Target-oriented wave-equation inversion with regularization in the prestack-image domain

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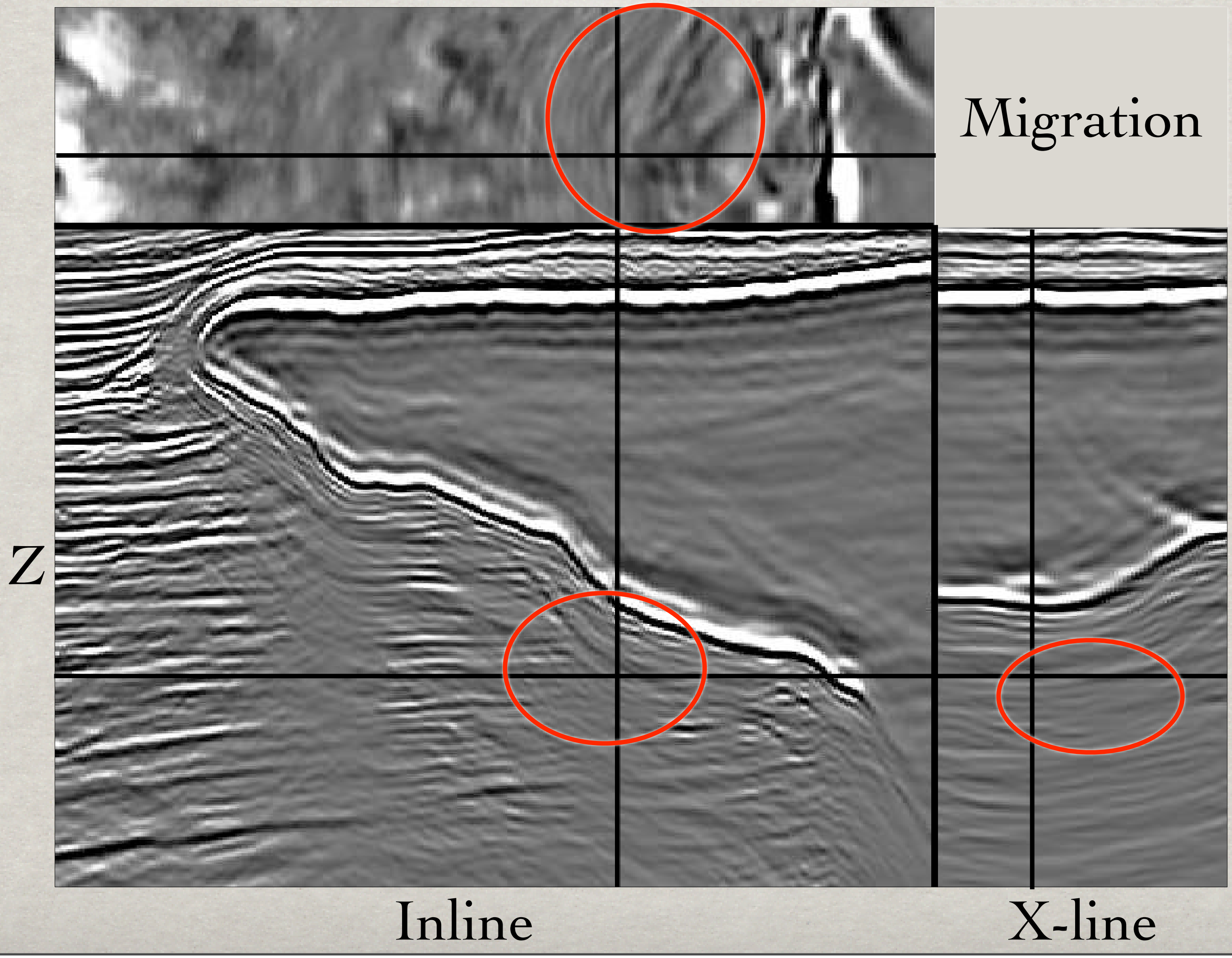
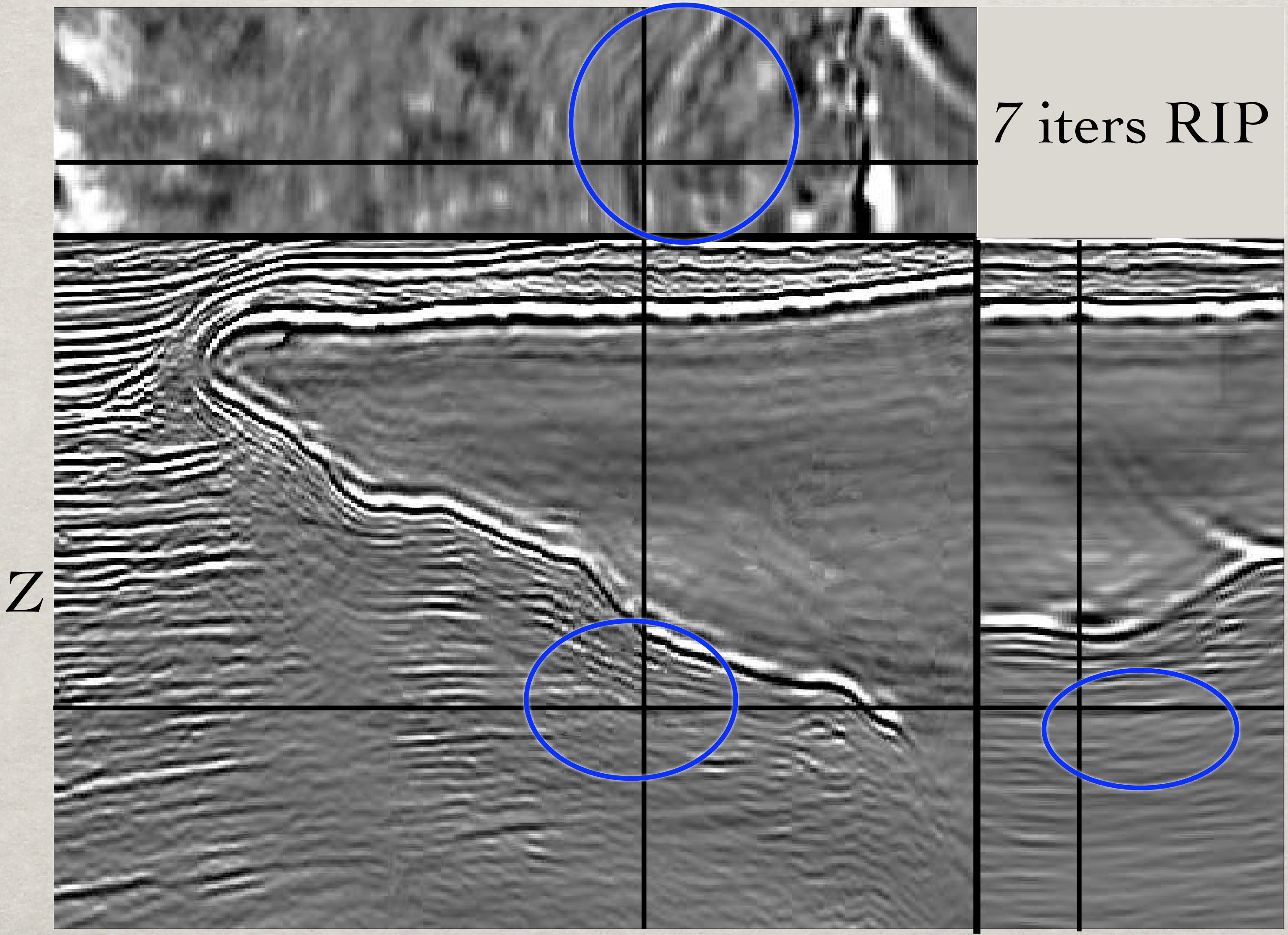
SEP123 pp 75-82, SEP124 pp 85-94

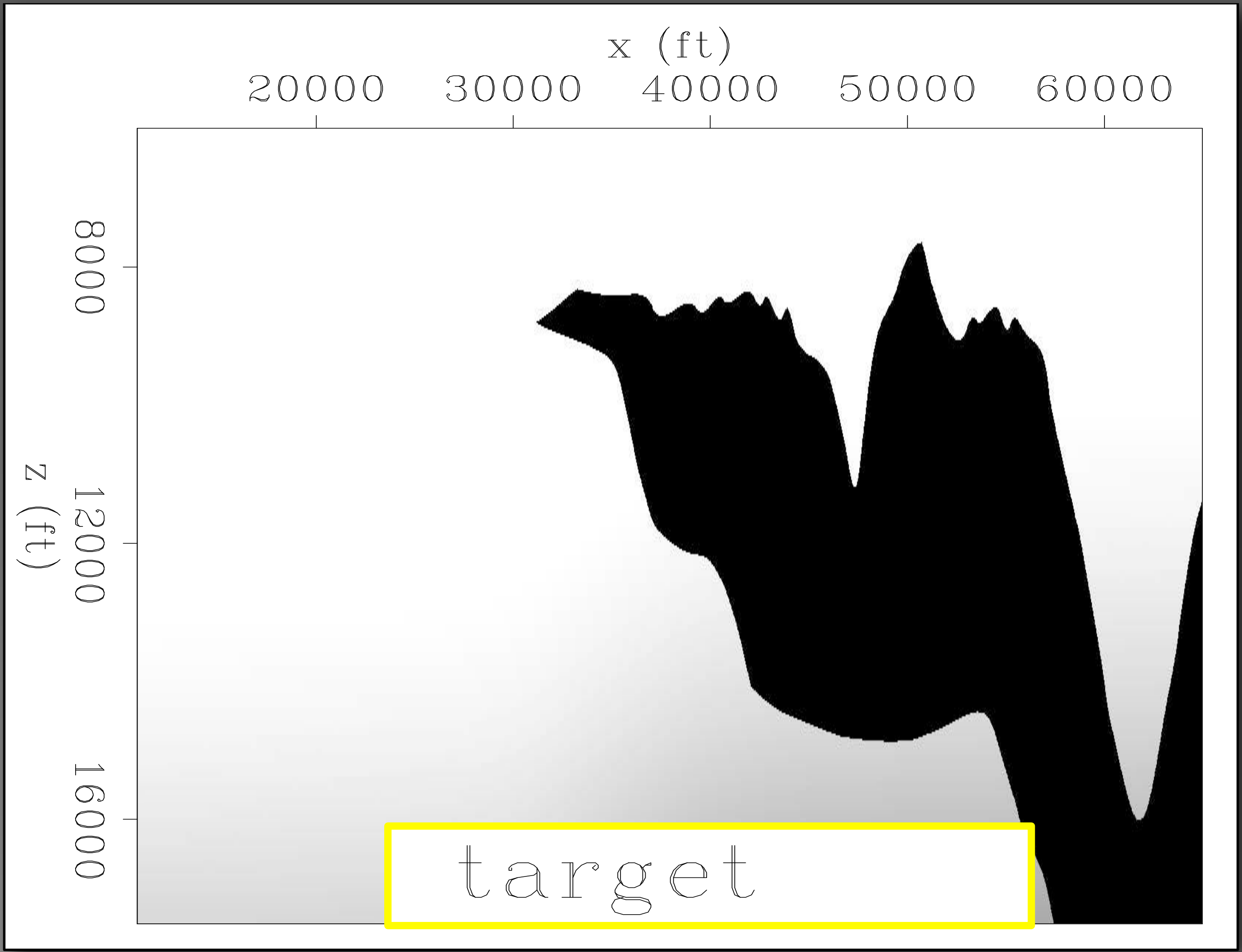
3-D Results After Stacking



3-D Results After Stacking



Target oriented inversion



4

Outline

- **Least-squares imaging** (explicit Hessian)

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- **Prestack image-domain Hessian**

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- Hessian construction cost

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- Inversion with regularization in the subsurface-offset domain: Sigsbee model

migration ≠ inversion

Non-unitary seismic imaging operators:

6

migration ≠ inversion

Non-unitary seismic imaging operators:

- **Irregular acquisition geometry**

migration ≠ inversion

Non-unitary seismic imaging operators:

- **Irregular acquisition geometry**
- **Complex velocity model**

migration ≠ inversion

Non-unitary seismic imaging operators:

- **Irregular acquisition geometry**
- **Complex velocity model**
- **Bandlimited seismic data**

Least-squares imaging

$$\mathbf{d} = \mathbf{L}\mathbf{m}$$

Least-squares imaging

$$\mathbf{d} = \mathbf{L}\mathbf{m}$$

$$\hat{\mathbf{m}} = (\mathbf{L}'\mathbf{L})^{-1}\mathbf{L}'\mathbf{d}_{obs}$$

7

Least-squares imaging

$$\mathbf{d} = \mathbf{L}\mathbf{m}$$

$$\hat{\mathbf{m}} = (\mathbf{L}'\mathbf{L})^{-1}\mathbf{L}'\mathbf{d}_{obs}$$

$$\hat{\mathbf{m}} = \mathbf{H}^{-1}\mathbf{m}_{mig}$$

7

Hessian approximations

Cross correlation imaging condition

$$\mathbf{H}(\mathbf{x}, \mathbf{y}) = \mathbf{I}$$

Hessian approximations

Cross correlation imaging condition

$$\mathbf{H}(\mathbf{x},\mathbf{y}) = \mathbf{I}$$

Division by shot illumination imaging condition

$$diag[\mathbf{H}(\mathbf{x},\mathbf{y})] = \sum_{\omega}\sum_{\mathbf{x}_s} \mathbf{G}'(\mathbf{x},\mathbf{x}_s;\omega)\mathbf{G}(\mathbf{y},\mathbf{x}_s;\omega)$$

Previous Hessian approximations

- Hu et al. (2001), horizontally invariant deconvolution operator

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- Rickett (2003), weighting functions from reference images

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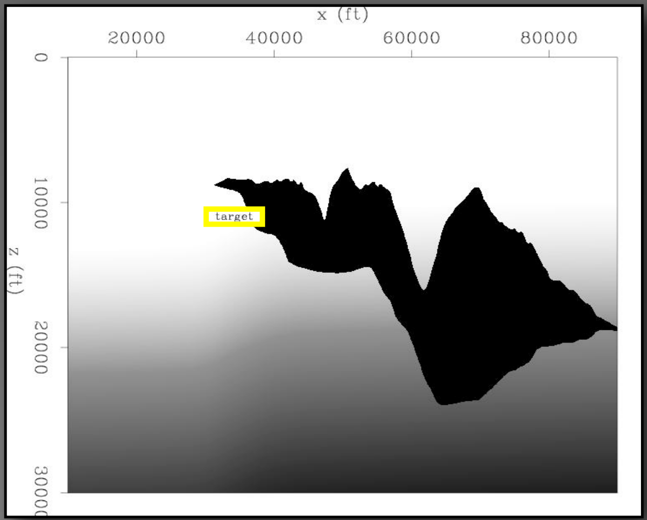
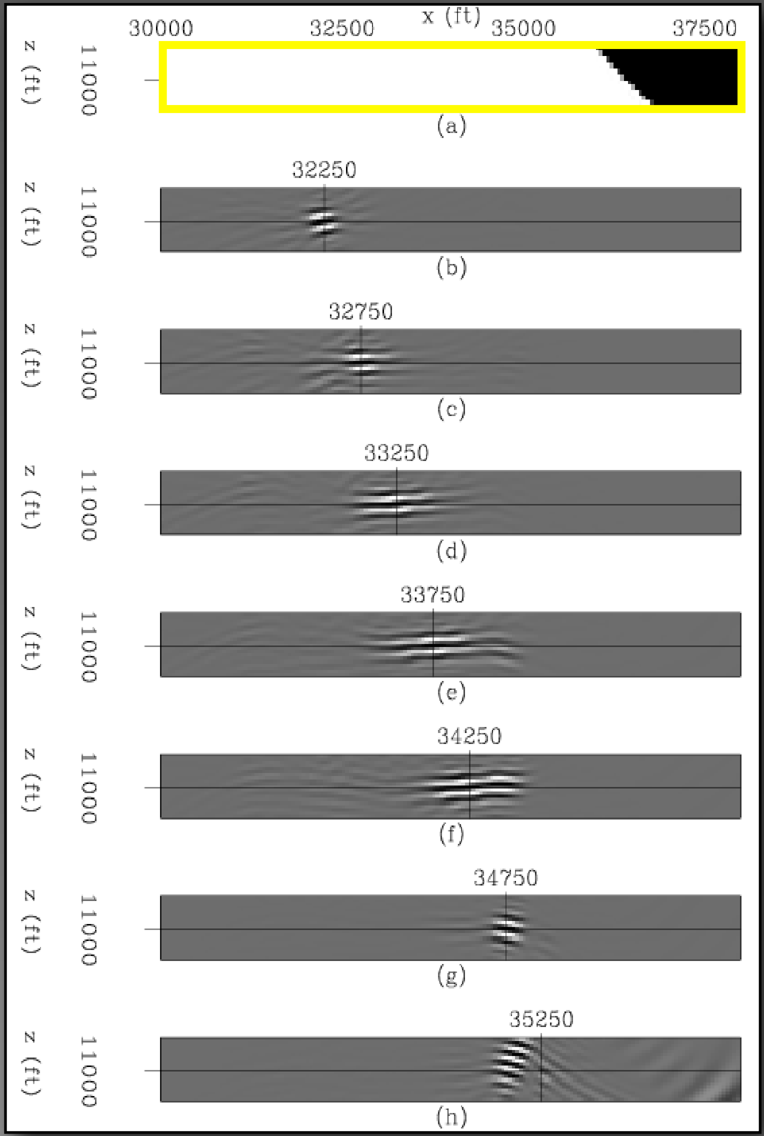
- Hu et al. (2001), horizontally invariant deconvolution operator
- Rickett (2003), weighting functions from reference images
- Guitton (2004), bank of matching filters

Row of the Hessian matrix





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- Least-squares imaging (explicit Hessian)
- Prestack image-domain Hessian
- Hessian construction cost
- Inversion with regularization in the subsurface-offset domain: Sigsbee model

Why prestack image domain?

Wave-equation inversion regularization schemes

12

Why prestack image domain?

Wave-equation inversion regularization schemes

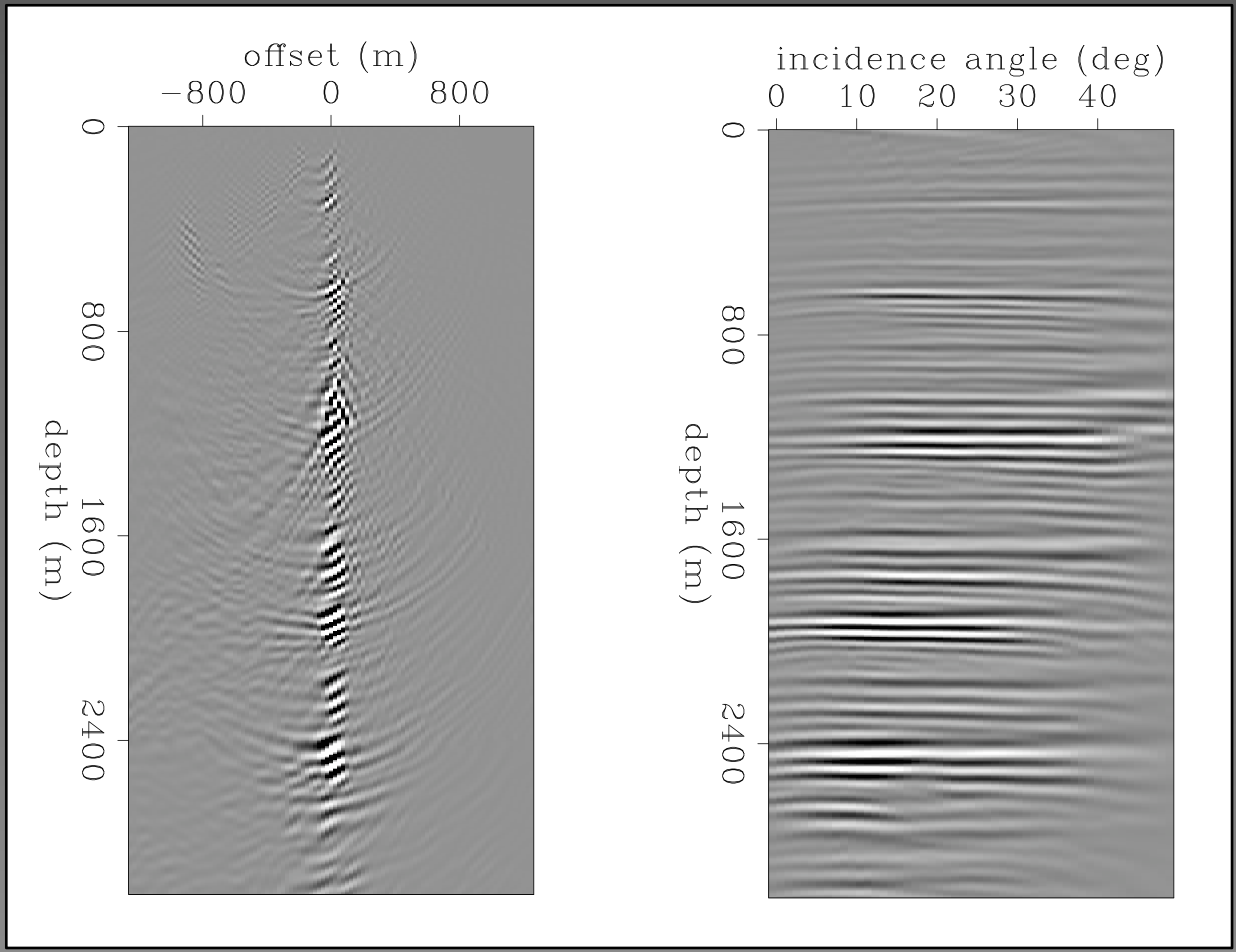
- **Clapp (2005), and Kuhl & Sacchi (2003), regularization in the offset-ray parameter**

Why prestack image domain?

Wave-equation inversion regularization schemes

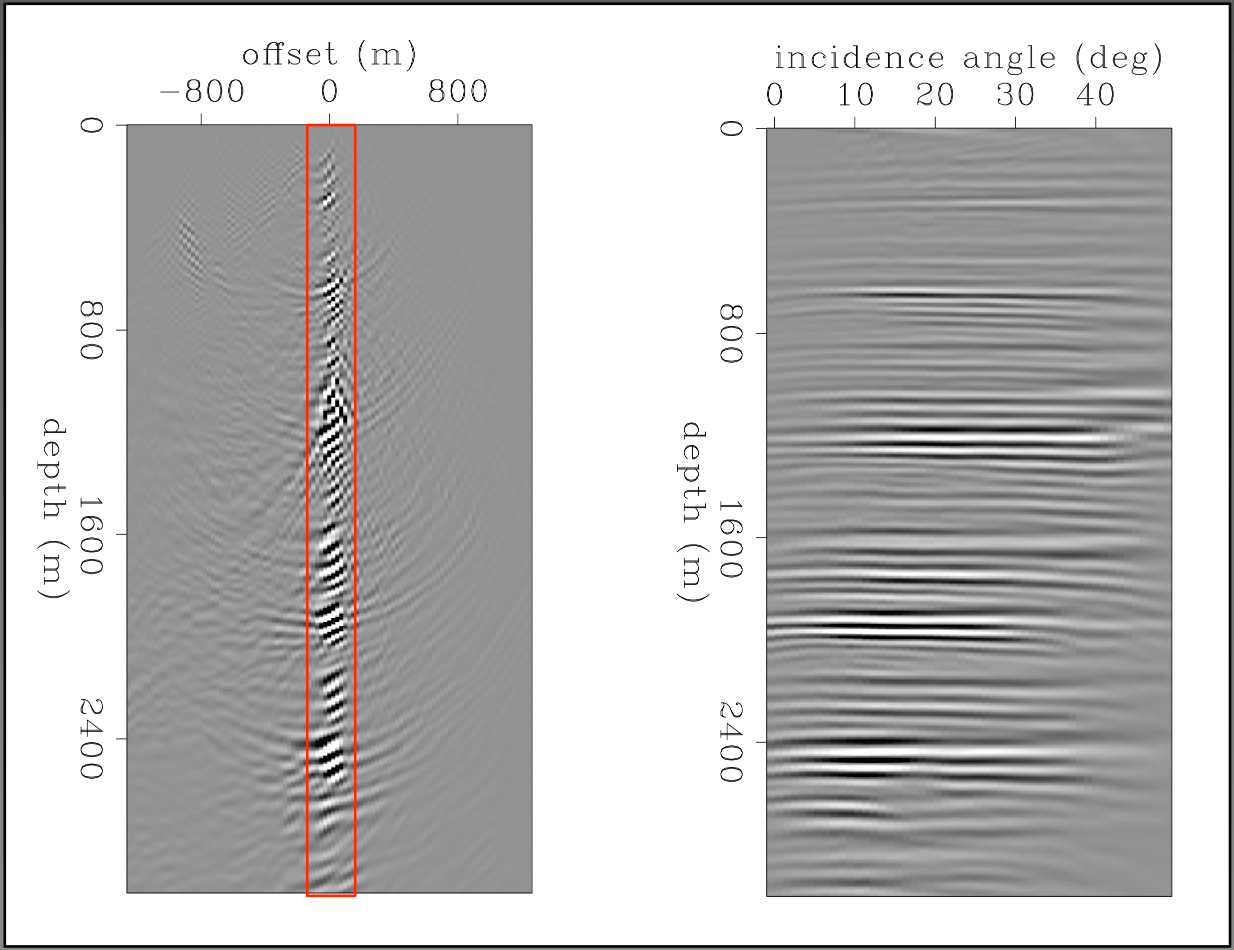
- Clapp (2005), and Kuhl & Sacchi (2003), regularization in the offset-ray parameter
- Shen et al. (2003), subsurface-offset differential semblance operators

Subsurface-offset vs. angle



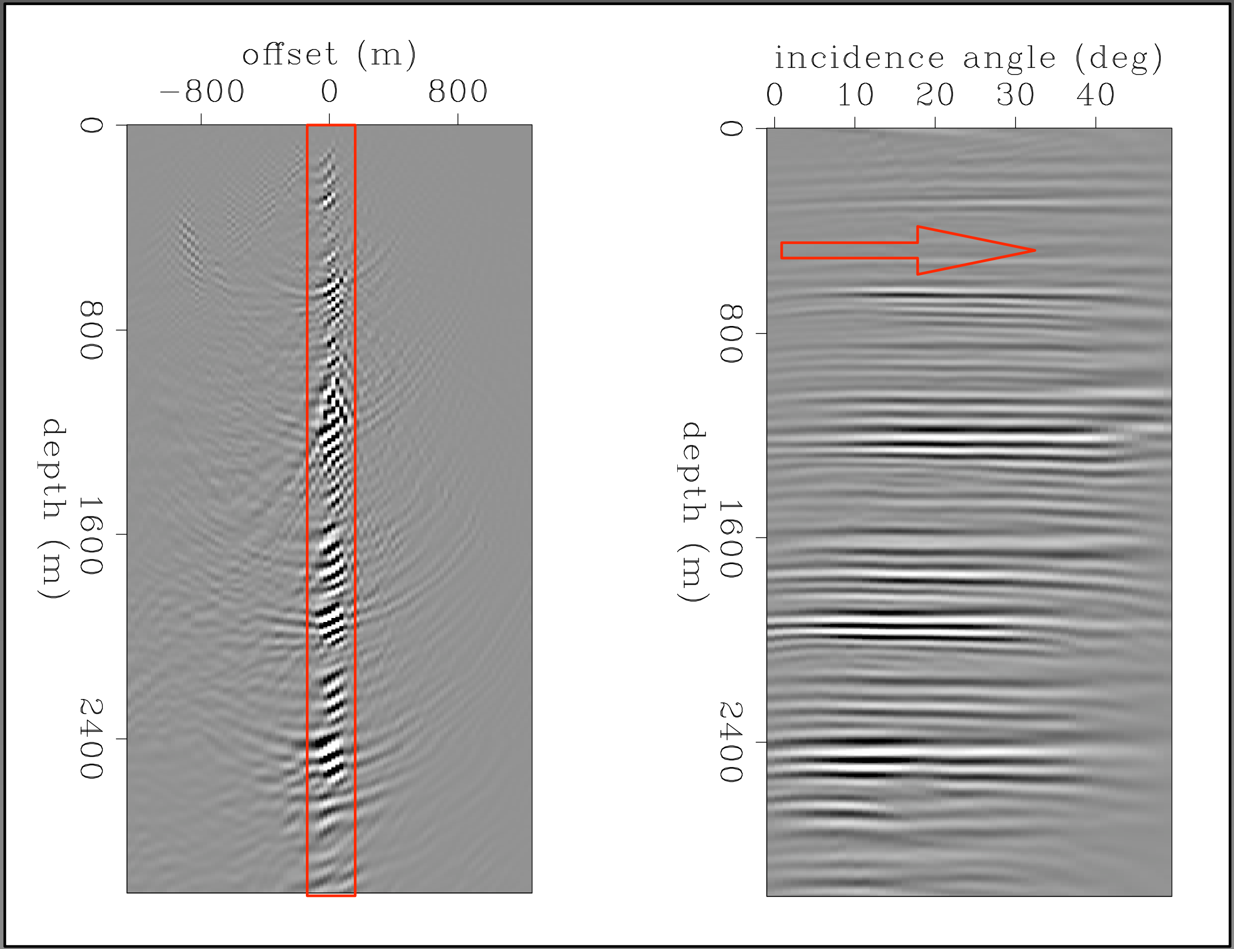
13

Subsurface-offset vs. angle



13

Subsurface-offset vs. angle



13

Shot profile Prestack migration

$$\mathbf{m}(\mathbf{x}, \mathbf{h}) = \sum_{\omega} \sum_{\mathbf{x}_s} \sum_{\mathbf{x}_r} \mathbf{G}'(\mathbf{x} + \mathbf{h}, \mathbf{x}_s; \omega) \mathbf{G}'(\mathbf{x} - \mathbf{h}, \mathbf{x}_r; \omega)$$
$$\sum_{\mathbf{h}}{}' \sum_{\mathbf{x}}{}' \mathbf{d}(\mathbf{x}_s, \mathbf{x}_r; \omega)$$

Shot profile Prestack migration

$$\mathbf{m}(\mathbf{x},\mathbf{h}) = \sum_{\omega}\sum_{\mathbf{x}_s}\sum_{\mathbf{x}_r} \mathrm{G}'(\mathbf{x}+\mathbf{h},\mathbf{x}_s;\omega)\mathrm{G}'(\mathbf{x}-\mathbf{h},\mathbf{x}_r;\omega)$$

$$\sum_{\mathbf{h}}{}'\sum_{\mathbf{x}}{}'\mathbf{d}(\mathbf{x}_s,\mathbf{x}_r;\omega)$$

$$\mathrm{G}(\mathbf{x},\mathbf{x}_s;\omega)$$
$$\mathrm{G}(\mathbf{x},\mathbf{x}_r;\omega)$$

one way wave equation Green functions

Shot profile Prestack migration

$$\mathbf{m}(\mathbf{x},\mathbf{h}) = \sum_{\omega}\sum_{\mathbf{x}_s}\sum_{\mathbf{x}_r} \mathbf{G}'(\mathbf{x}+\mathbf{h},\mathbf{x}_s;\omega)\mathbf{G}'(\mathbf{x}-\mathbf{h},\mathbf{x}_r;\omega)$$

$$\sum_{\mathbf{h}}{}'\sum_{\mathbf{x}}{}' \mathbf{d}(\mathbf{x}_s,\mathbf{x}_r;\omega)$$

$\mathrm{G}(\mathrm{x},\mathrm{x}_s;\omega)$ **one way wave equation**

$\mathrm{G}(\mathrm{x},\mathrm{x}_r;\omega)$ **Green functions**

h **subsurface-offset**

Linear modeling operator

$$\mathbf{d}(\mathbf{x}_s, \mathbf{x}_r; \omega) = \sum_{\mathbf{x}} \sum_{\mathbf{h}} \mathbf{G}(\mathbf{x} - \mathbf{h}, \mathbf{x}_r; \omega) \mathbf{G}(\mathbf{x} + \mathbf{h}, \mathbf{x}_s; \omega) \sum_{\mathbf{x}_r}{}' \sum_{\mathbf{x}_s}{}' \sum_{\omega}{}' \mathbf{m}(\mathbf{x}, \mathbf{h})$$

Subsurface-offset Hessian

$$\mathbf{H}(\mathbf{x},\mathbf{h};\mathbf{x}',\mathbf{h}') = \sum_{\omega}\sum_{\mathbf{x}_s} \mathbf{G}'(\mathbf{x}+\mathbf{h},\mathbf{x}_s;\omega)\mathbf{G}(\mathbf{x}'+\mathbf{h}',\mathbf{x}_s;\omega)$$

$$\sum_{\mathbf{x}_r} \mathbf{G}'(\mathbf{x}-\mathbf{h},\mathbf{x}_r;\omega)\mathbf{G}(\mathbf{x}'-\mathbf{h}',\mathbf{x}_r;\omega)$$

16

Subsurface-offset Hessian

source term

$$\mathbf{H}(\mathbf{x},\mathbf{h};\mathbf{x}',\mathbf{h}') = \sum_{\omega} \sum_{\mathbf{x}_s} \mathbf{G}'(\mathbf{x}+\mathbf{h},\mathbf{x}_s;\omega)\mathbf{G}(\mathbf{x}'+\mathbf{h}',\mathbf{x}_s;\omega)$$

$$\sum_{\mathbf{x}_r} \mathbf{G}'(\mathbf{x}-\mathbf{h},\mathbf{x}_r;\omega)\mathbf{G}(\mathbf{x}'-\mathbf{h}',\mathbf{x}_r;\omega)$$

Subsurface-offset Hessian

source term

$$\mathbf{H}(\mathbf{x},\mathbf{h};\mathbf{x}',\mathbf{h}') = \sum_{\omega} \sum_{\mathbf{x}_s} \mathbf{G}'(\mathbf{x}+\mathbf{h},\mathbf{x}_s;\omega)\mathbf{G}(\mathbf{x}'+\mathbf{h}',\mathbf{x}_s;\omega)$$

receiver term

$$\sum_{\mathbf{x}_r} \mathbf{G}'(\mathbf{x}-\mathbf{h},\mathbf{x}_r;\omega)\mathbf{G}(\mathbf{x}'-\mathbf{h}',\mathbf{x}_r;\omega)$$

From offset to angle

$$\mathbf{m}(\mathbf{x}, \boldsymbol{\Theta}) = \mathbf{T}_{\mathbf{h} \rightarrow \boldsymbol{\Theta}} \mathbf{m}(\mathbf{x}, \mathbf{h})$$

$\Theta = (\theta, \alpha)$ **reflection and azimuth angles**

$\mathbf{T}_{\mathbf{h} \rightarrow \boldsymbol{\Theta}}$ **slant stack operator**

Sava & Fomel (2003)

Angle domain Hessian

$$\mathrm{H}(\mathbf{x},\Theta;\mathbf{x}',\Theta') = \mathrm{T}_{\mathbf{h}\rightarrow\Theta}\mathrm{H}(\mathbf{x},\mathbf{h};\mathbf{x}',\mathbf{h}')\mathrm{T}_{\Theta\rightarrow\mathbf{h}}$$

$\mathrm{H}(\mathbf{x},\mathbf{h};\mathbf{x}',\mathbf{h}')$ **can be pre-computed**

Outline

- Least-squares imaging (explicit Hessian)
- Prestack image-domain Hessian
- **Hessian construction cost**
- Inversion with regularization in the subsurface-offset domain: Sigsbee model

Thanks to frequency independence

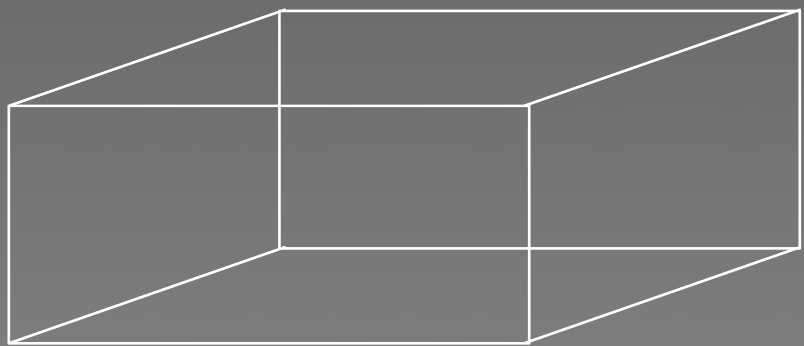
Embarrassingly parallel in the frequency dimension

$$\mathbf{H}(\mathbf{x},\mathbf{h};\mathbf{x}',\mathbf{h}') = \boxed{\sum_{\omega}} \sum_{\mathbf{x}_s} \mathbf{G}'(\mathbf{x}+\mathbf{h},\mathbf{x}_s;\omega)\mathbf{G}(\mathbf{x}'+\mathbf{h}',\mathbf{x}_s;\omega)$$

$$\sum_{\mathbf{x}_r} \mathbf{G}'(\mathbf{x}-\mathbf{h},\mathbf{x}_r;\omega)\mathbf{G}(\mathbf{x}'-\mathbf{h}',\mathbf{x}_r;\omega)$$

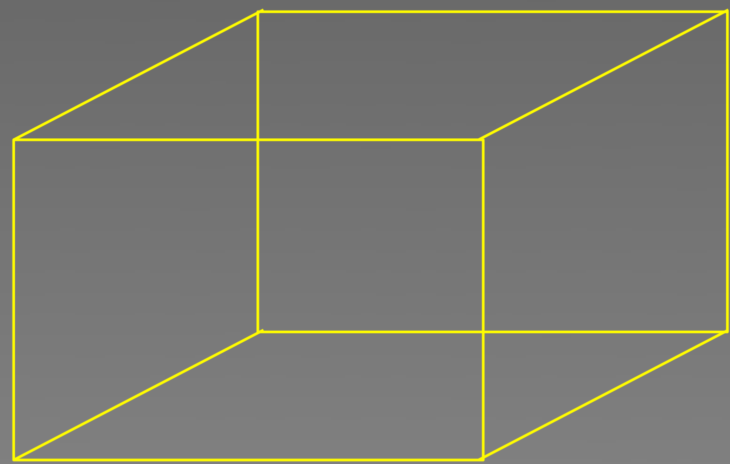
Toy example

$n_x = 400$
$n_y = 200$
$n_z = 200$



model space

$n_{sx} = 400$
$n_{sy} = 200$
$n_w = 200$



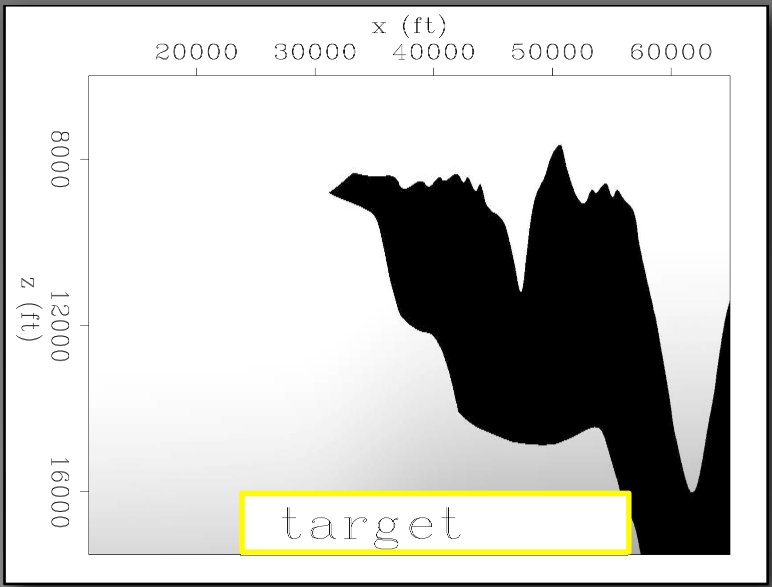
data space

Problem dimensions

Approximation	G		H
	Serial	Parallel	
NA	$2x10^{15}$ (petabytes)	10^{13} (terabytes)	10^{15} (petabyte)

Target-oriented Hessian

$$\mathbf{H}(\mathbf{x}_T, \mathbf{y}_T) = \sum_{\omega} \sum_{\mathbf{x}_s} \mathbf{G}'(\mathbf{x}_T, \mathbf{x}_s; \omega) \mathbf{G}(\mathbf{y}_T, \mathbf{x}_s; \omega) \sum_{\mathbf{x}_r} \mathbf{G}'(\mathbf{x}_T, \mathbf{x}_r; \omega) \mathbf{G}(\mathbf{y}_T, \mathbf{x}_r; \omega)$$



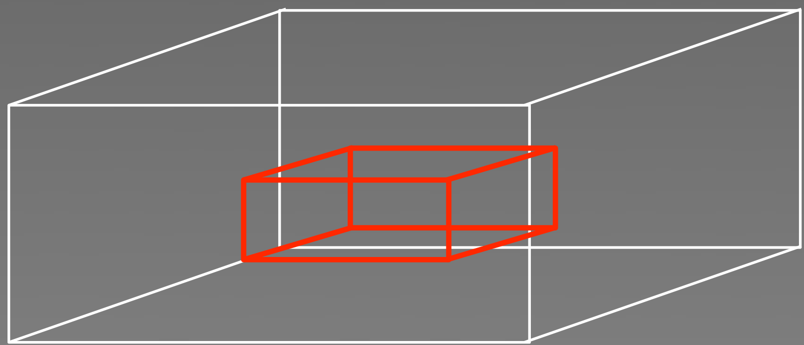
Potential computational savings if shooting from bottom to top

23

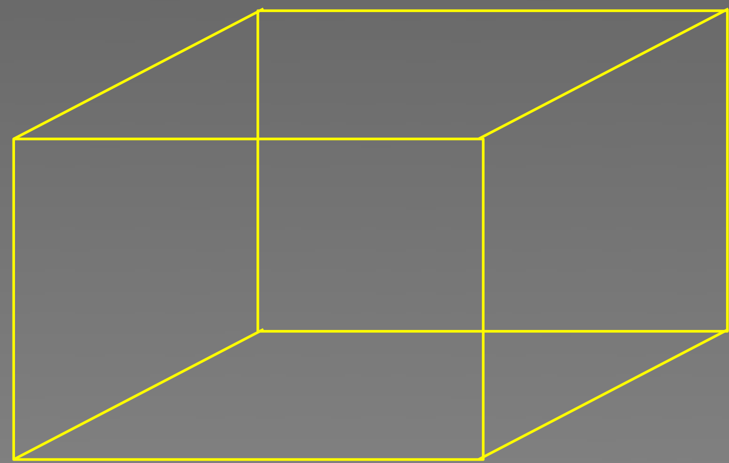
Toy example

$n_{xT} = 40$
$n_{yT} = 20$
$n_{zT} = 20$

$n_{sx} = 400$
$n_{sy} = 200$
$n_w = 200$



model space



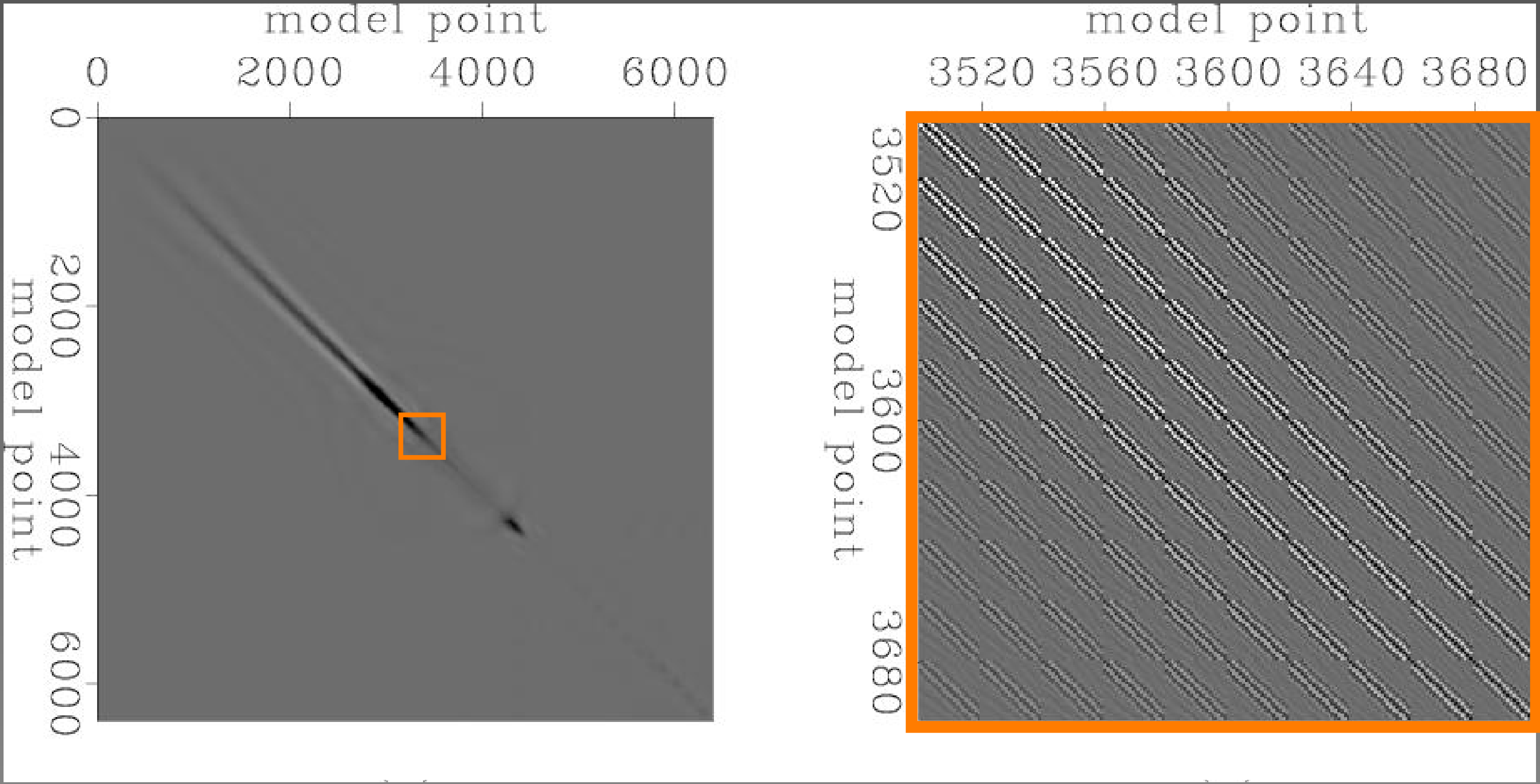
data space

24

Problem dimensions

Approximation	G		H
	Serial	Parallel	
NA	$2x10^{15}$ (petabytes)	10^{13} (terabytes)	10^{15} (petabyte)
target	$2x10^{12}$ (terabytes)	10^{10} (gigabytes)	10^{9} (gigabyte)

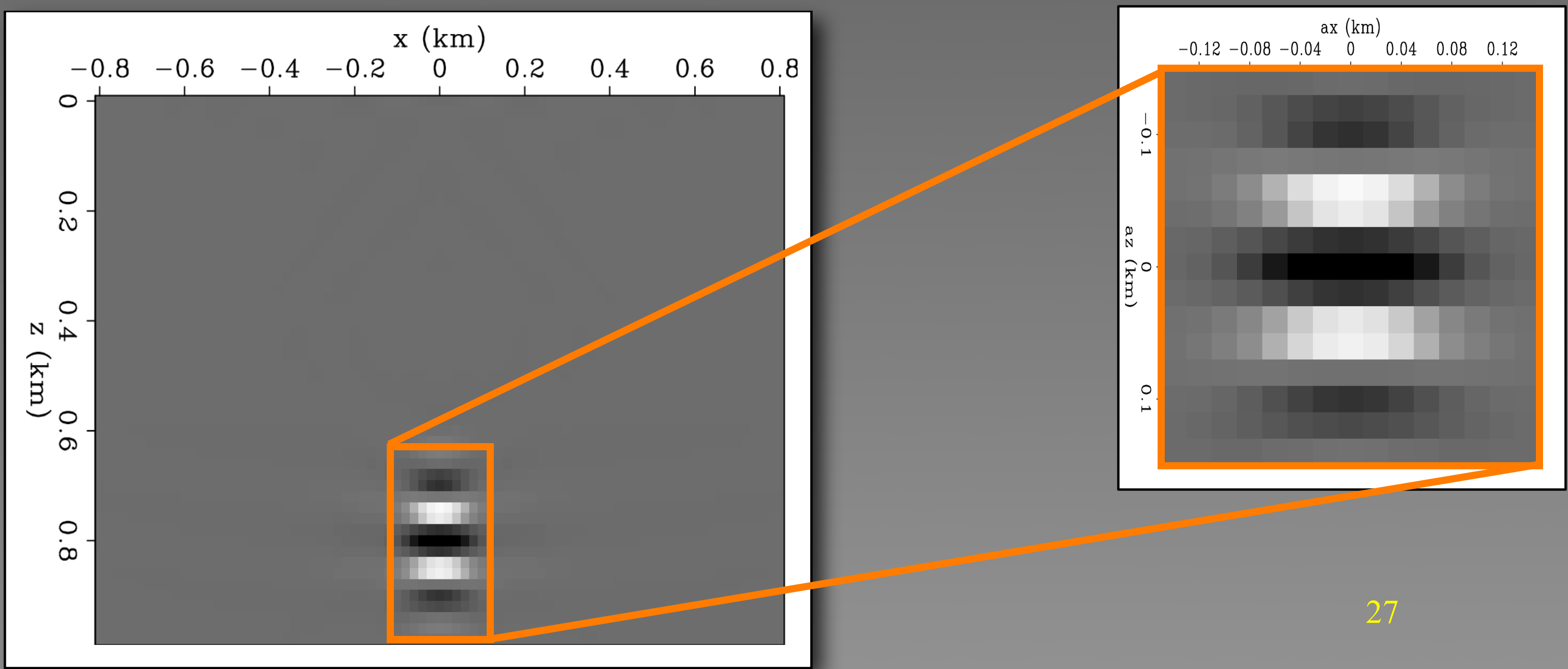
Hessian matrix



Sparse symmetric matrix

Hessian sparsity and structure

$$\mathbf{H}(\mathbf{x}_T, \mathbf{x}_T + \mathbf{a_x}) = \sum_{\omega}\sum_{\mathbf{x}_s} \mathbf{G}'(\mathbf{x}_T, \mathbf{x}_s; \omega)\mathbf{G}(\mathbf{x}_T + \mathbf{a_x}, \mathbf{x}_s; \omega)$$
$$\sum_{\mathbf{x}_r} \mathbf{G}'(\mathbf{x}_T, \mathbf{x}_r; \omega)\mathbf{G}(\mathbf{x}_T + \mathbf{a_x}, \mathbf{x}_r; \omega)$$



27

Problem dimensions

Approximation	G Serial	G Parallel	H
NA	$2x10^{15}$ (petabytes)	10^{13} (terabytes)	10^{15} (petabyte)
target	$2x10^{12}$ (terabytes)	10^{10} (gigabytes)	10^{9} (gigabyte)
structure	$2x10^{12}$ (terabytes)	10^{10} (gigabytes)	$6x10^{7}$(megabytes)

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- Least-squares imaging (explicit Hessian)
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- **Inversion with regularization in the subsurface-offset domain: Sigsbee model**

29

Inversion with damping regularization

$$\mathbf{H}(\mathbf{x},\mathbf{h};\mathbf{x}',\mathbf{h}')\hat{\mathbf{m}}(\mathbf{x},\mathbf{h}) - \mathbf{m}_{mig}(\mathbf{x},\mathbf{h}) \approx 0$$

$$\varepsilon \mathbf{I}\hat{\mathbf{m}}(\mathbf{x},\mathbf{h}) \approx 0$$

Iterative inversion algorithm

Inversion with differential semblance regularization

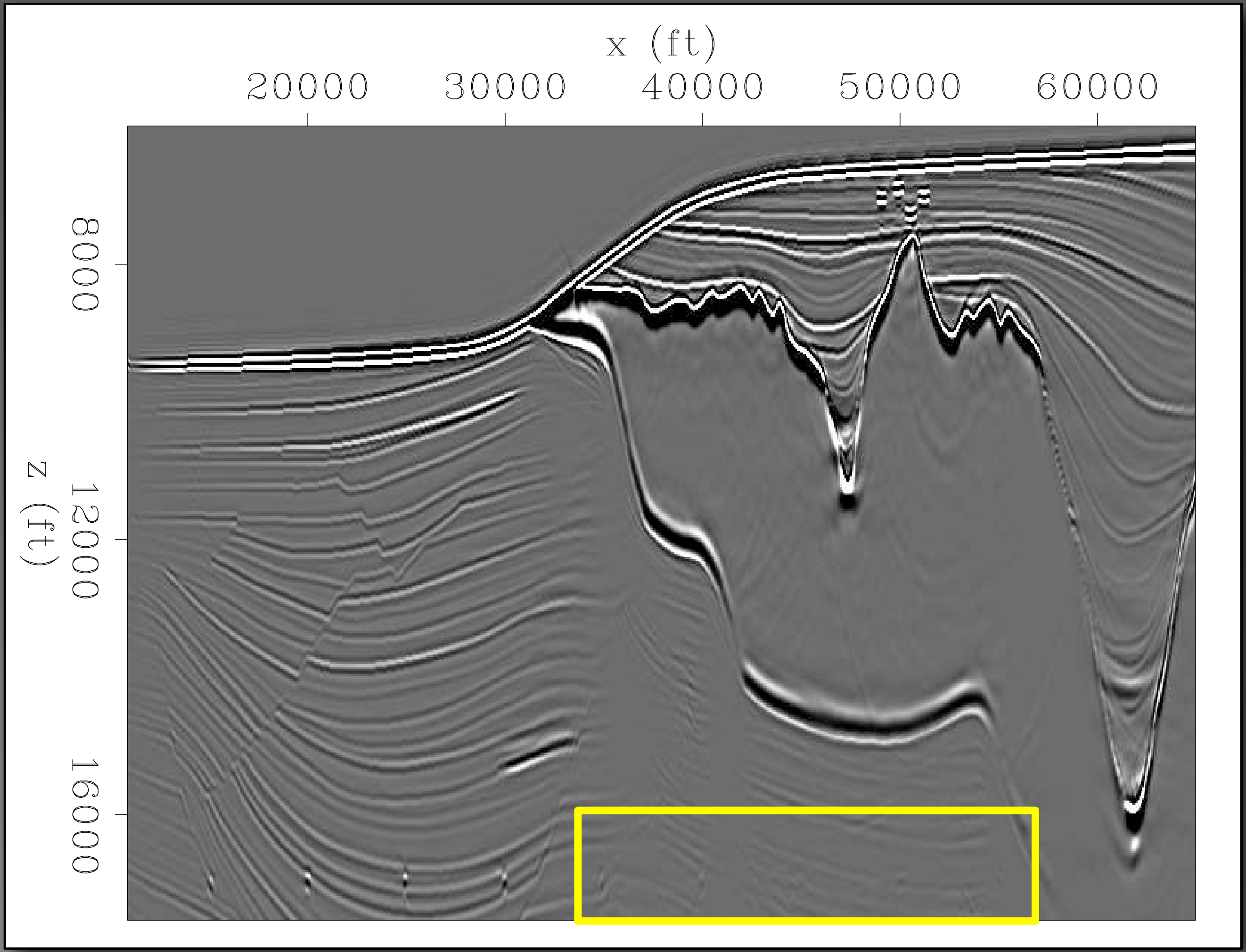
$$\mathbf{H}(\mathbf{x},\mathbf{h};\mathbf{x}',\mathbf{h}')\hat{\mathbf{m}}(\mathbf{x},\mathbf{h}) - \mathbf{m}_{mig}(\mathbf{x},\mathbf{h}) \approx 0$$

$$\varepsilon \mathbf{P}_{\mathbf{h}}\hat{\mathbf{m}}(\mathbf{x},\mathbf{h}) \approx 0$$

$\mathbf{P}_{\mathbf{h}} = |\mathbf{h}|$ **differential semblance operator**

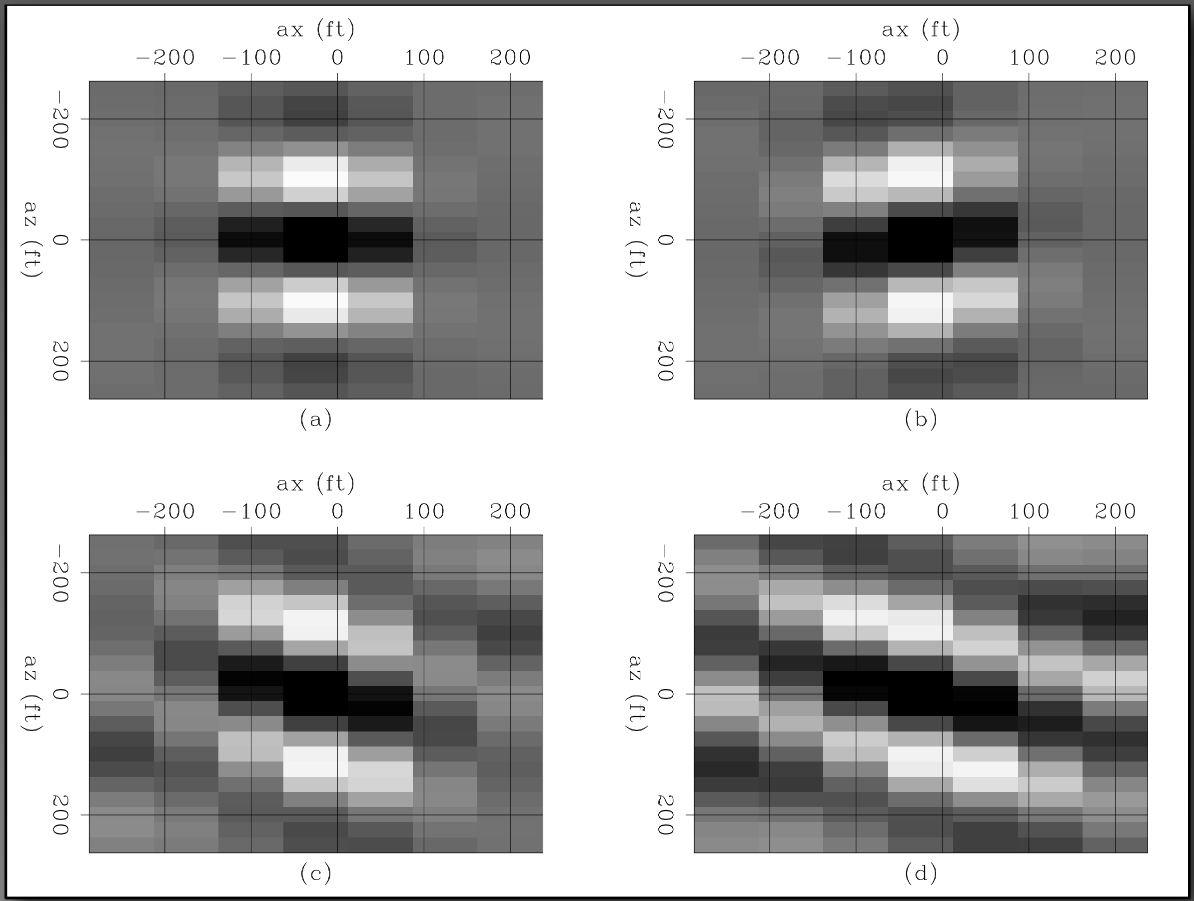
Iterative inversion algorithm

Sigsbee model migration



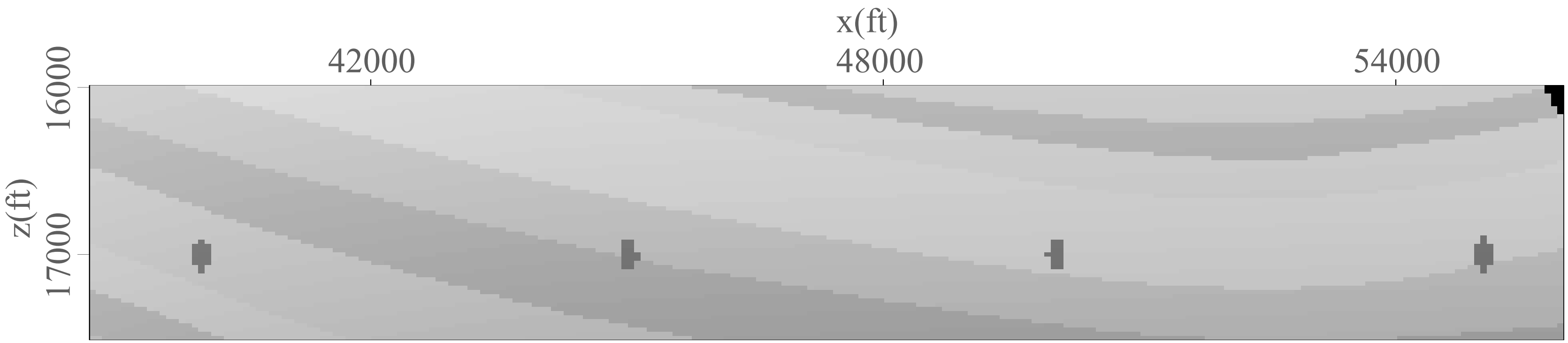
32

Non-stationary filters



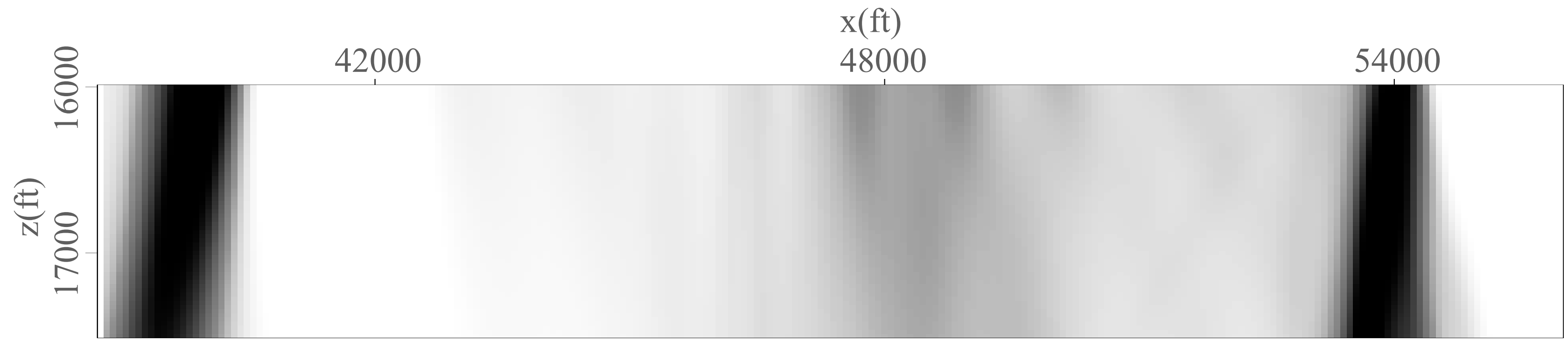
33

Target migration



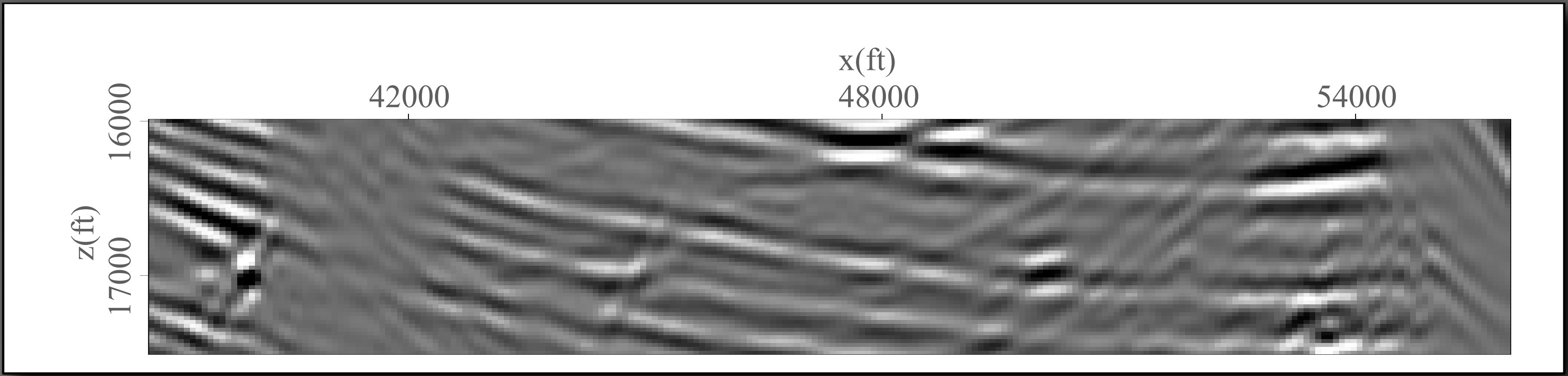
34

Target migration

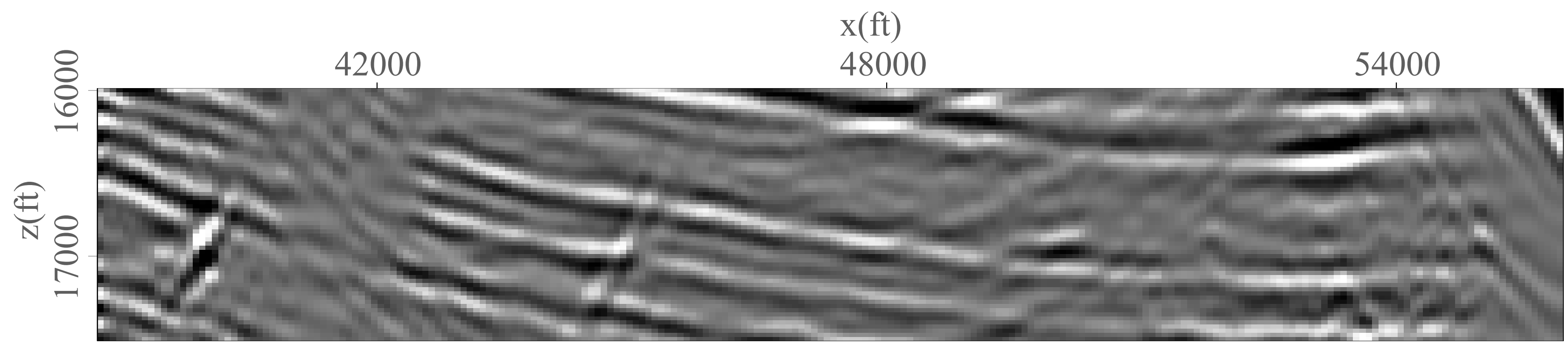


35

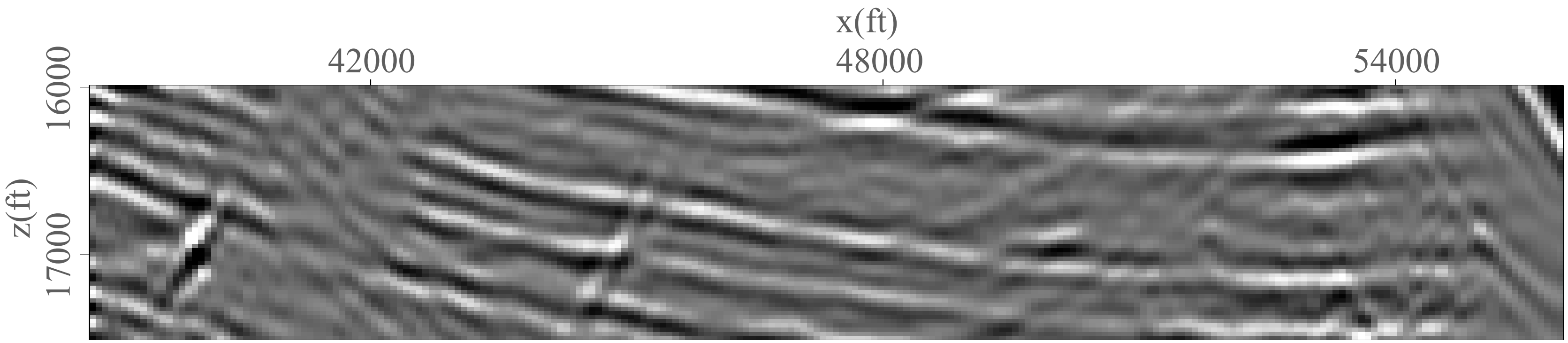
Target migration



Target inversion with damping regularization

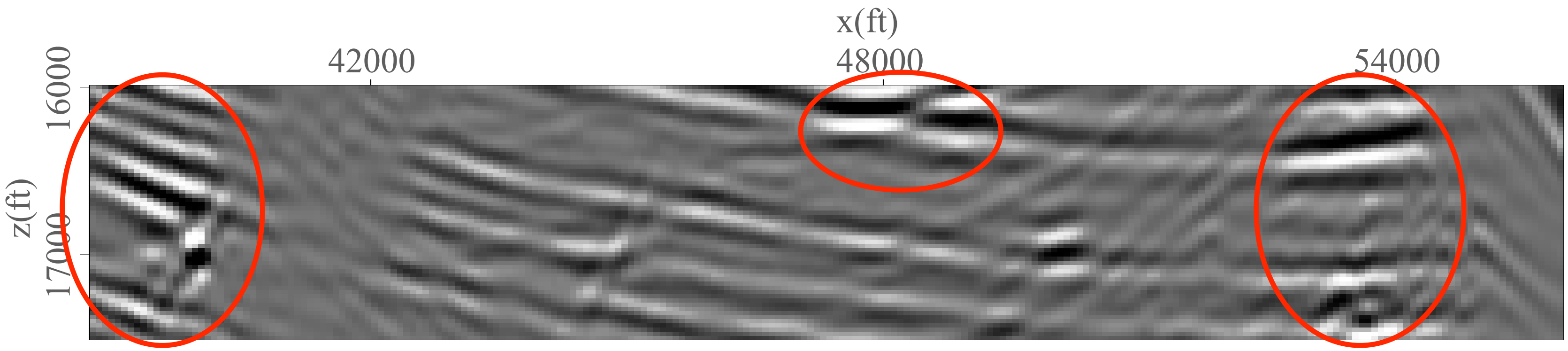


Target inversion with differential semblance regularization



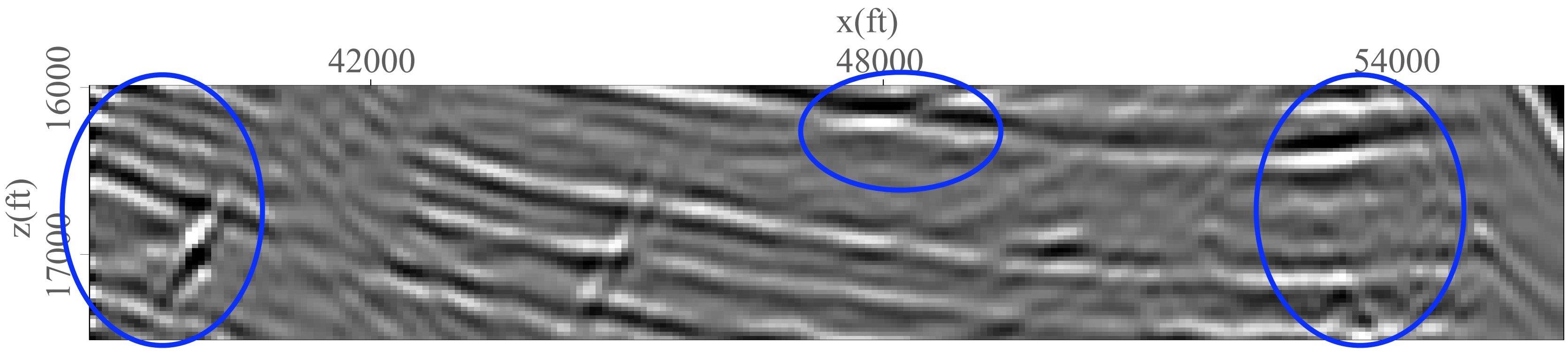
38

Target migration



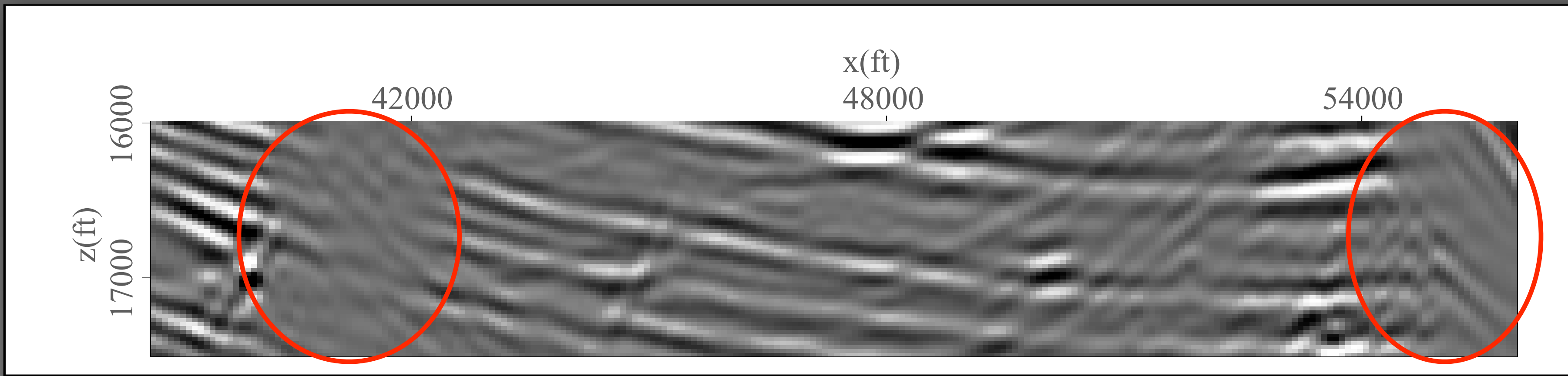
39

Target inversion with differential semblance regularization



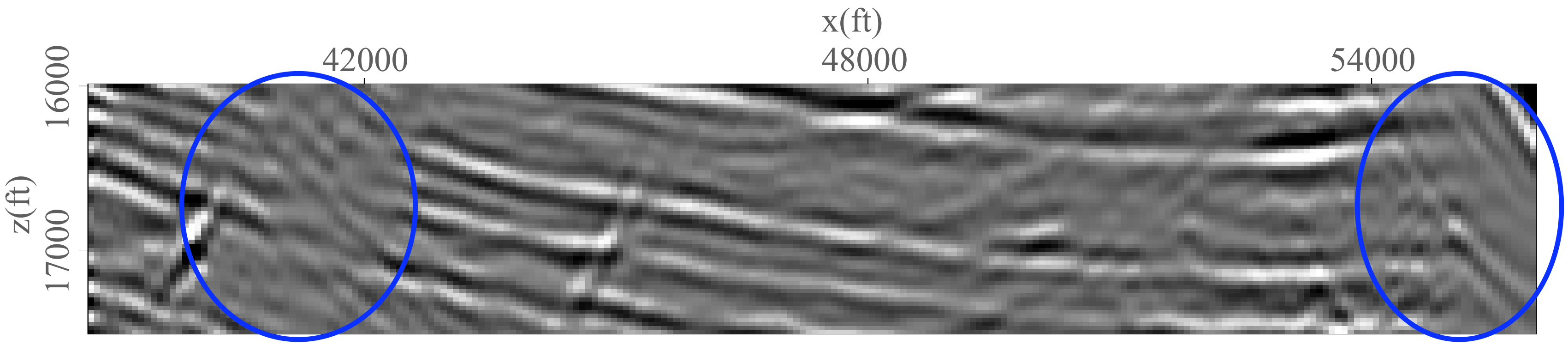
40

Target migration

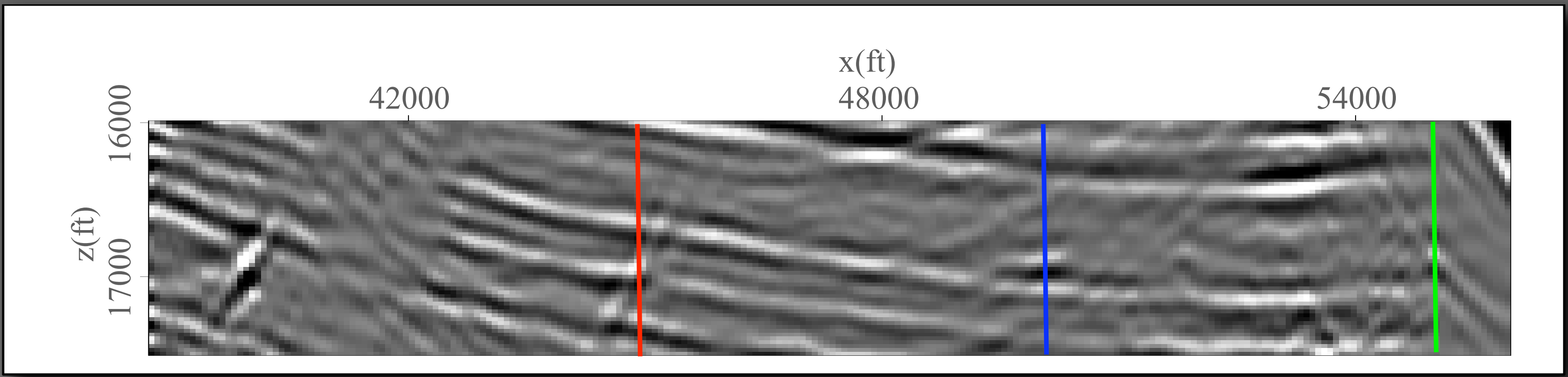


41

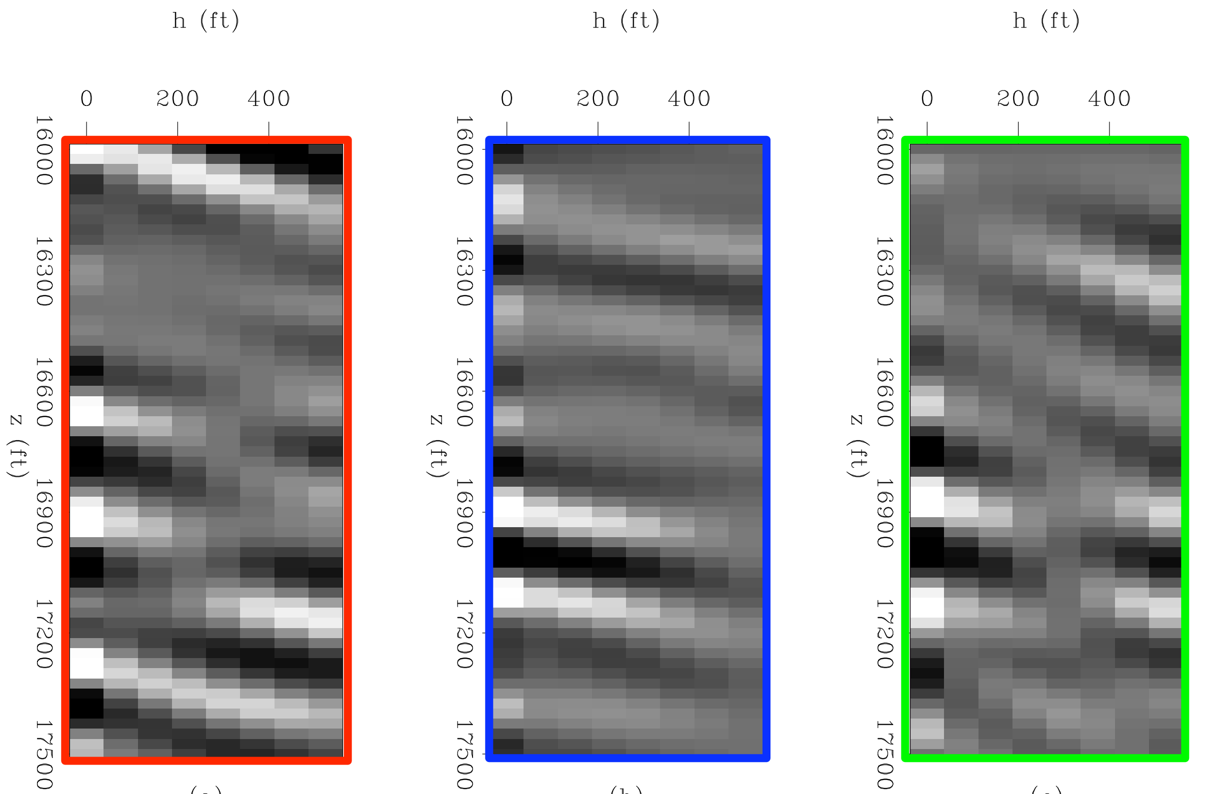
Target inversion with differential semblance regularization



Target inversion with differential semblance regularization



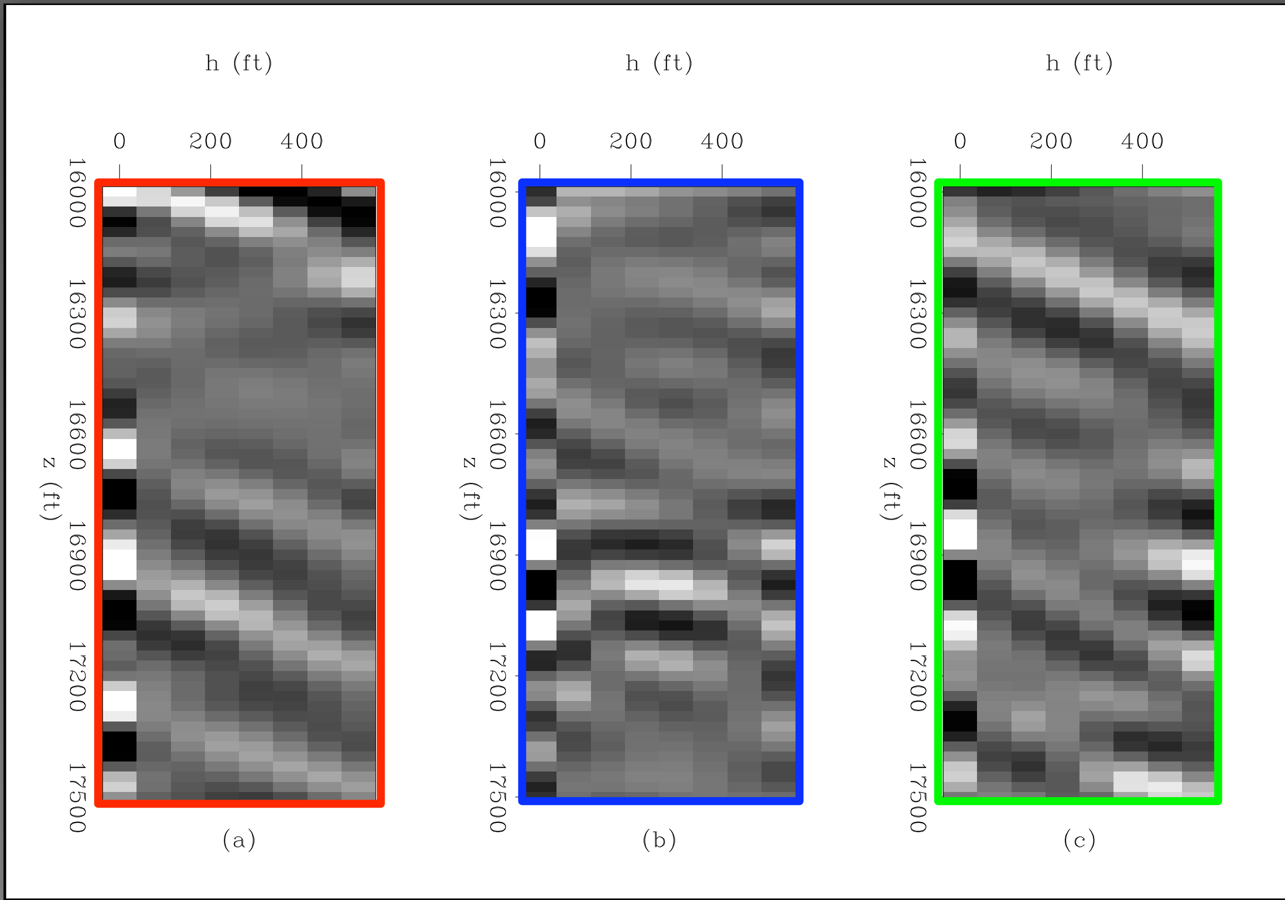
Target migration



(a) (b) (c)

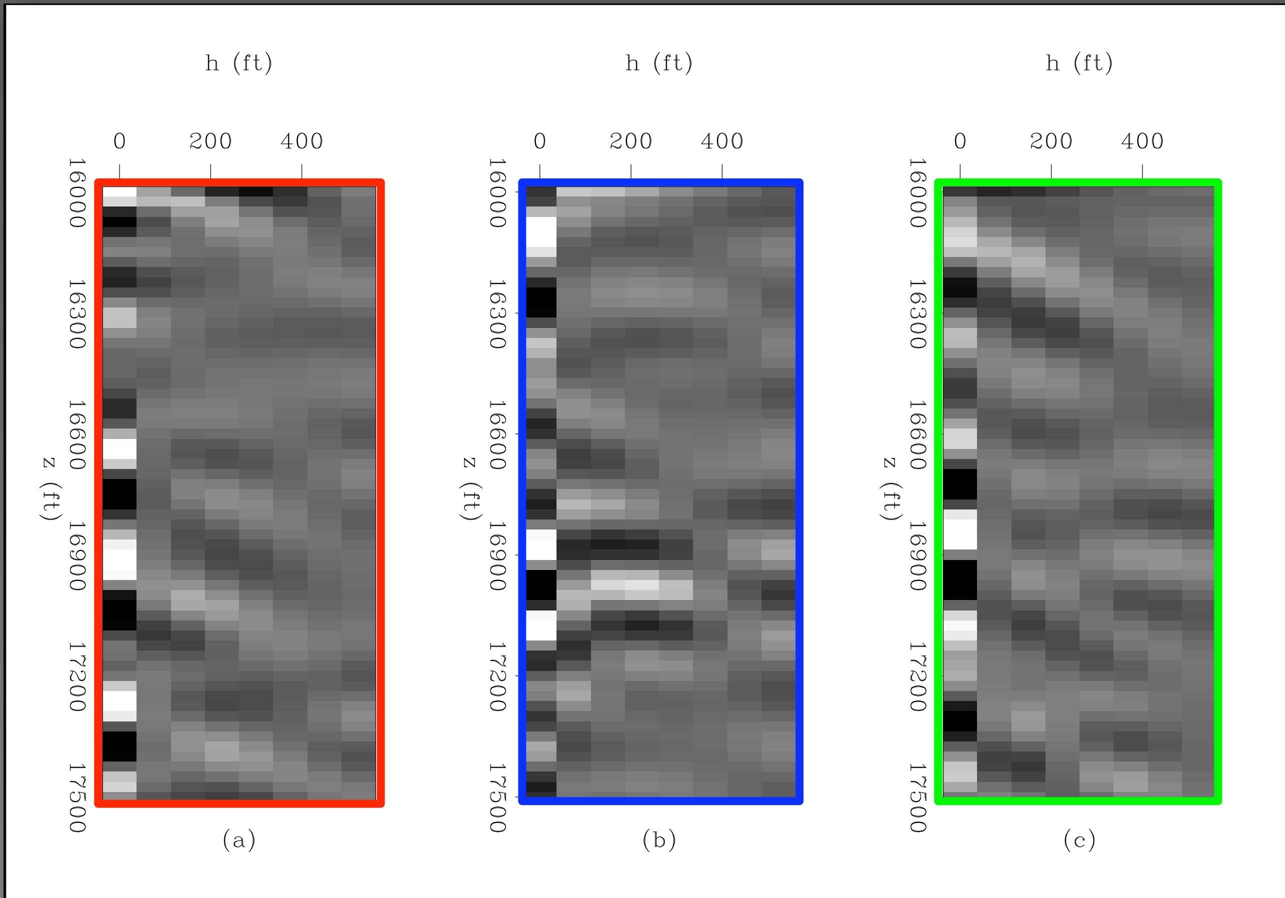
44

Target inversion with damping regularization



45

Target inversion with differential semblance regularization



46

Conclusions

- A **diagonal-matrix** approximation of the Hessian is an **oversimplification**.
- A **target-oriented** strategy can be applied to explicitly compute the inversion Hessian.
- The Hessian **sparse structure** allows big computational savings.

47

Conclusions (cont.)

- **The regularization in the subsurface-offset results looks promising.**

Acknowledgments

- Thanks to SMAART JV for the Sigsbee model

50

Green's function dimensions

$$\mathbf{G}(\mathbf{x}, \mathbf{x}_{surf}; \omega)$$

$$\mathbf{G}(x, y, z, x_{surf}, y_{surf}; \omega)$$

$$(n_x, n_y, n_z, n_{x_{surf}}, n_{y_{surf}}; n_\omega)$$

$$8 \times 4 \times 10^2 \times 2 \times 10^2 \times 2 \times 10^2 \times 4 \times 10^2 \times 2 \times 10^2 \times 2 \times 10^2 = \boxed{2 \times 10^{15}}$$

(petabyte)

In parallel $\dfrac{2 \times 10^{15}}{2 \times 10^2} = 10^{13}$

(terabytes)

Hessian dimensions

$$\mathbf{H}(\mathbf{x}, \mathbf{y})$$

$$\mathbf{H}(x, y, z, x', y', z')$$

$$(n_x, n_y, n_z, n_x, n_y, n_z)$$

$$4 \times 4 \times 10^2 \times 2 \times 10^2 \times 2 \times 10^2 \times 4 \times 10^2 \times 2 \times 10^2 \times 2 \times 10^2 = \boxed{10^{15}}$$

(petabyte)

Green's function dimensions

$$\mathbf{G}(\mathbf{x}_T, \mathbf{x}_s; \omega)$$

$$\mathbf{G}(x_T, y_T, z_T, x_{surf}, y_{surf}; \omega)$$

$$(n_{x_T}, n_{y_T}, n_{z_T}, n_{x_{surf}}, n_{y_{surf}}; n_\omega)$$

$$8 \times 4 \times 10 \times 2 \times 10 \times 2 \times 10 \times 4 \times 10^2 \times 2 \times 10^2 \times 2 \times 10^2 = \boxed{2 \times 10^{12}}$$

(terabytes)

In parallel $\dfrac{2 \times 10^{12}}{2 \times 10^2} = 10^{10}$

(gigabytes)

53

Hessian dimensions

$$\mathbf{H}(x_T, y_T)$$

$$\mathbf{H}(x_T, y_T, z_T, x'_T, y'_T, z'_T)$$

$$(n_{x_T}, n_{y_T}, n_{z_T}, n_{x_T}, n_{y_T}, n_{z_T})$$

$$4 \times 4 \times 10 \times 2 \times 10 \times 2 \times 10 \times 4 \times 10 \times 2 \times 10 \times 2 \times 10 = \boxed{10^9}$$

(gigabyte)

54

Hessian dimensions

$$\mathbf{H}(\mathbf{x}_T, \mathbf{x}_T + \mathbf{a}_\mathbf{x})$$

$$\mathbf{H}(x_T, y_T, z_T, x_T + a_x, y_T + a_T, z_T + a_T)$$

$$(n_{x_T}, n_{y_T}, n_{z_T}, n_{a_x}, n_{a_y}, n_{a_z})$$

$$4 \times (4 \times 10) \times (2 \times 10) \times (2 \times 10) \times 10 \times 10 \times 10 = \boxed{6 \times 10^7}$$

(megabytes)