Automatic wave-equation migration velocity analysis
or
Autofocusing Migration

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SEP 134
pp. 65-77

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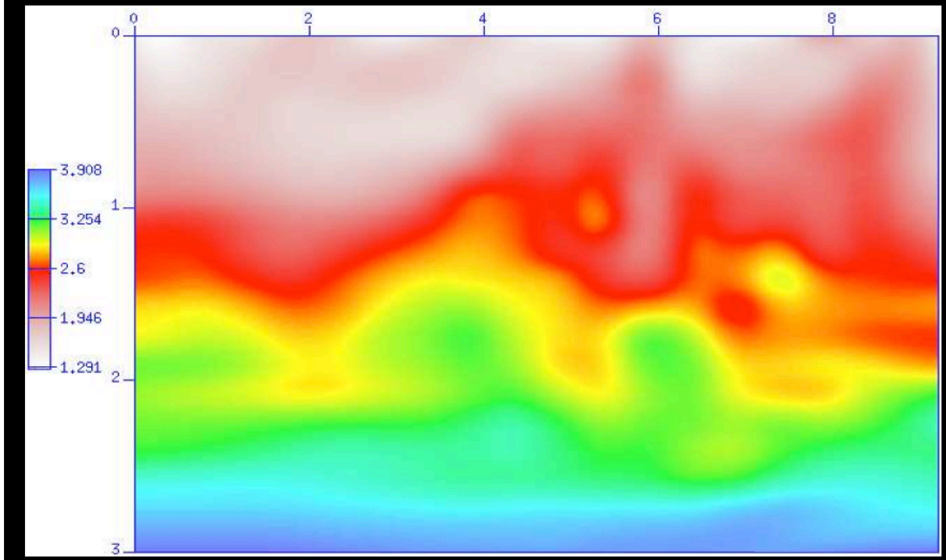
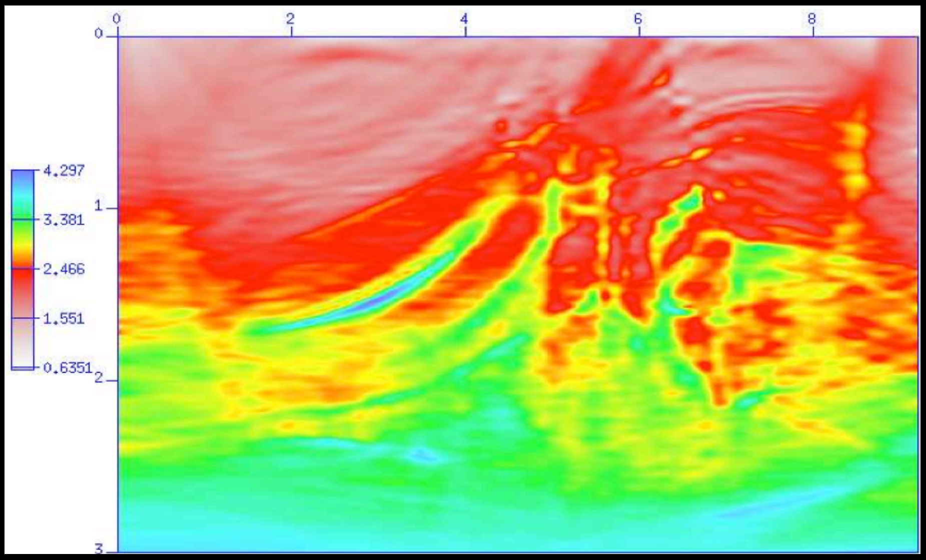
Differential Semblance Optimization with WEM



Initial velocity model



Final velocity model

Peng and Calandra (Total) - SEG 2005

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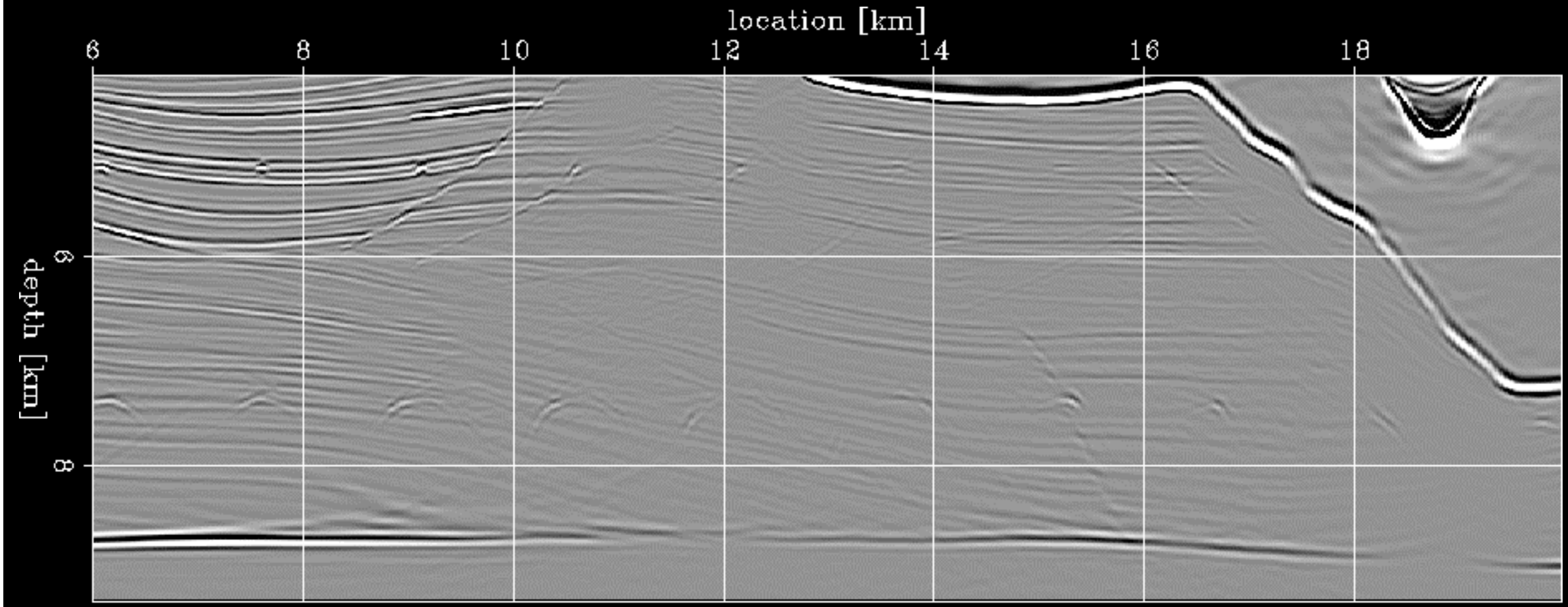
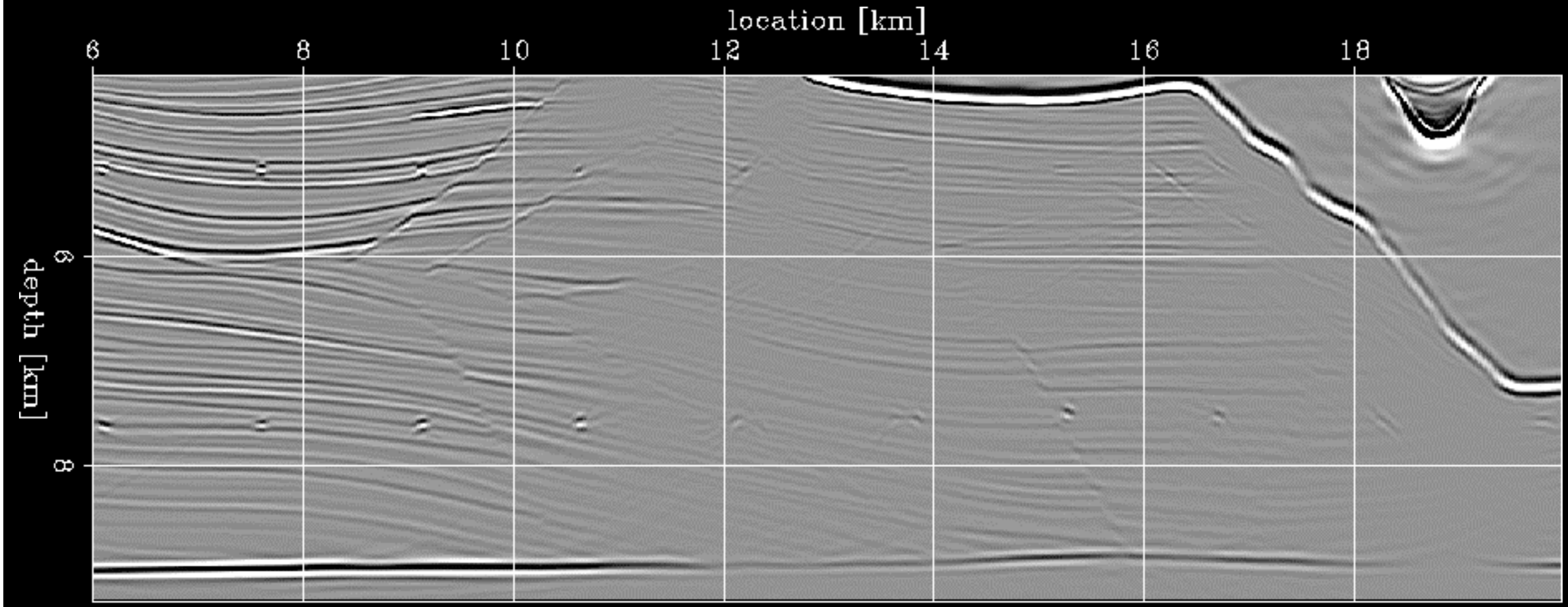
WEMVA - Image with initial velocity



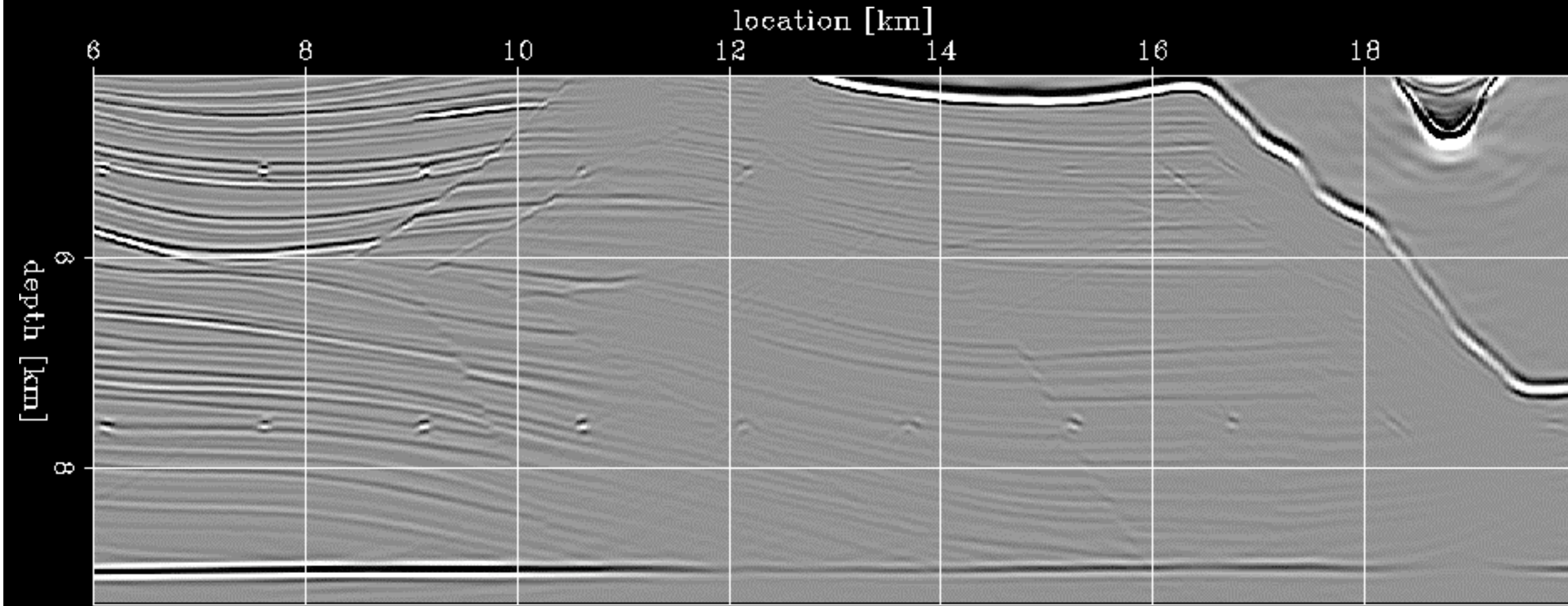
Courtesy of Paul Sava (SEP, now CSM)

WEMVA - Image with estimated velocity



Courtesy of Paul Sava (SEP, now CSM)

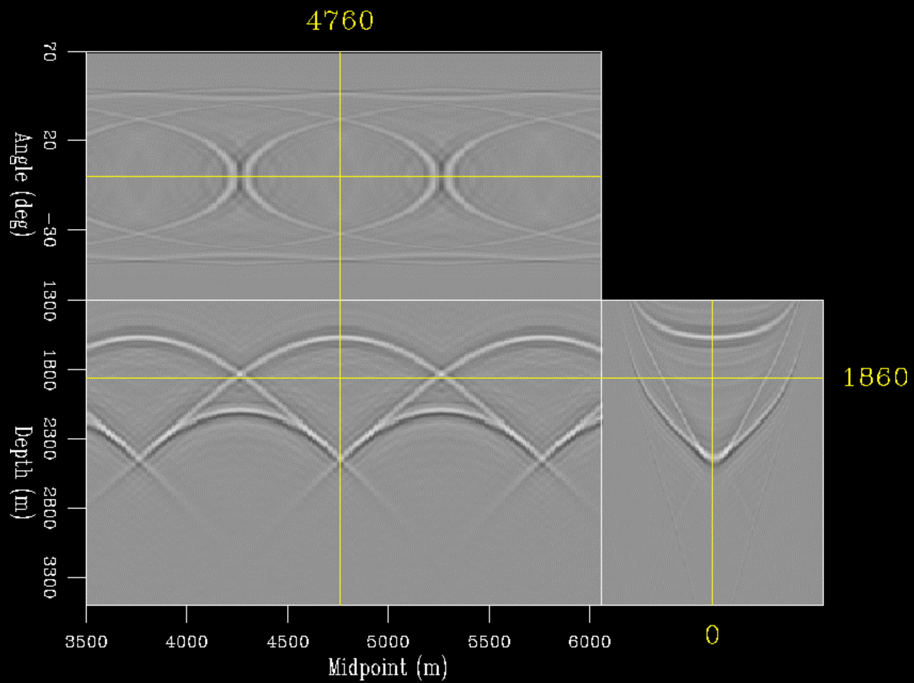
WEMVA - Image with correct velocity



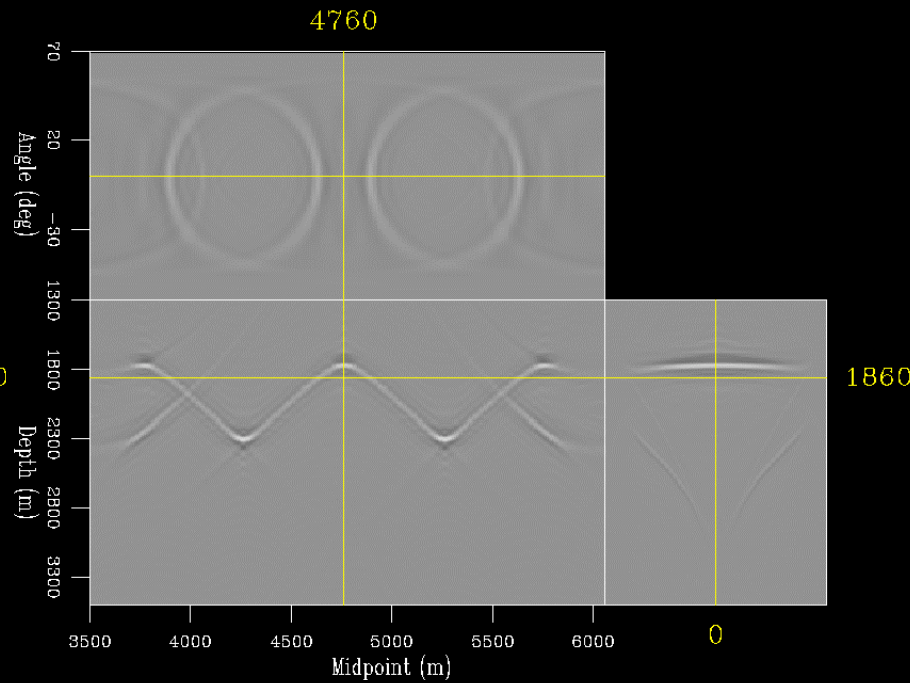
Courtesy of Paul Sava (SEP, now CSM)

Potential of residual migration for autofocusing

Migration with slow velocity

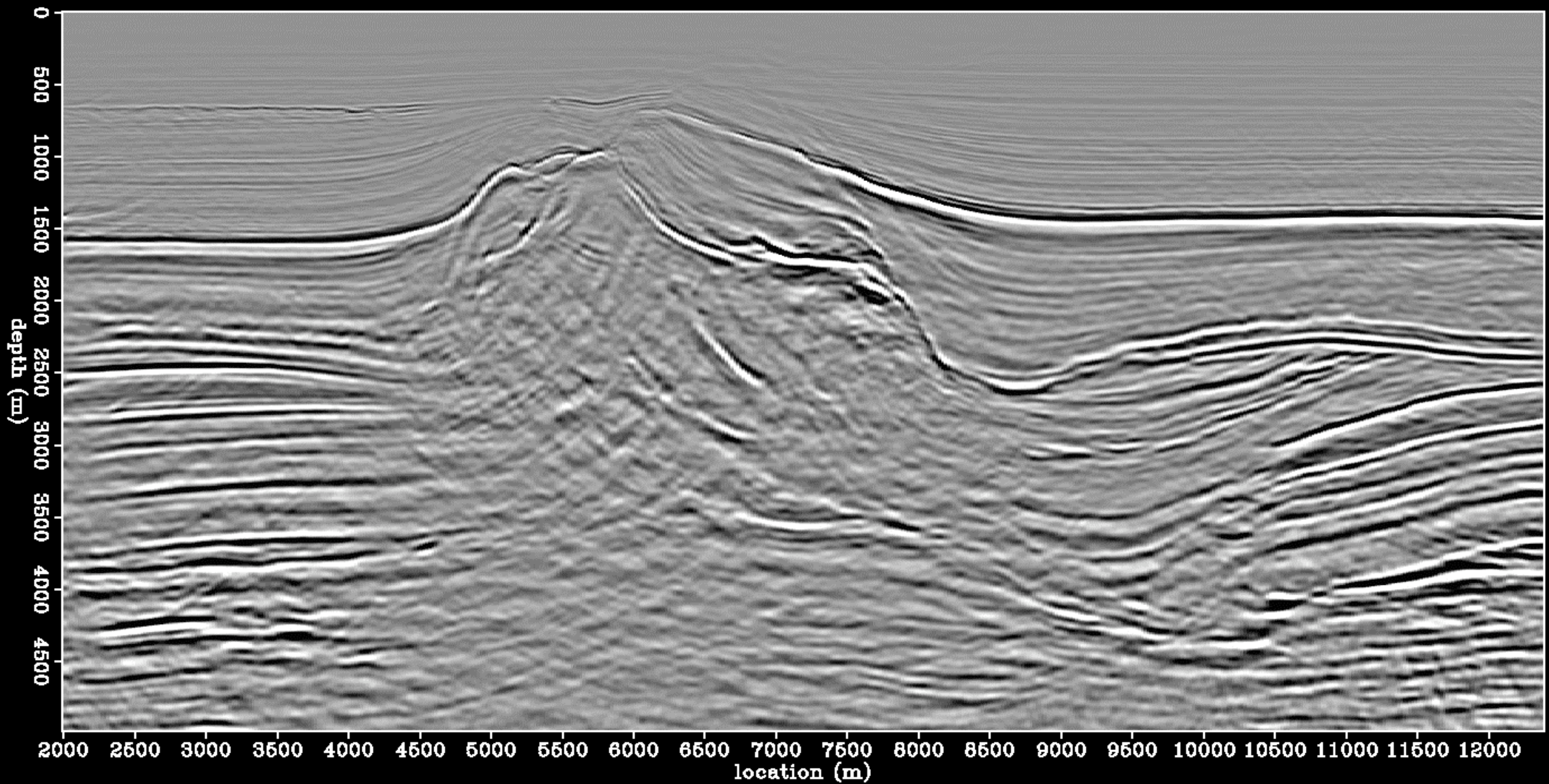


Residual migration* image



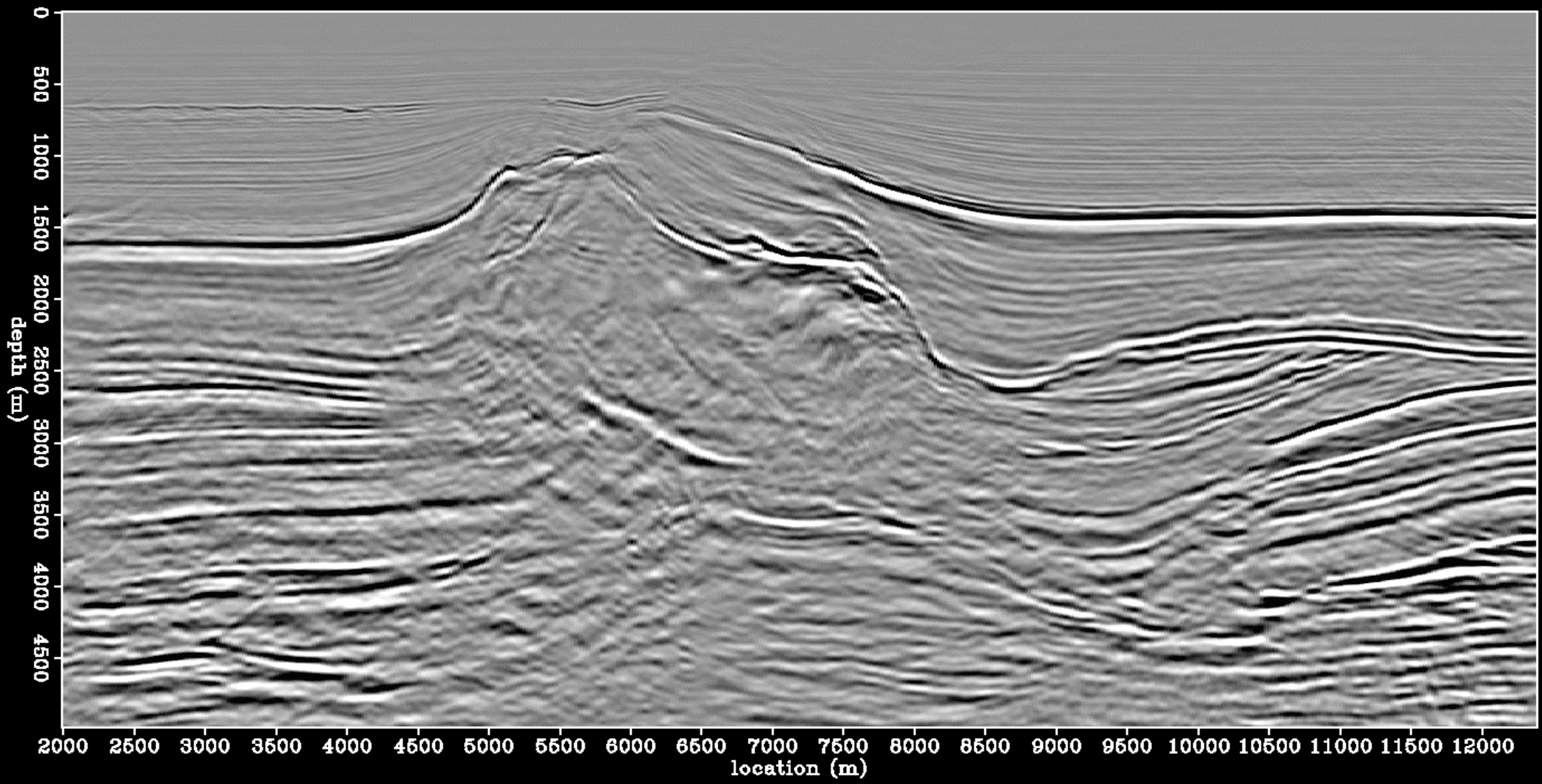
(*) Residual migration in the angle domain (SEP 134 - Appendix A)

Where is the salt flank?



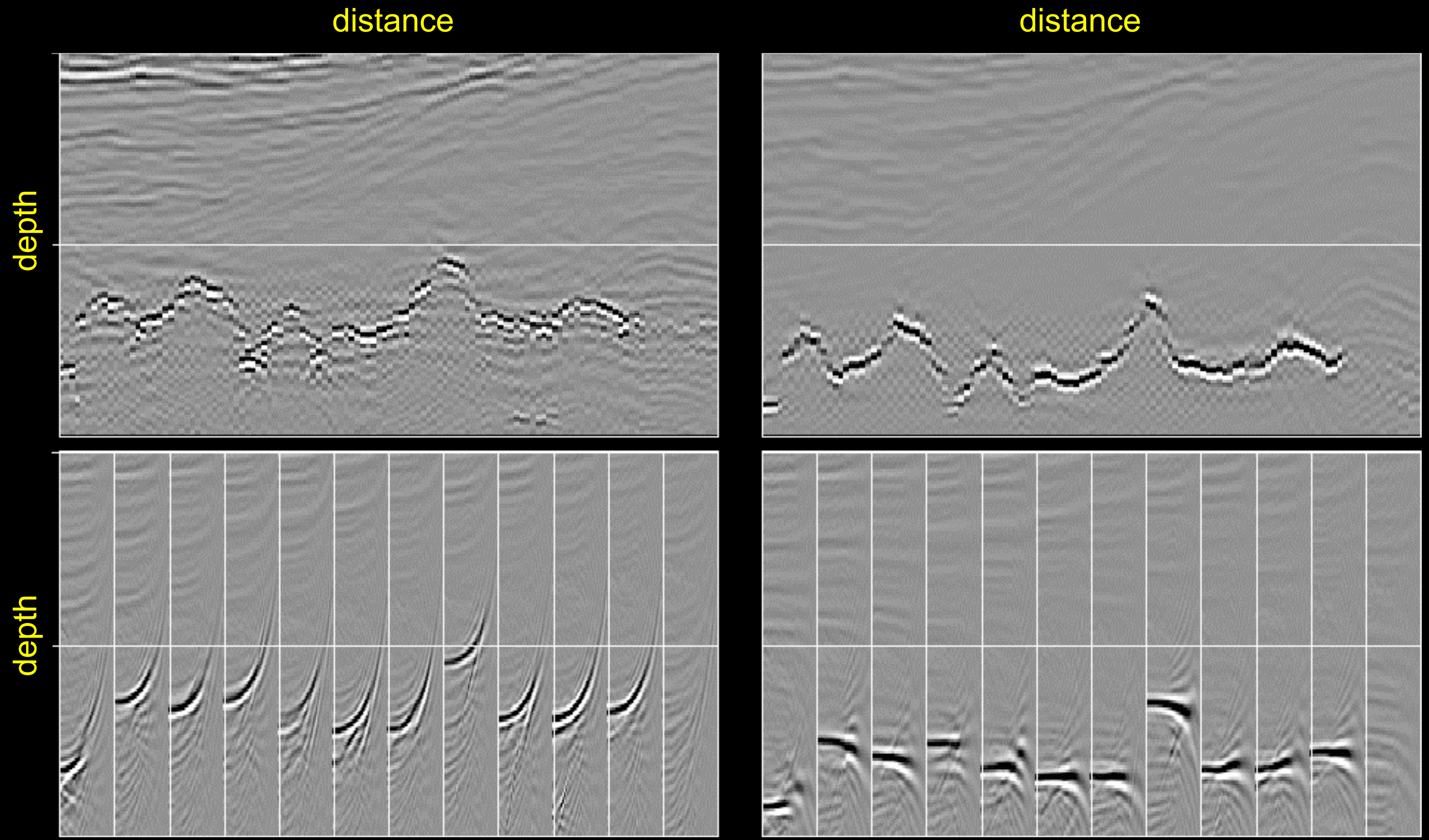
Courtesy of Paul Sava (SEP now CSM)

After prestack residual migration



Courtesy of Paul Sava (SEP now CSM)

WEMVA velocity updating using diffractions



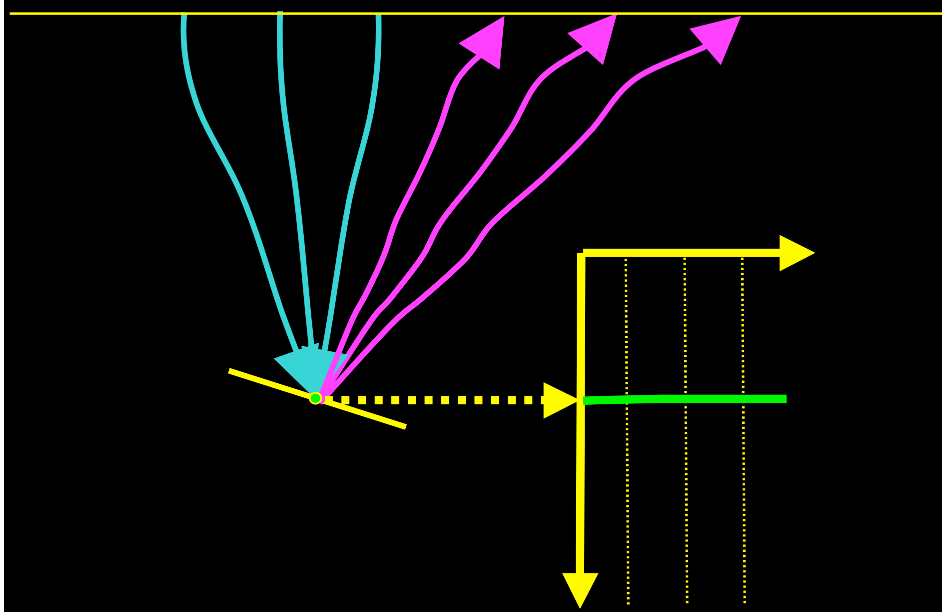
Courtesy of Paul Sava (SEP now CSM)

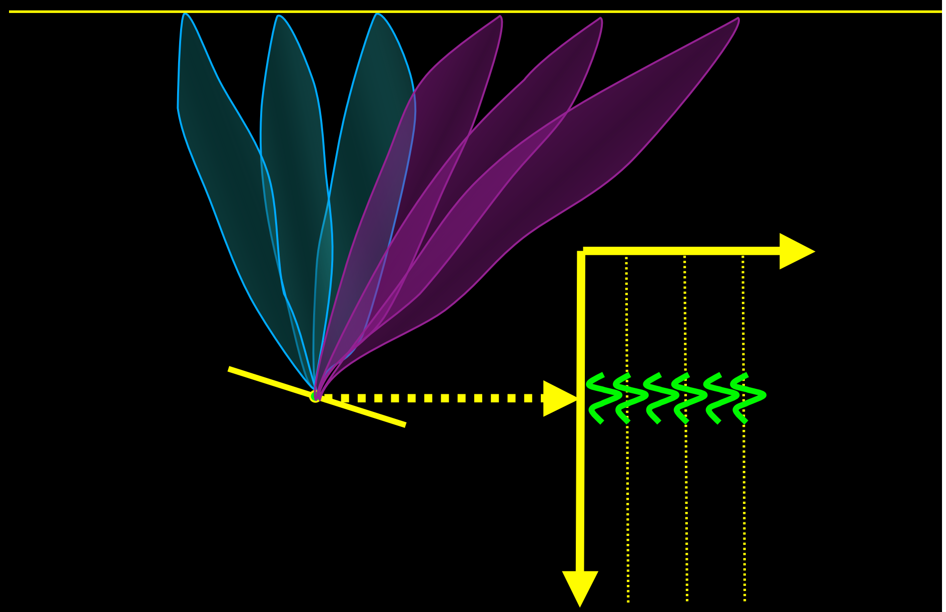
WEMVA vs. DSO for autofocusing migration

WEMVA:

- \+ Exploits the “autofocusing” power of residual migration for MVA.
- \+ Each iteration is cheaper by at least a factor of two (as explained in a moment).
- — Requires the picking of residual migration (or moveout) parameters ($\Delta\rho$).
- — Cannot use quasi-Newton optimization methods.

Ray tomography MVA ⇔ Wave-Equation MVA





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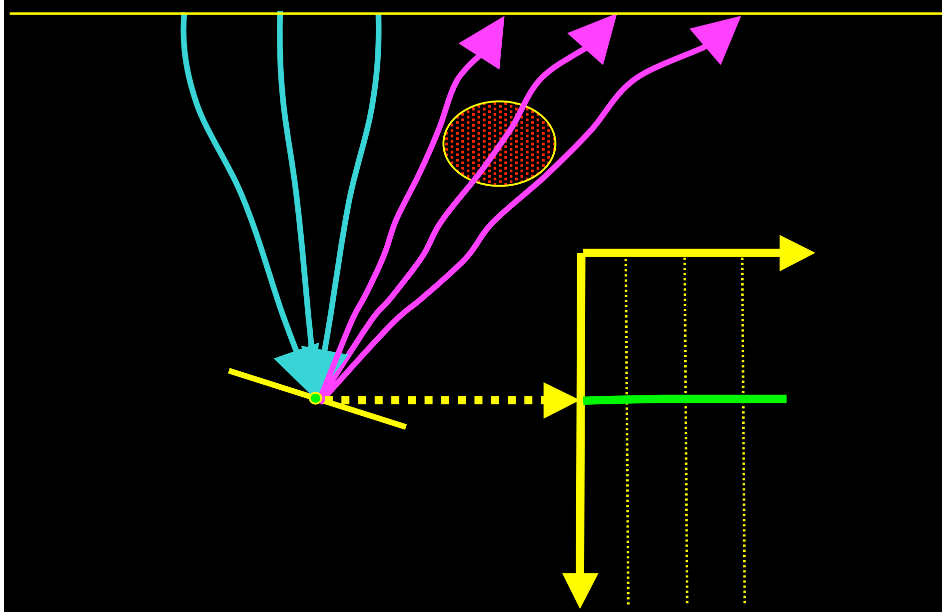
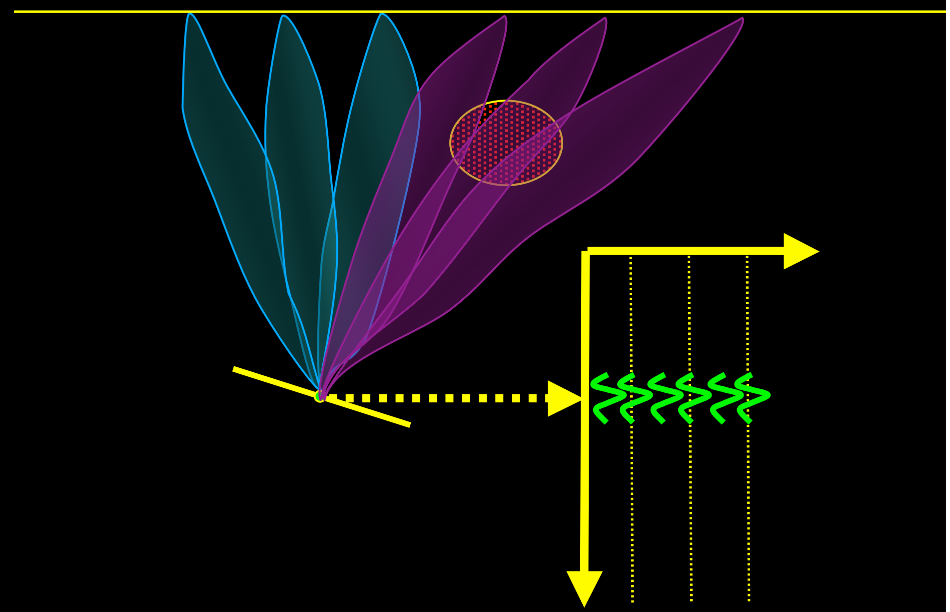
Ray tomography MVA ⇔ Wave-Equation MVA





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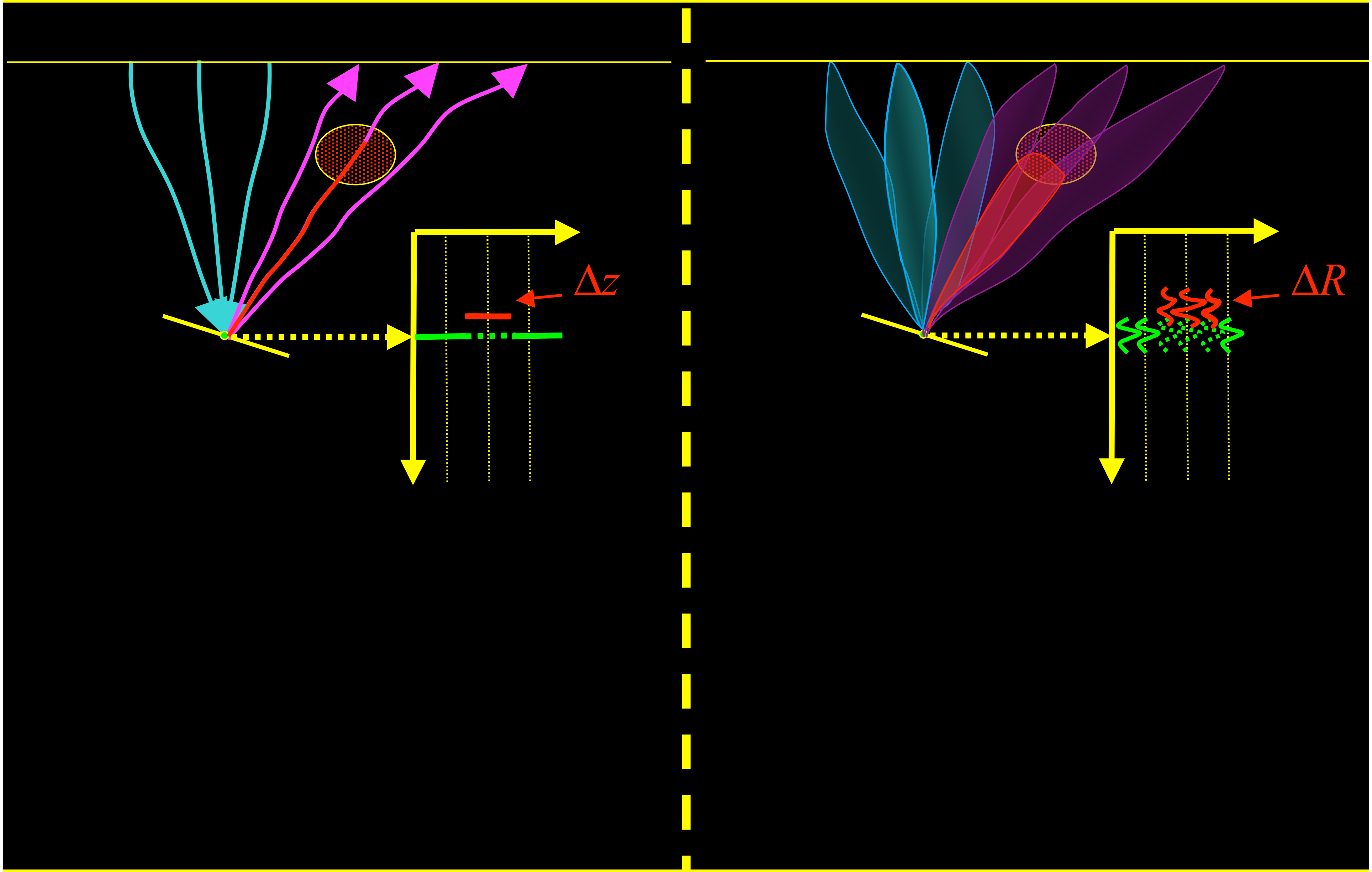
Ray tomography MVA ⇔ Wave-Equation MVA



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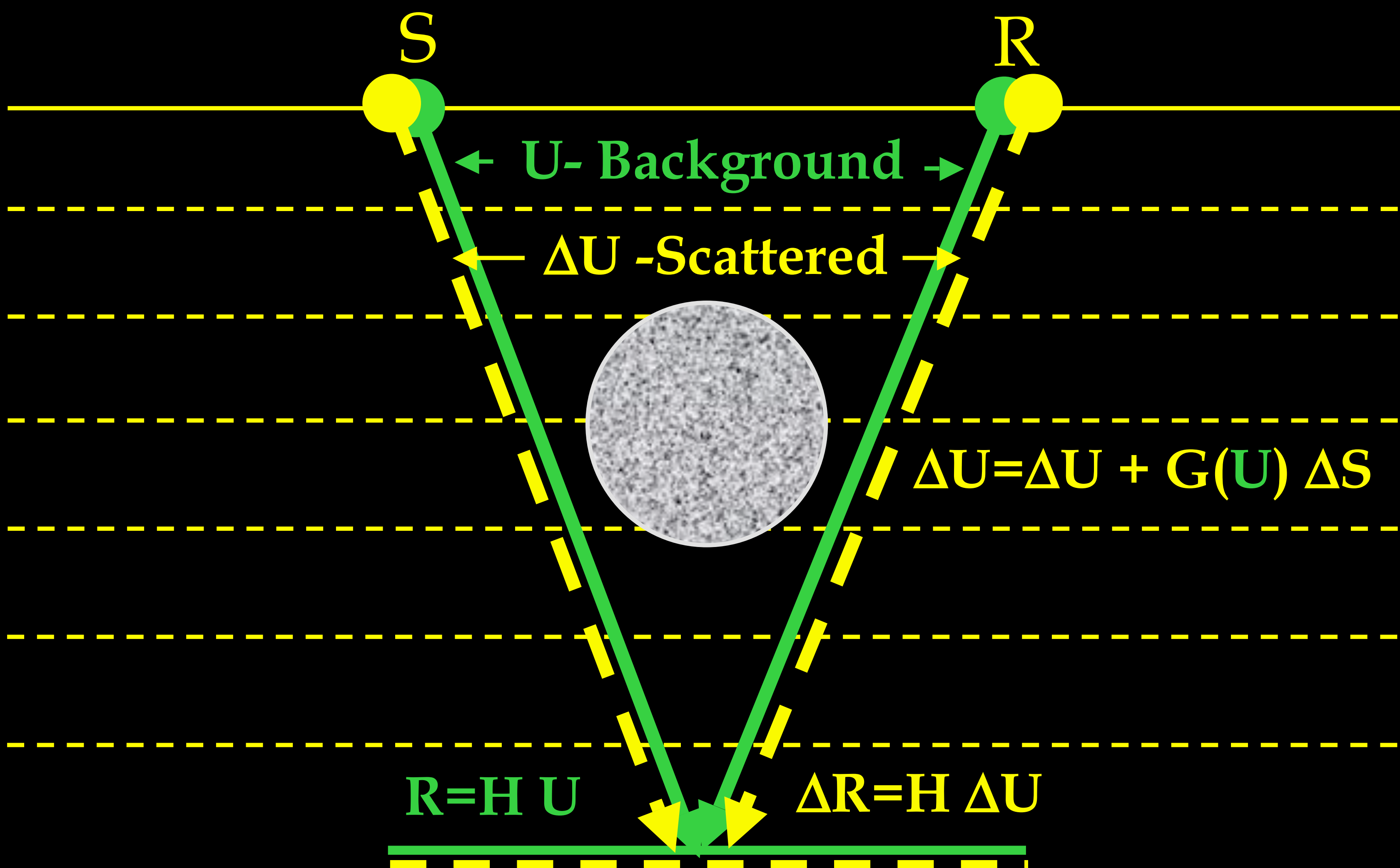
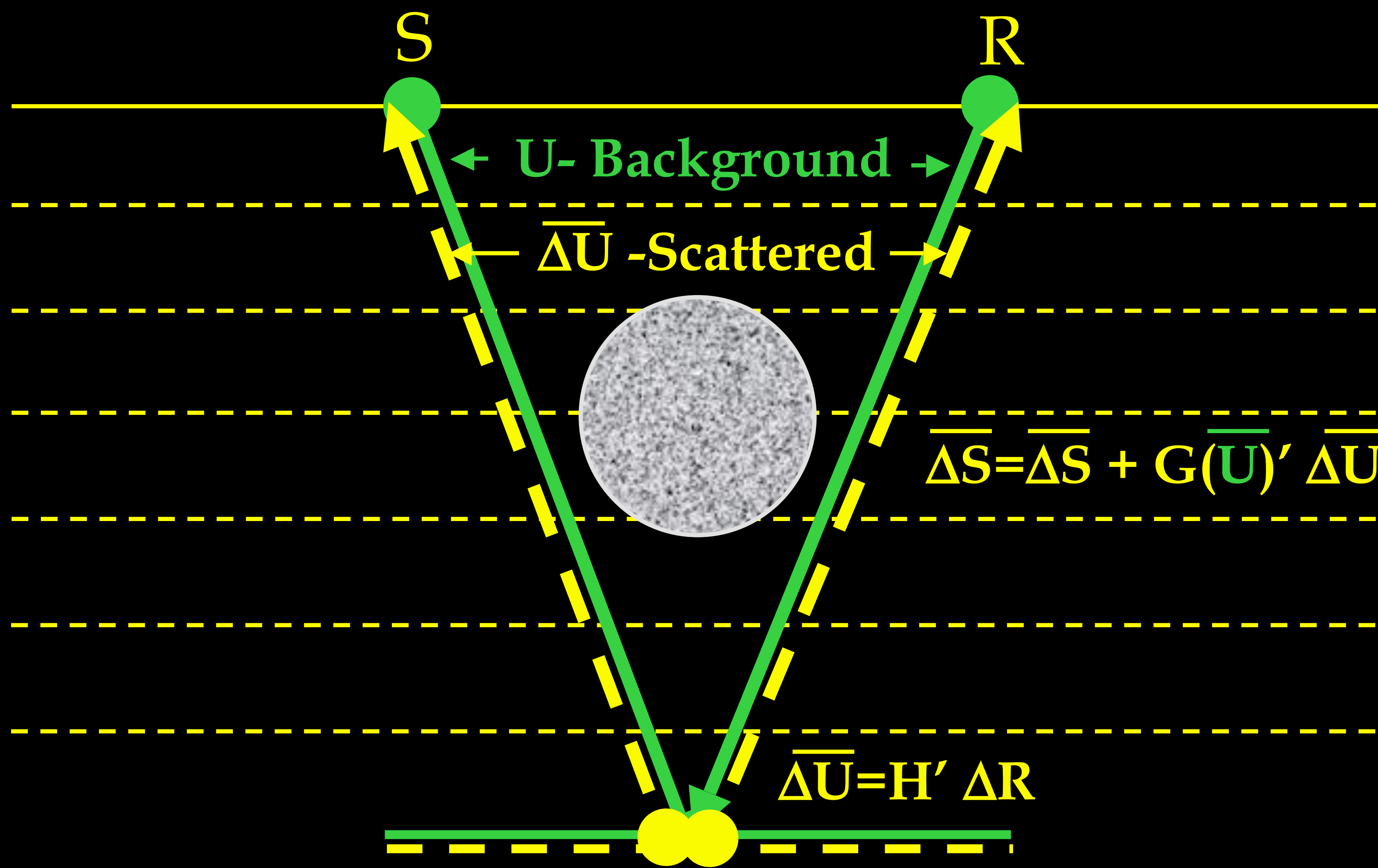
Linear operator L – Δs ⇒ΔR (forward)



Linear operator L′ – Δs ⇐ΔR (adjoint)



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Objective function for autofocusing migration

$$J_{\mathbf{s}} = \left\| \mathbf{R}(\mathbf{s}) - \mathbf{F}(\mathbf{R}(\mathbf{s})) \right\|_2^2$$

$$\min_{\mathbf{s}} J_{\mathbf{s}} \text{ subject to } \mathbf{R} = \text{PreMig}(\text{Data}; \mathbf{s})$$

where: **R** is the prestack image,

F is a "residual focusing" operator,

s is the slowness function.

Two well-known examples of residual focusing

DSO $\mathbf{F}(\mathbf{R}) = (\mathbf{I} - \mathbf{D})\mathbf{R}$

$$\Rightarrow J_{\mathbf{s}} = \left\| \mathbf{DR(s)} \right\|_2^2$$

where: **D** is the Differential Semblance (DSO) operator.

WEMVA $\mathbf{F}(\mathbf{R}) = (\mathbf{I} + \mathbf{K}[\Delta\rho])\mathbf{R}_0$

$$\Rightarrow J_{\mathbf{s}} = \left\| (\mathbf{R(s)} - \mathbf{R_0}) - \mathbf{K}[\Delta\rho]\mathbf{R_0} \right\|_2^2$$

where: **K** is a differential residual migration (or residual moveout) operator.

$\Delta\rho$ = gradient of auxiliary objective function $J_{\Delta\rho}$

$$J_{\Delta\rho} = \left\|\mathbf{DF}\left(\Delta\rho,\mathbf{R}\right)\right\|_2^2 =$$

$$\Downarrow$$

$$\widehat{\Delta\rho} = \mathbf{G}\nabla J_{\Delta\rho} = \mathbf{GM}'\left[\mathbf{R}\right]\mathbf{D}'\mathbf{DR}$$

$$\text{where: } \mathbf{M}\left[\mathbf{R}\right] = \frac{\partial\mathbf{K}\left[\Delta\rho\right]\mathbf{R}}{\partial\Delta\rho},$$

G is a smoother along depth.

A third option for autofocusing migration

AWEMVA

$$\mathbf{F}(\mathbf{R}) = \left(\mathbf{I} - \mathbf{K}\left[\widehat{\Delta\rho}\right]\right)\mathbf{R}$$

$$\Rightarrow J_{\mathrm{S}} = \left\|\mathbf{K}\left[\widehat{\Delta\rho}\right]\mathbf{R}\right\|_2^2$$

where: $\widehat{\Delta\rho} = \nabla J_{\Delta\rho}$ (previous slide).

Gradients and back-projections

DSO $-\nabla J_s = -\mathbf{L}'\left(\mathbf{D}'\mathbf{D}\mathbf{R}\right) = \mathbf{L}'\Delta\mathbf{R}_{\mathrm{DSO}}$

WEMVA $-\nabla J_s = \mathbf{L}'\left(\mathbf{K}[\Delta\rho]\mathbf{R}_0\right) = \mathbf{L}'\Delta\mathbf{R}_{\mathrm{WEMVA}}$

AWEMVA

$$-\nabla J_s = -\mathbf{L}'\left(\mathbf{K}'[\Delta\rho]\mathbf{K}[\Delta\rho] + \mathbf{D}'\mathbf{D}\mathbf{M}[\mathbf{R}]\mathbf{G}'\mathbf{M}'[\mathbf{R}]\mathbf{K}[\Delta\rho]\right)\mathbf{R} = \mathbf{L}'\Delta\mathbf{R}_{\mathrm{AWEMVA}}$$

where: **L** is the linearization of PreMig introduced at the beginning.

Gradients and back-projections

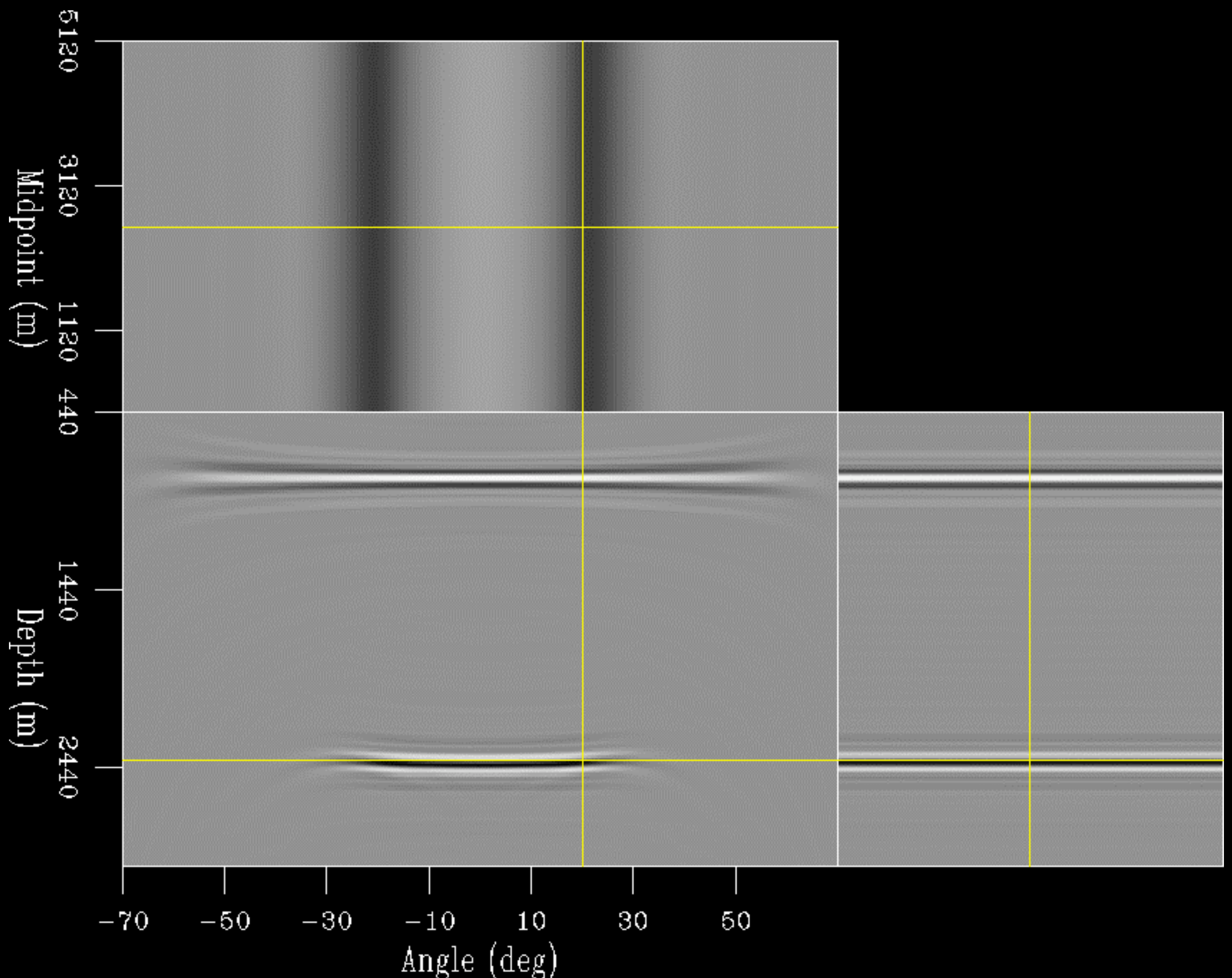
DSO $\quad -\nabla J_S = -\mathbf{L}'\left(\mathbf{D}'\mathbf{D}\mathbf{R}\right) = \mathbf{L}'\Delta\mathbf{R}_{\text{DSO}}$

WEMVA $\quad -\nabla J_S = \mathbf{L}'\left(\mathbf{K}[\Delta\rho]\mathbf{R}_0\right) = \mathbf{L}'\Delta\mathbf{R}_{\text{WEMVA}}$

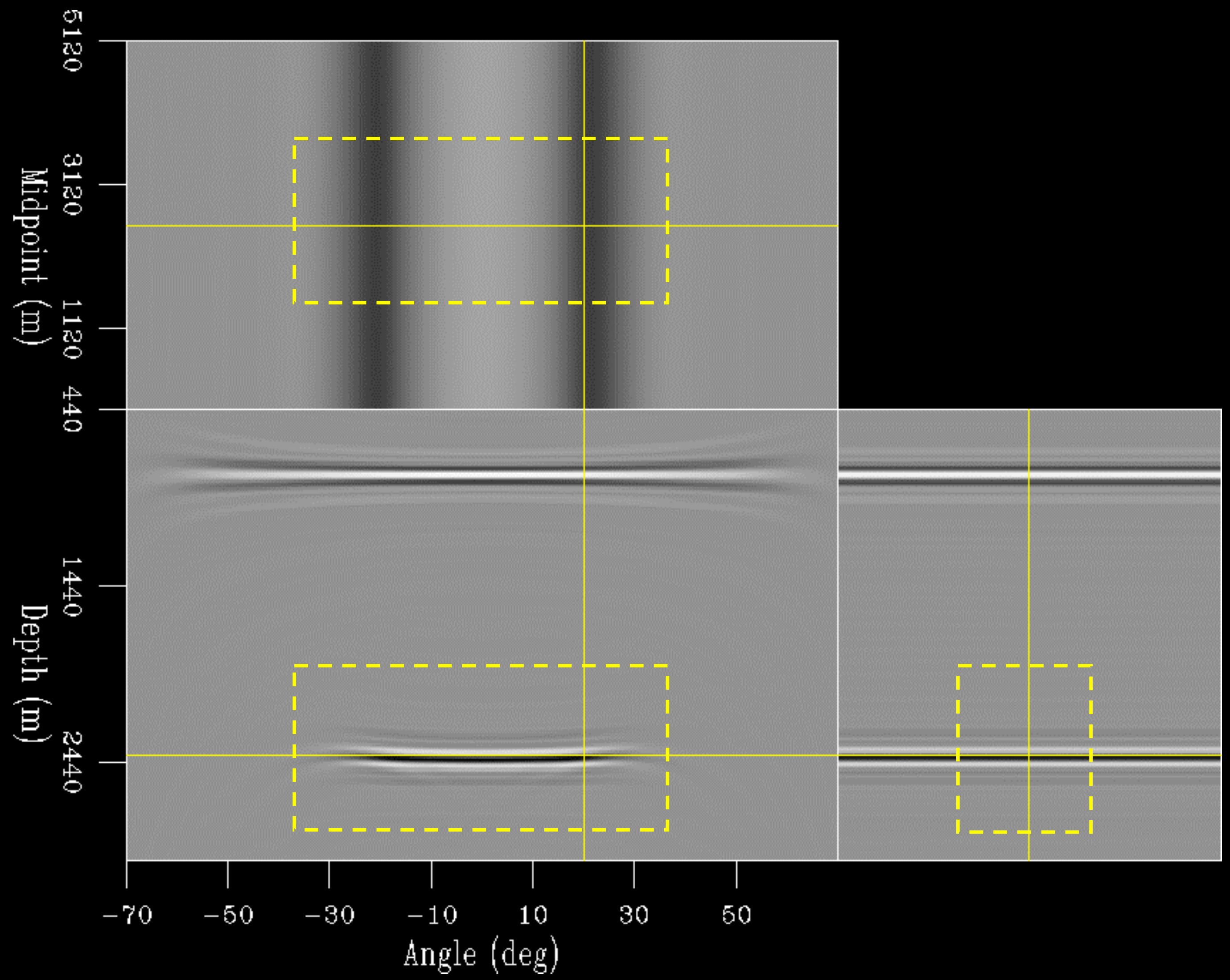
AWEMVA $\quad -\nabla J_S = -\mathbf{L}'\left(\mathbf{K}'[\Delta\rho]\mathbf{K}[\Delta\rho] + \mathbf{D}'\mathbf{D}\mathbf{M}[\mathbf{R}]\mathbf{G}'\mathbf{M}'[\mathbf{R}]\mathbf{K}[\Delta\rho]\right)\mathbf{R} = \mathbf{L}'\widetilde{\Delta\mathbf{R}}_{\text{AWEMVA}}$

where: **L** is the linearization of PreMig introduced at the beginning.

Simple synthetic data example



Simple synthetic data example



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Simple synthetic data example - Zoom

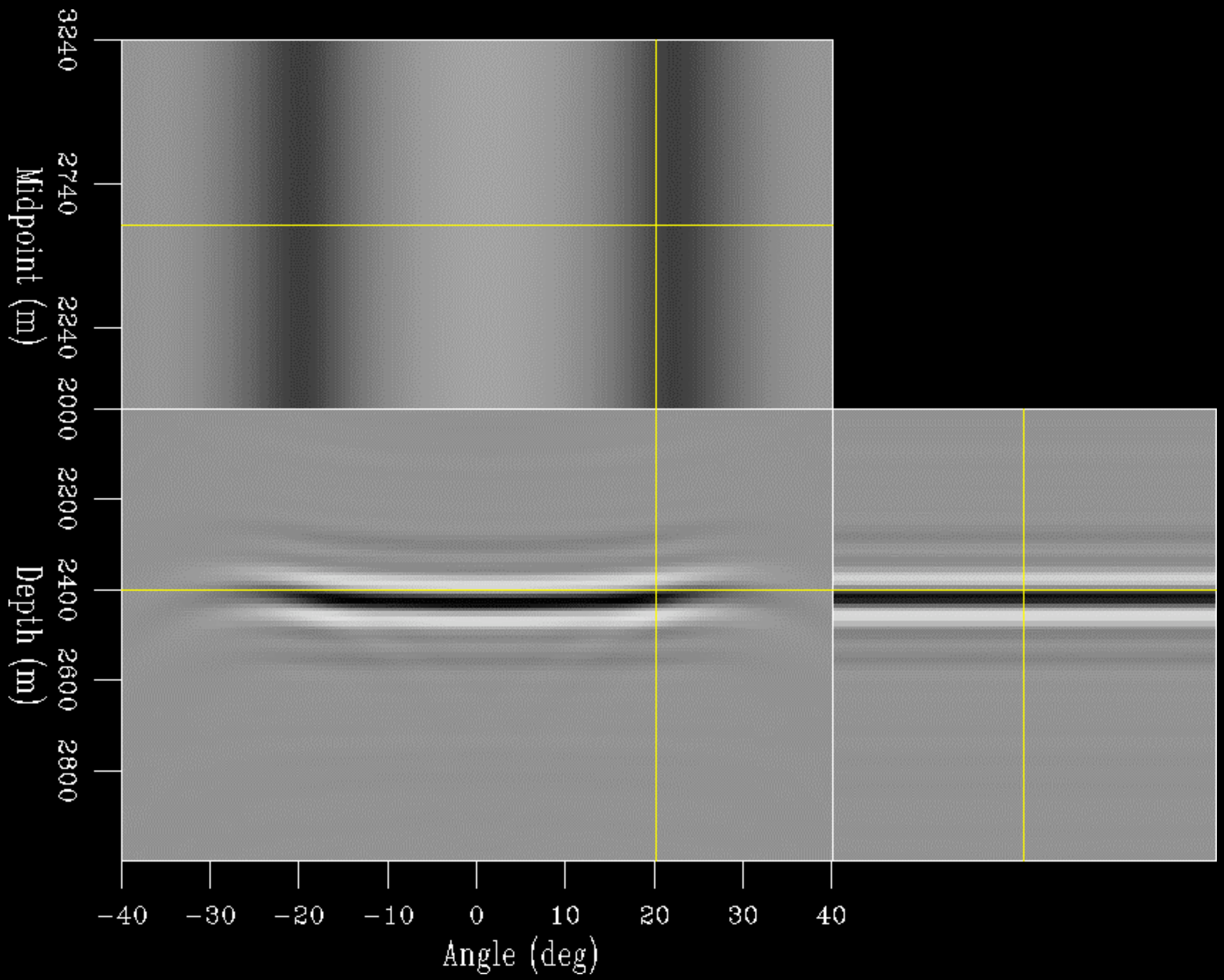

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Image perturbation by DSO - ΔR_{DSO}

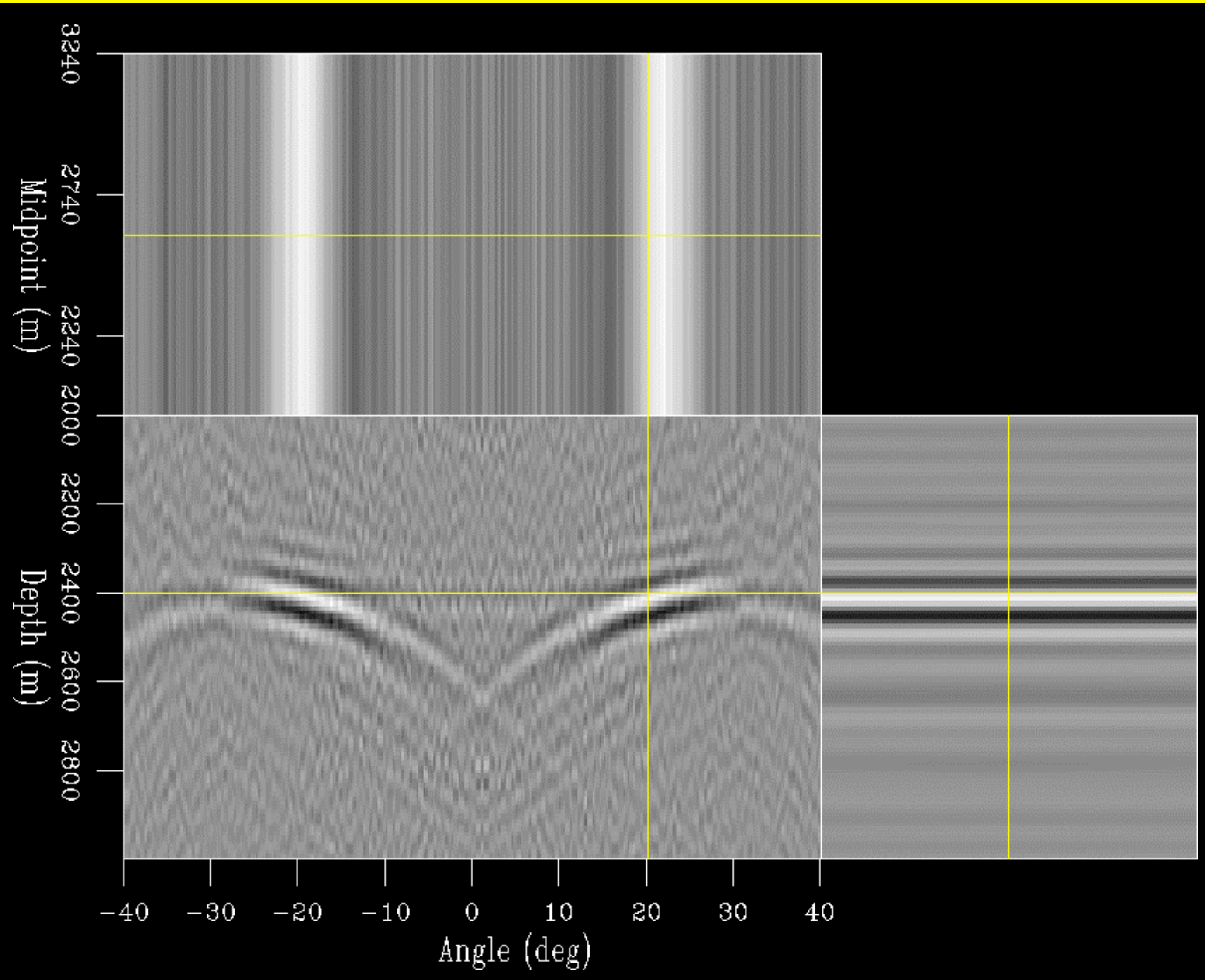

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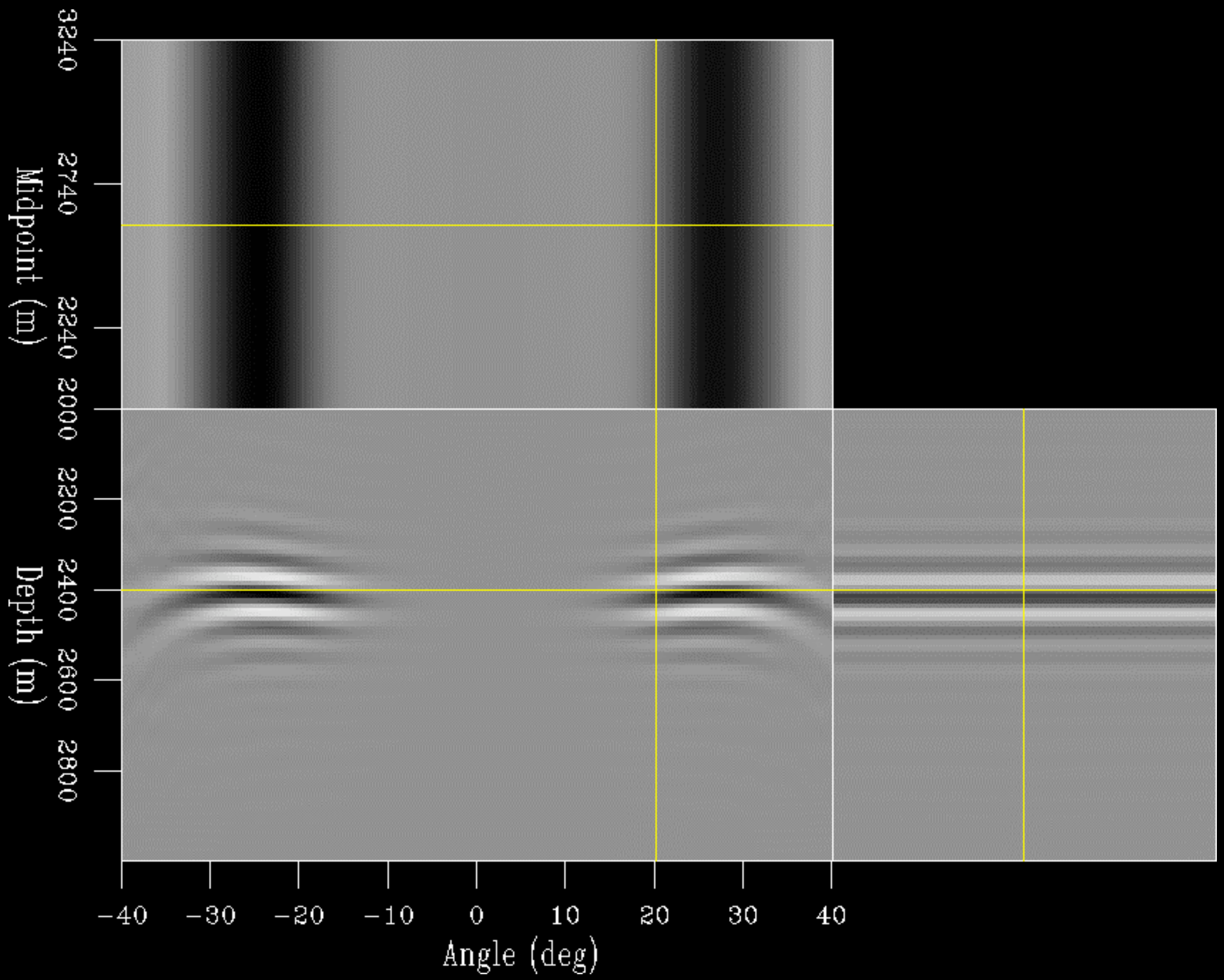
Image perturbation by RMO - $\Delta\widetilde{R}_{AWEMVA}$



Background image - R_0

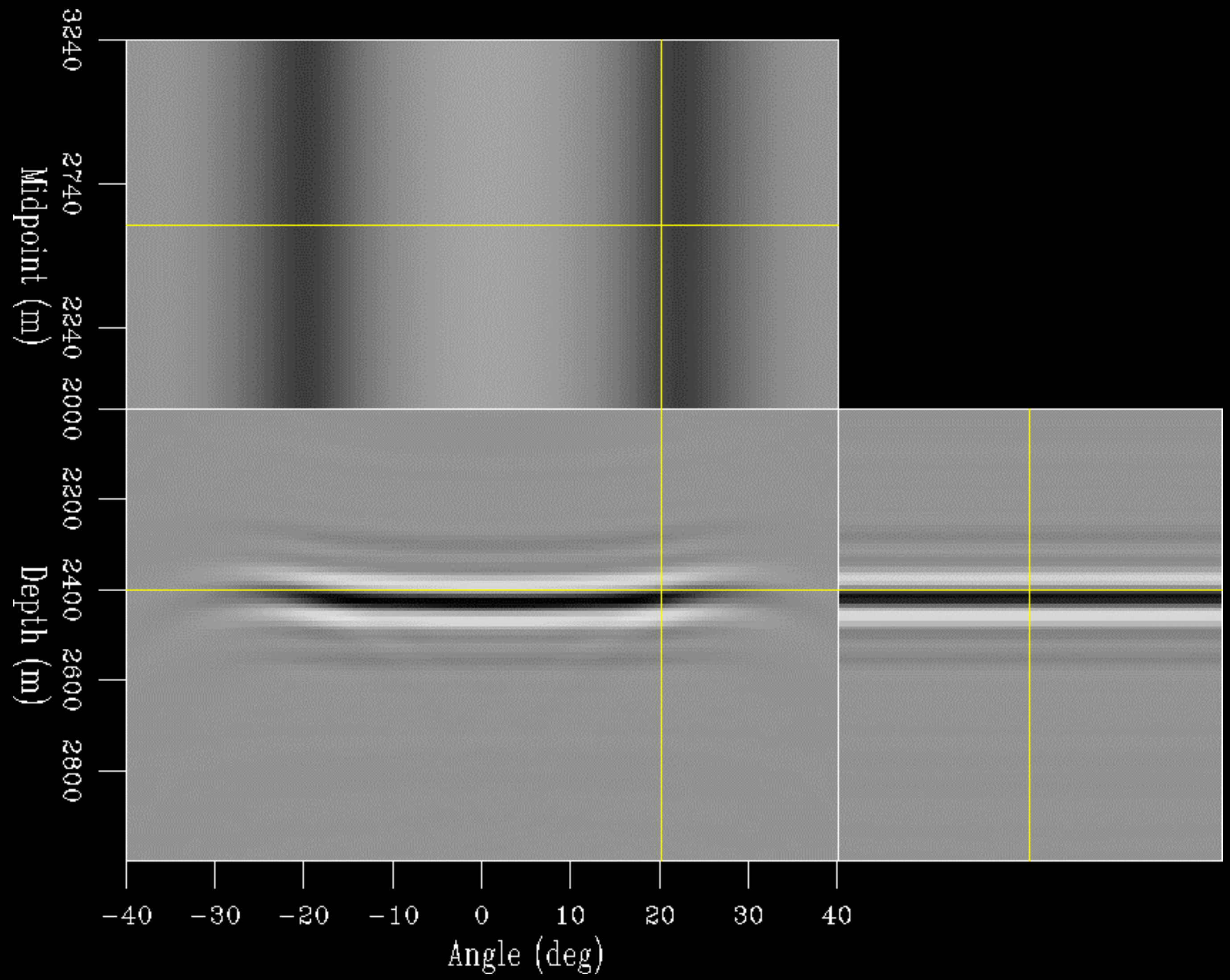


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Image perturbation by WEMVA - ΔR_{WEMVA}

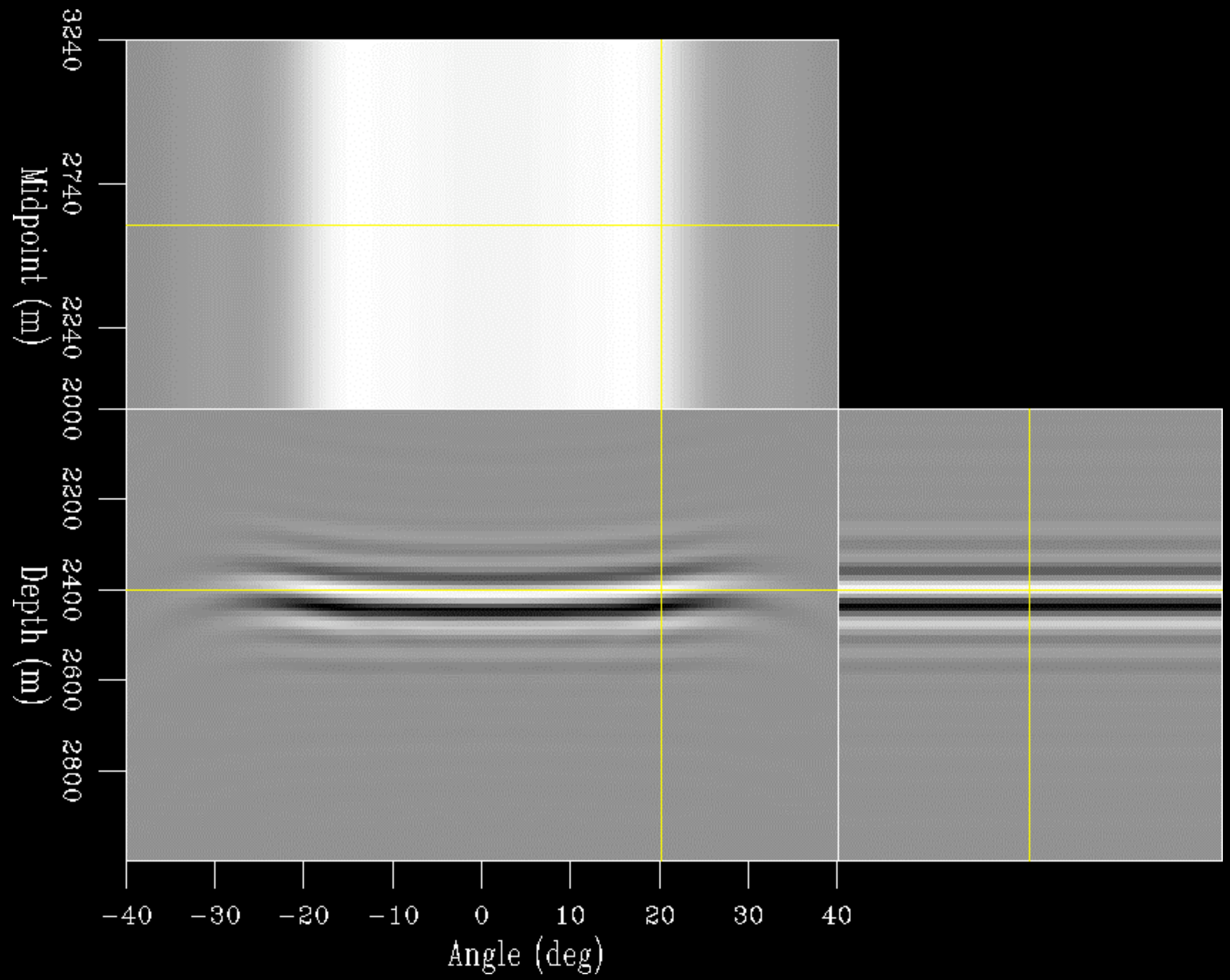

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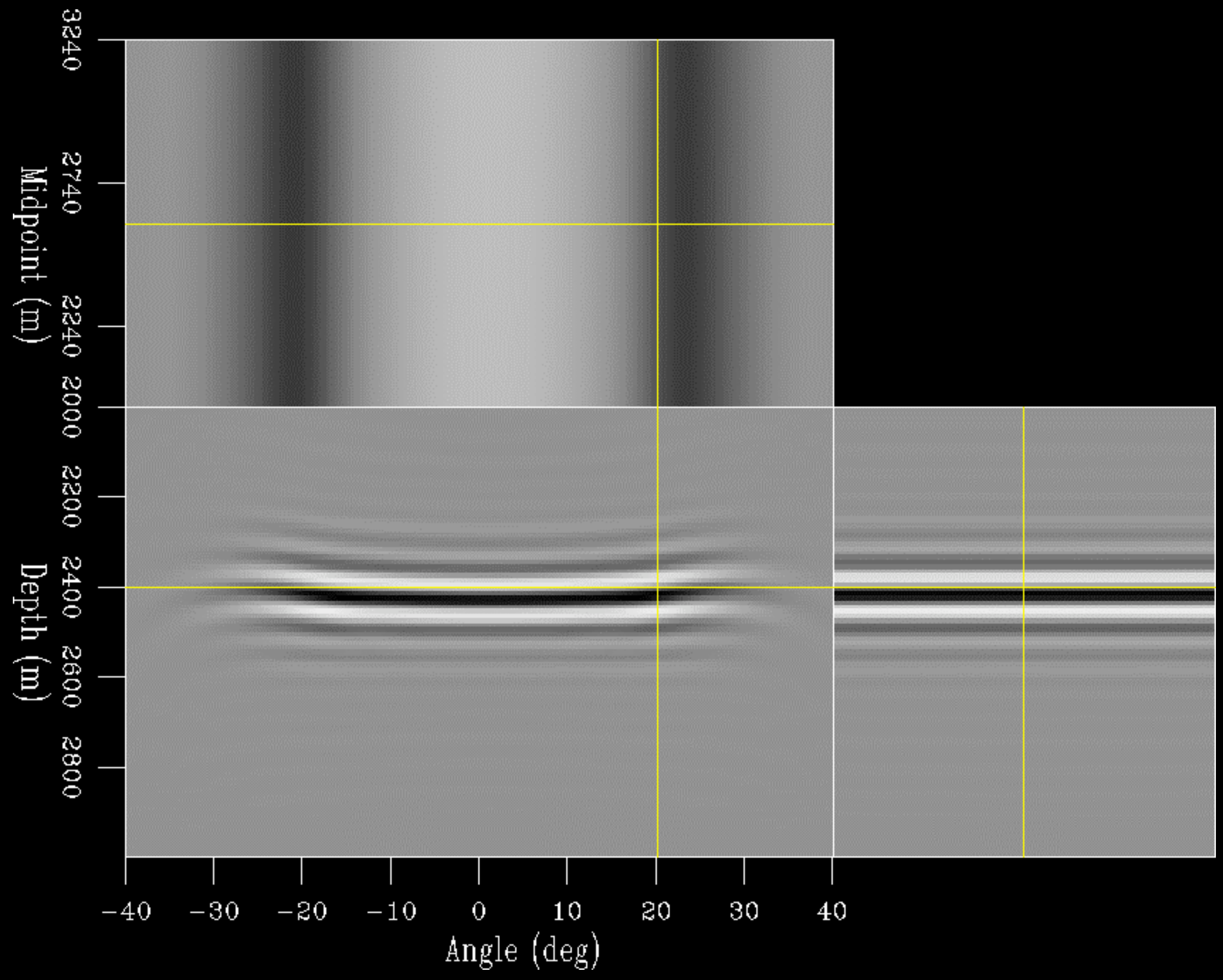
Image perturbation by AWEMVA - $\widetilde{\Delta R}_{AWEMVA}$



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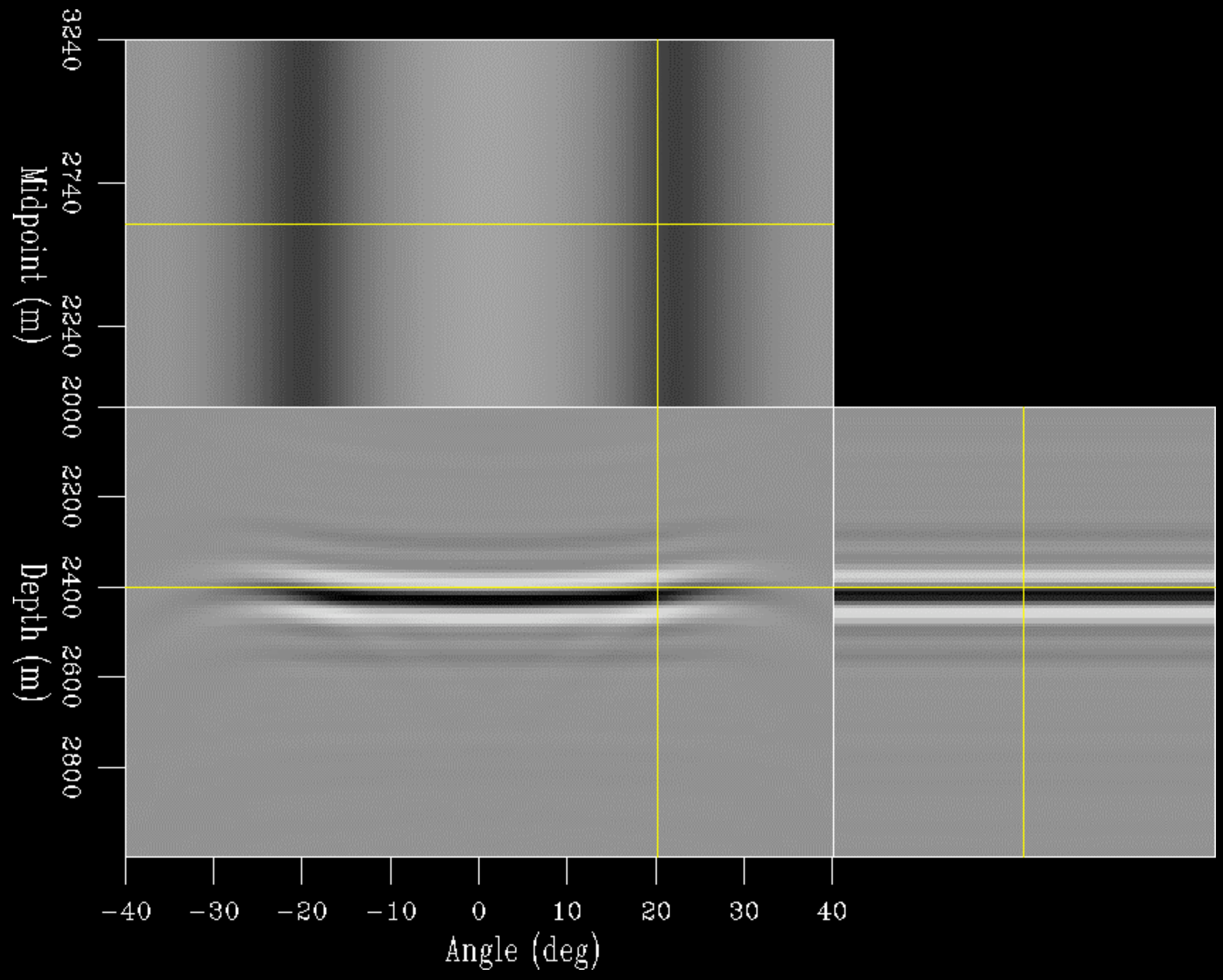
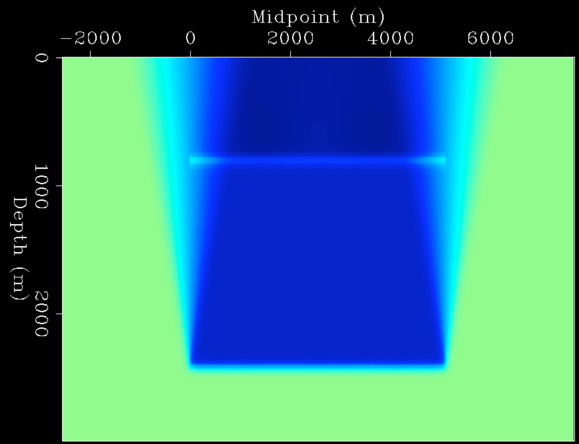
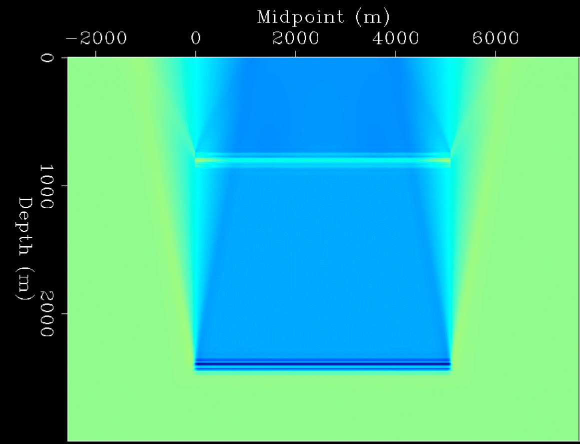
Background image - R_0



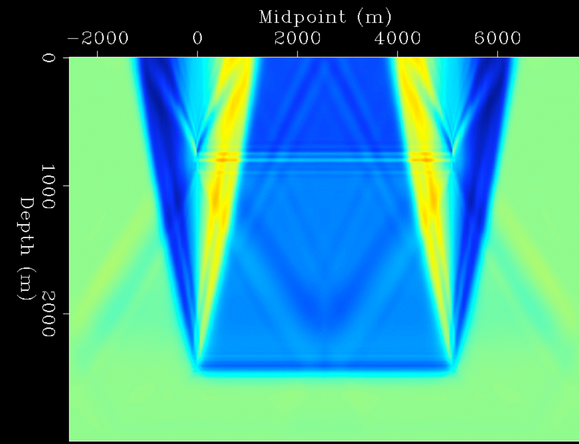
Backprojection of image perturbations $\Delta S = L'\Delta R$



$L'\Delta R_{WEMVA}$

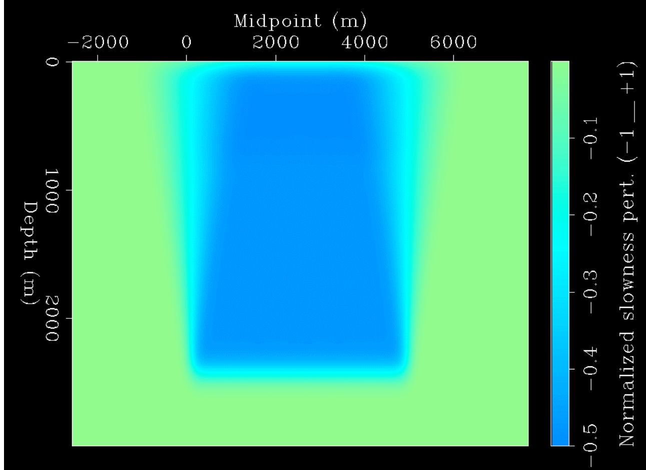


$L'\widetilde{\Delta R}_{AWEMVA}$

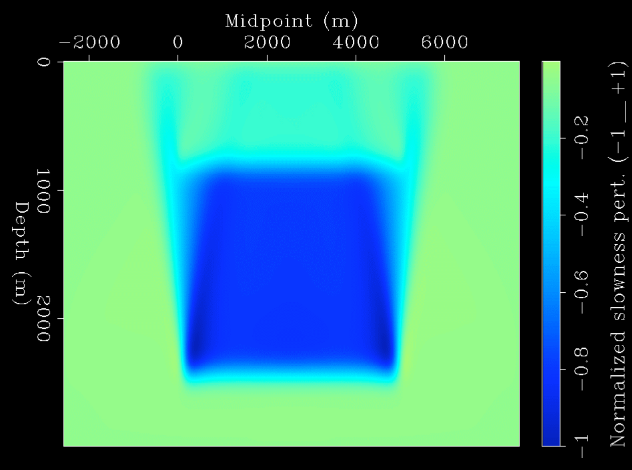


$L'\Delta R_{DSO}$

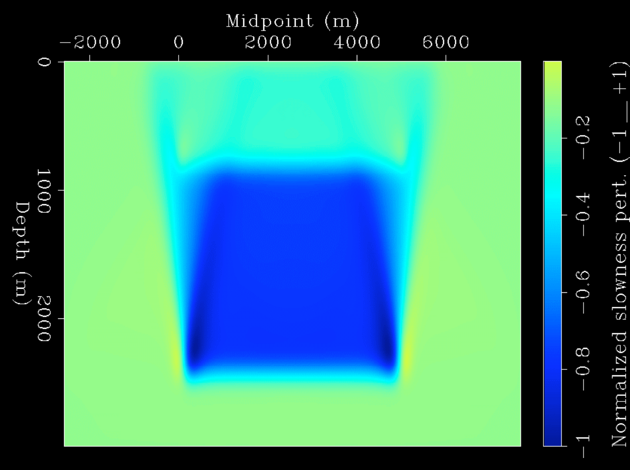
Regularized inversion of ΔR_{WEMVA}



Δs after 1 iter.



Δs after 3 iter.



Δs after 6 iter.

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WEMVA vs. DSO for autofocusing migration

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Discussions and future directions

- A close form solution for $\Delta\rho$ is an important step forward toward autofocusing migration.
- Preliminary numerical experiments are encouraging but inconclusive.
- Relative advantages of DSO, WEMVA and AWEMVA must to be assessed on relevant MVA problems.

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