifts to positive or negative θ
- Keep in mind during inversions based on ADCIG constraints



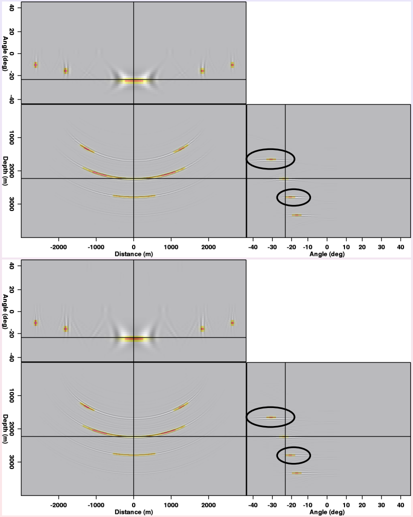
Conclusions

Generalized coordinate ADCIG extension

- ADCIGs can be calculated with Fourier-based methods in certain coordinate examples
  - 2D elliptic
- Generalized coordinates have variable ADCIG sensitivity to structural dip
- ADCIGs not always correctly centered
- Geometric dependence $\leftrightarrow$ ADCIGs calculated via slant stacks
  - 2D Polar, 3D Elliptic cylindrical and ellipsoidal coordinates

Acknowledgments

- Those people who asked ”So, how do you calculate ADCIGs in generalized coordinates?”
- Ben Witten, Brad Artman and Alejandro Valenciano for helpful discussions
- SEP Sponsors



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