Wavefield summation and direct migration

passive seismic processing as a migration problem

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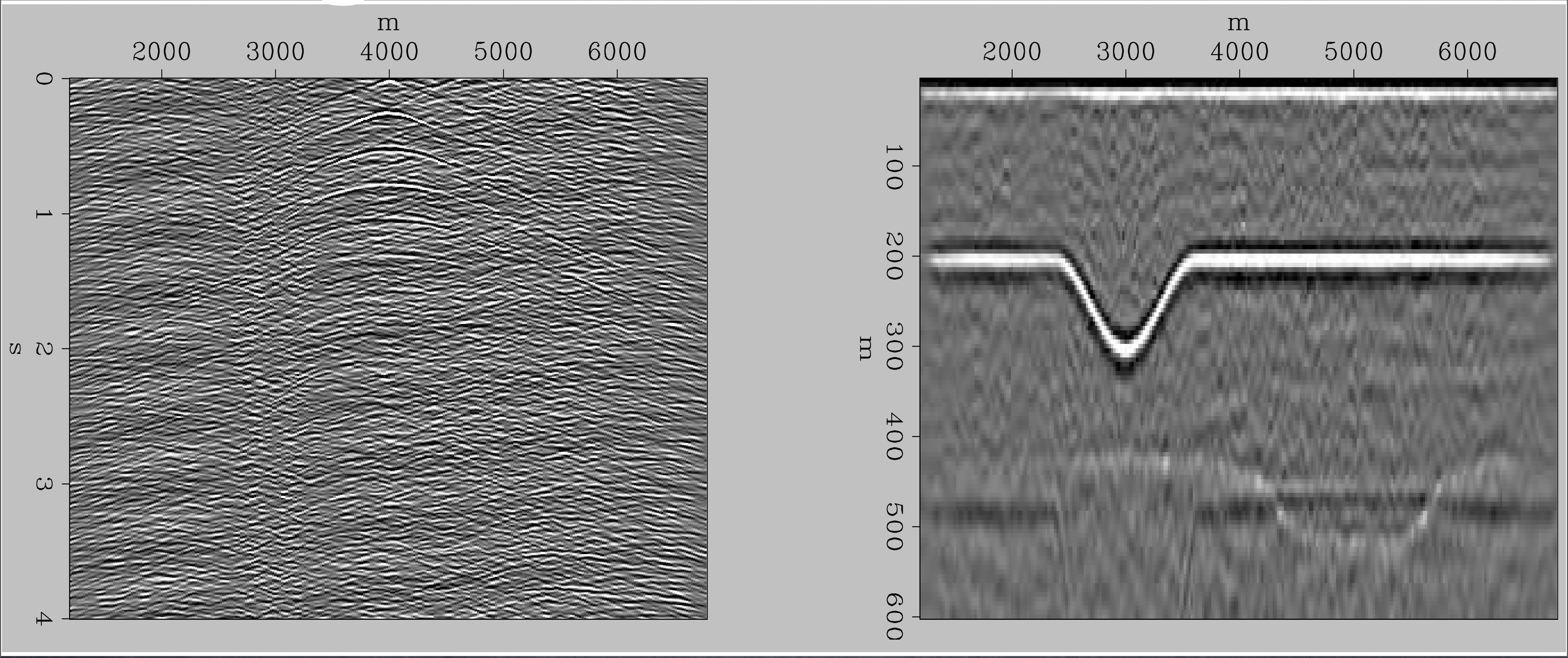
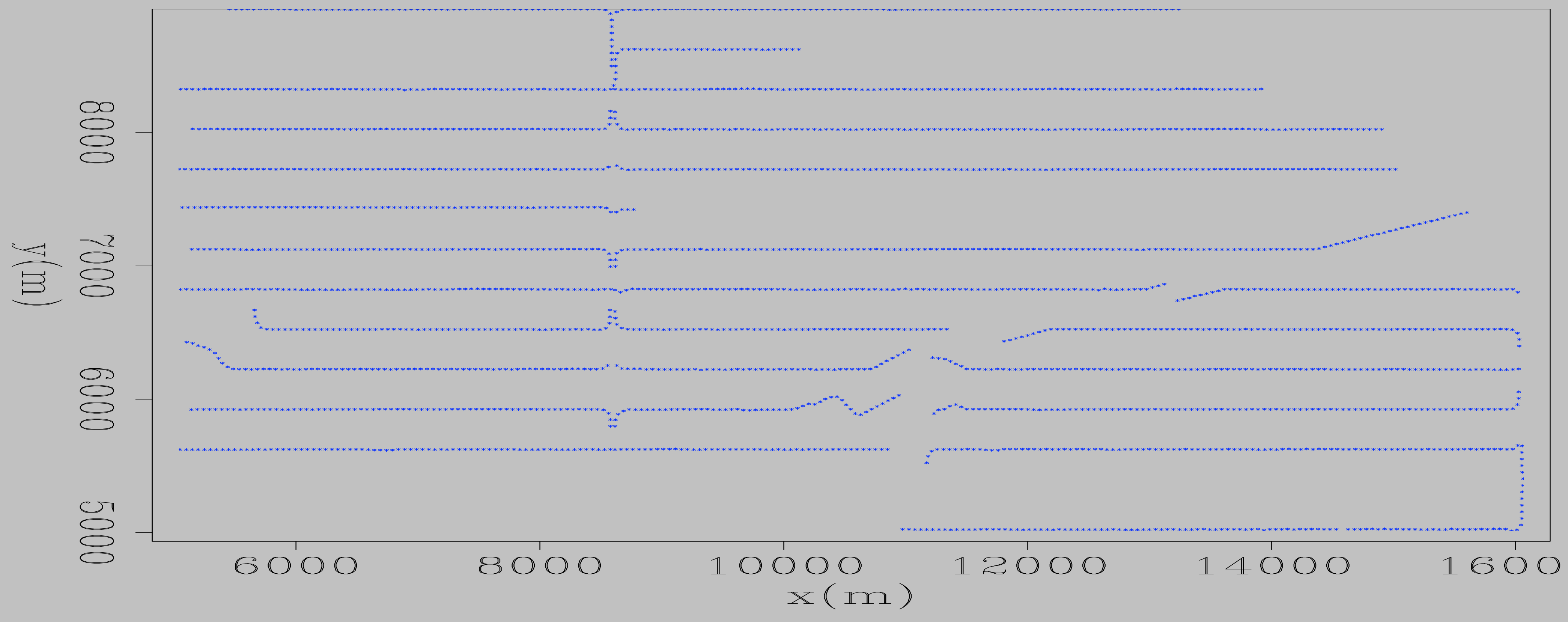
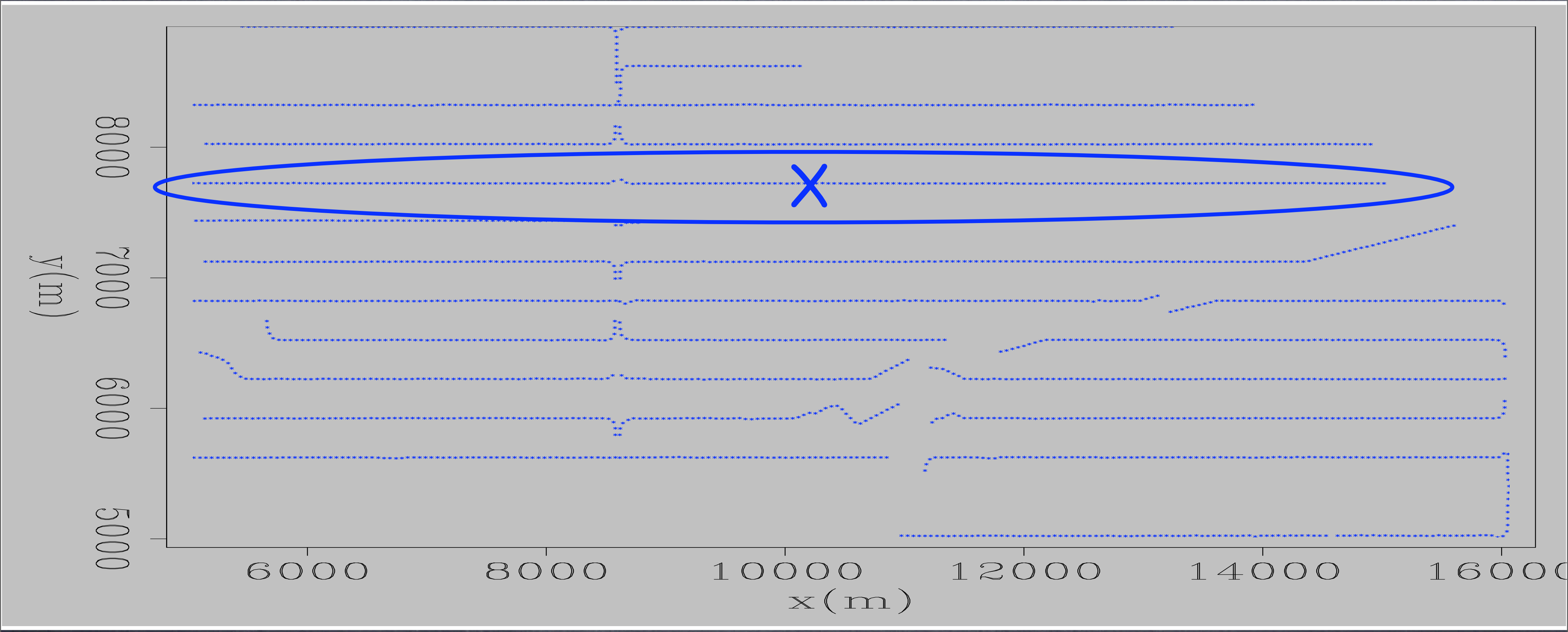
Migration to increase S/N



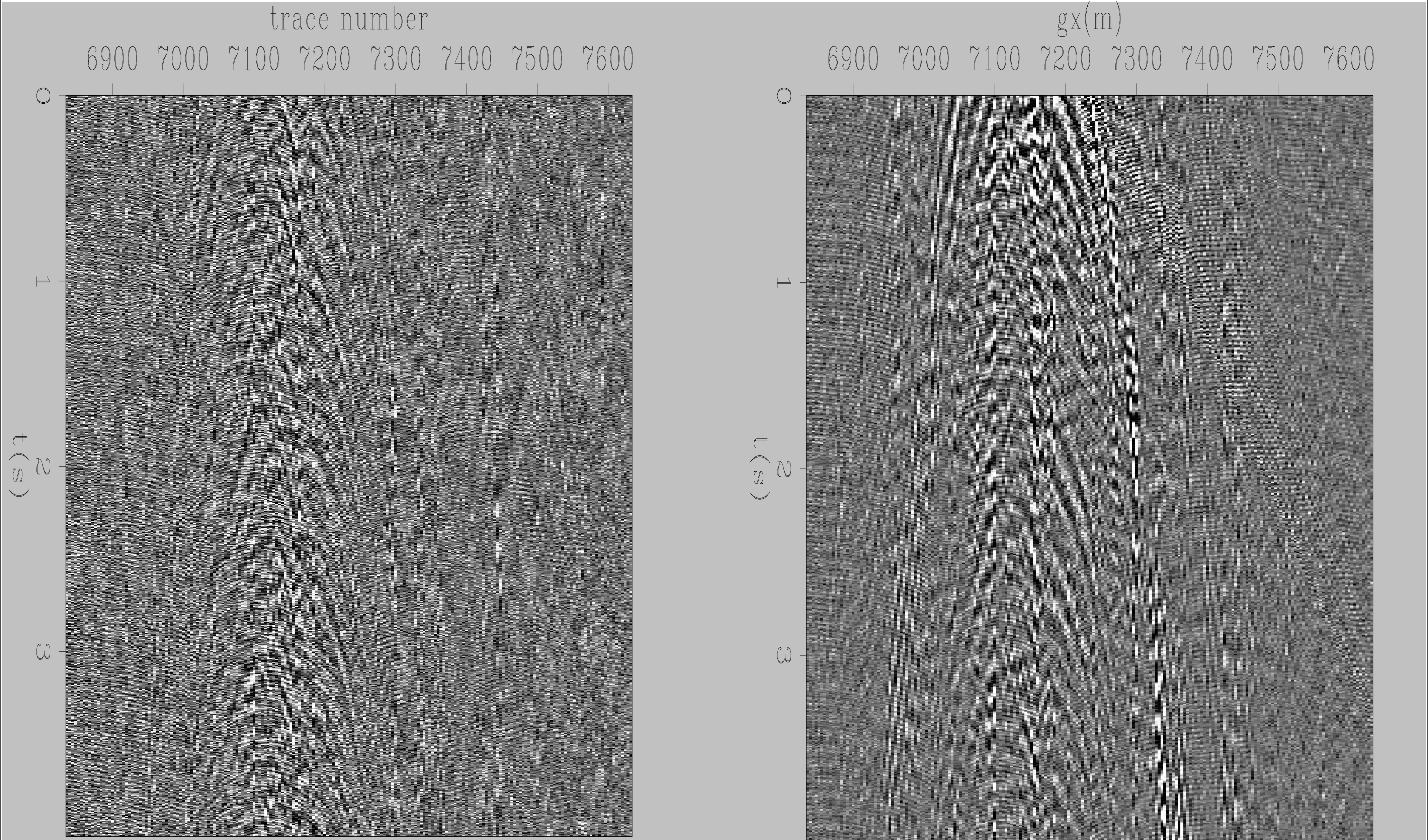
Valhall LoFS



Valhall LoFS



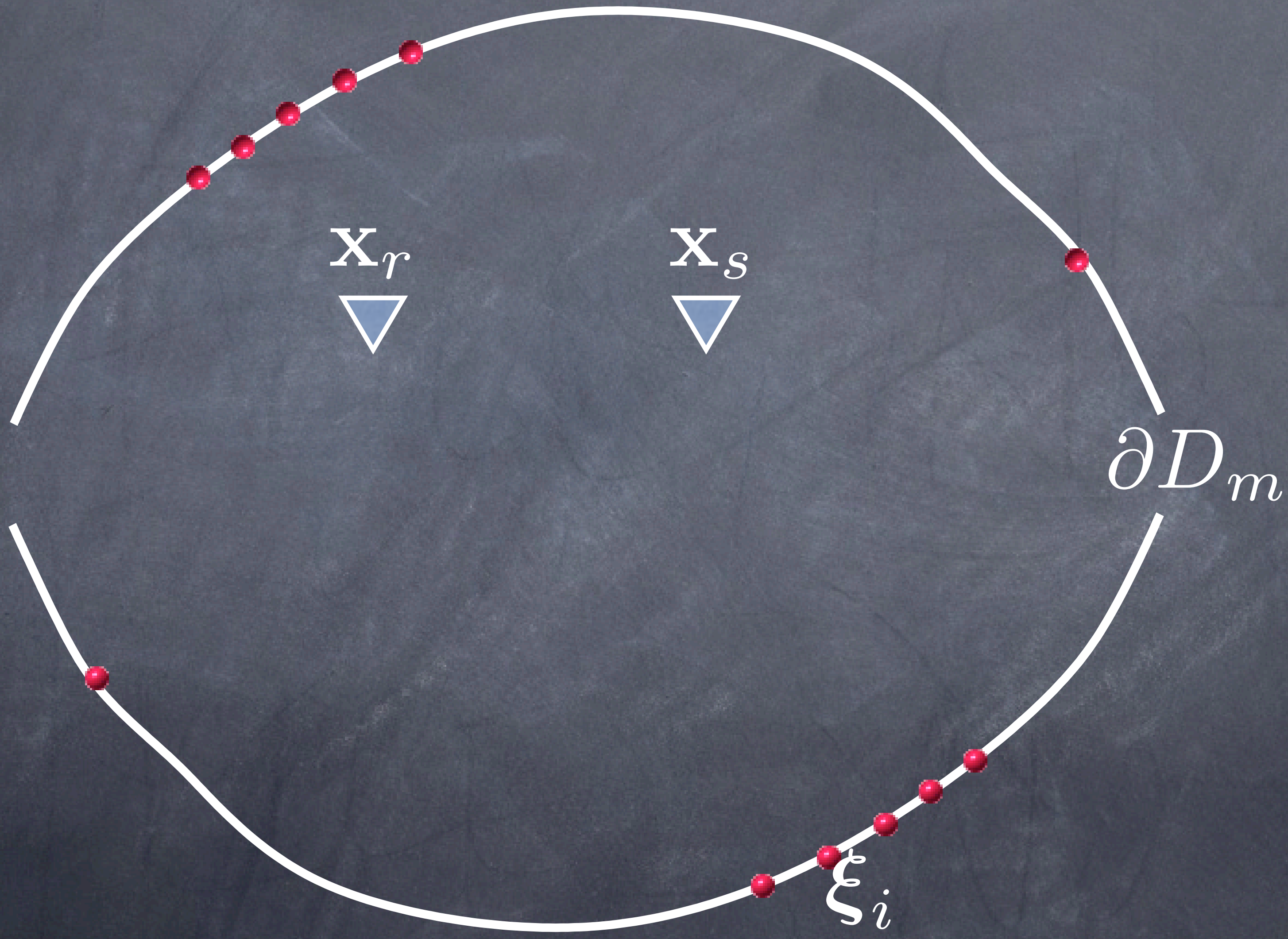
choose!



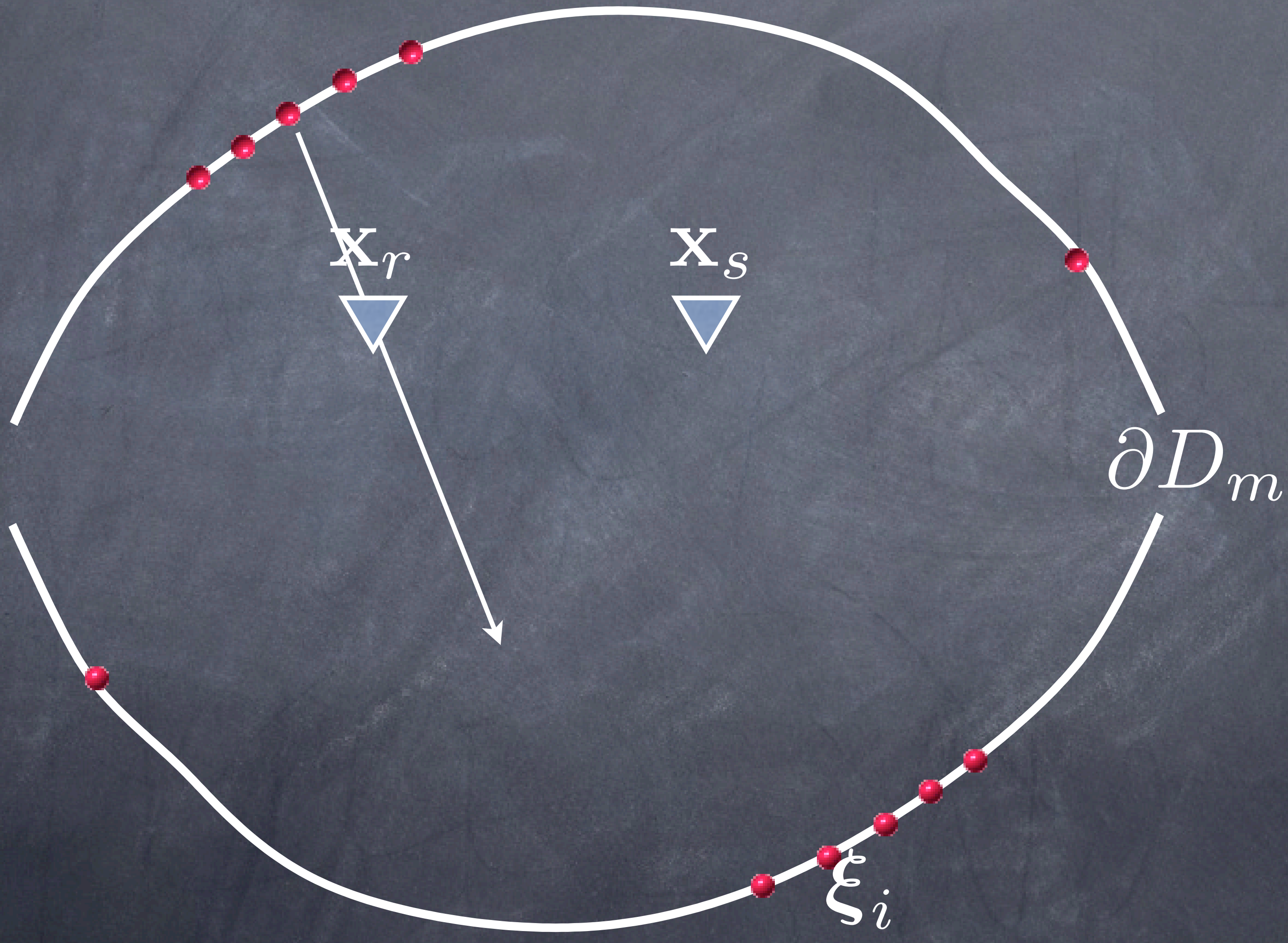
trace number
6900 7000 7100 7200 7300 7400 7500 7600
0 1 2 3
t(s)
gx(m)
6900 7000 7100 7200 7300 7400 7500 7600
0 1 2 3
t(s)

What correlations do

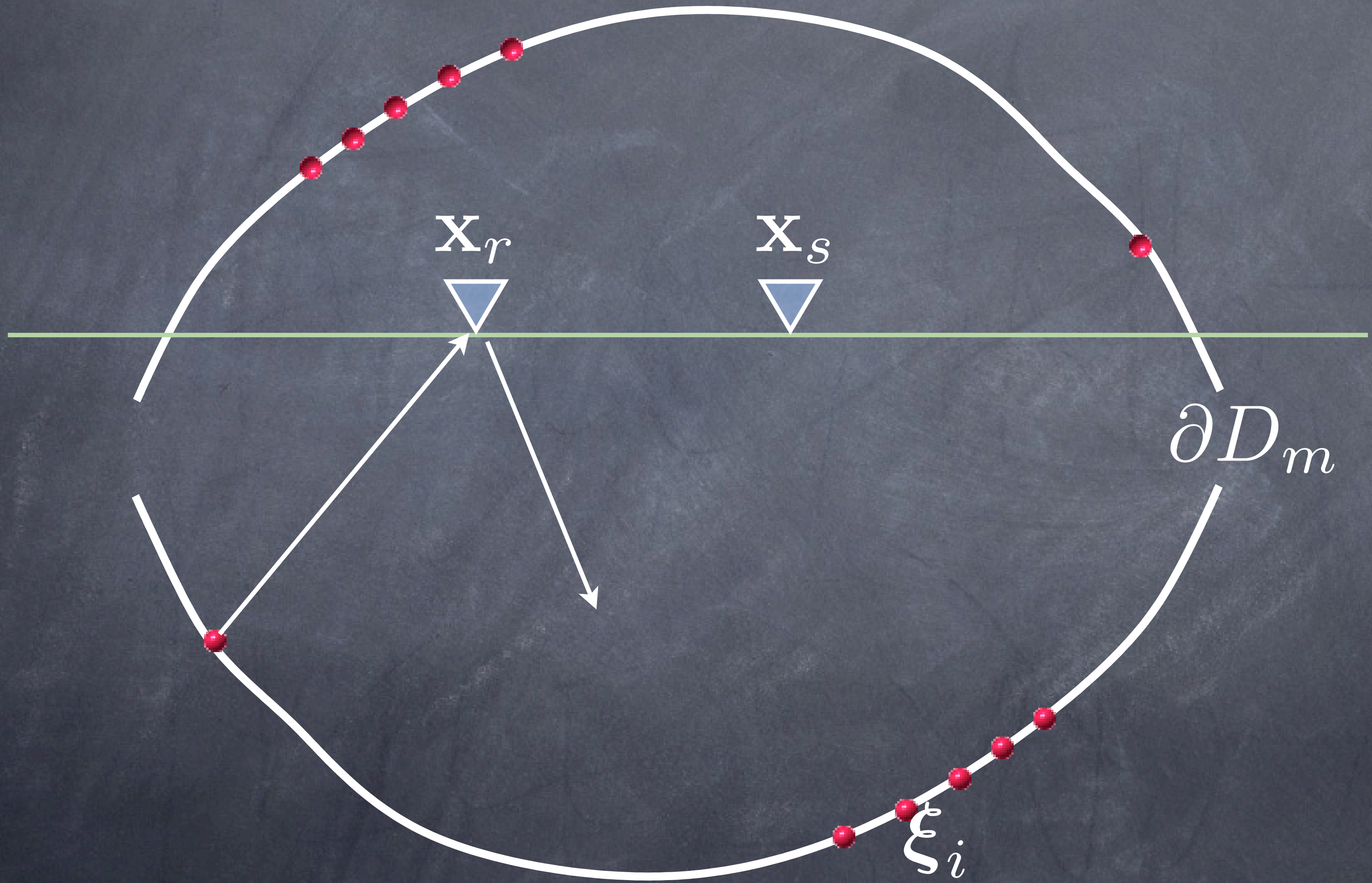
$$2\Re[R(\mathbf{x}_r, \mathbf{x}_s, \omega)] = \delta(\mathbf{x}_s - \mathbf{x}_r) - \int_{\partial D_m} T(\mathbf{x}_r, \boldsymbol{\xi}, \omega) T^*(\mathbf{x}_s, \boldsymbol{\xi}, \omega)\, \mathrm{d}^2\boldsymbol{\xi}$$



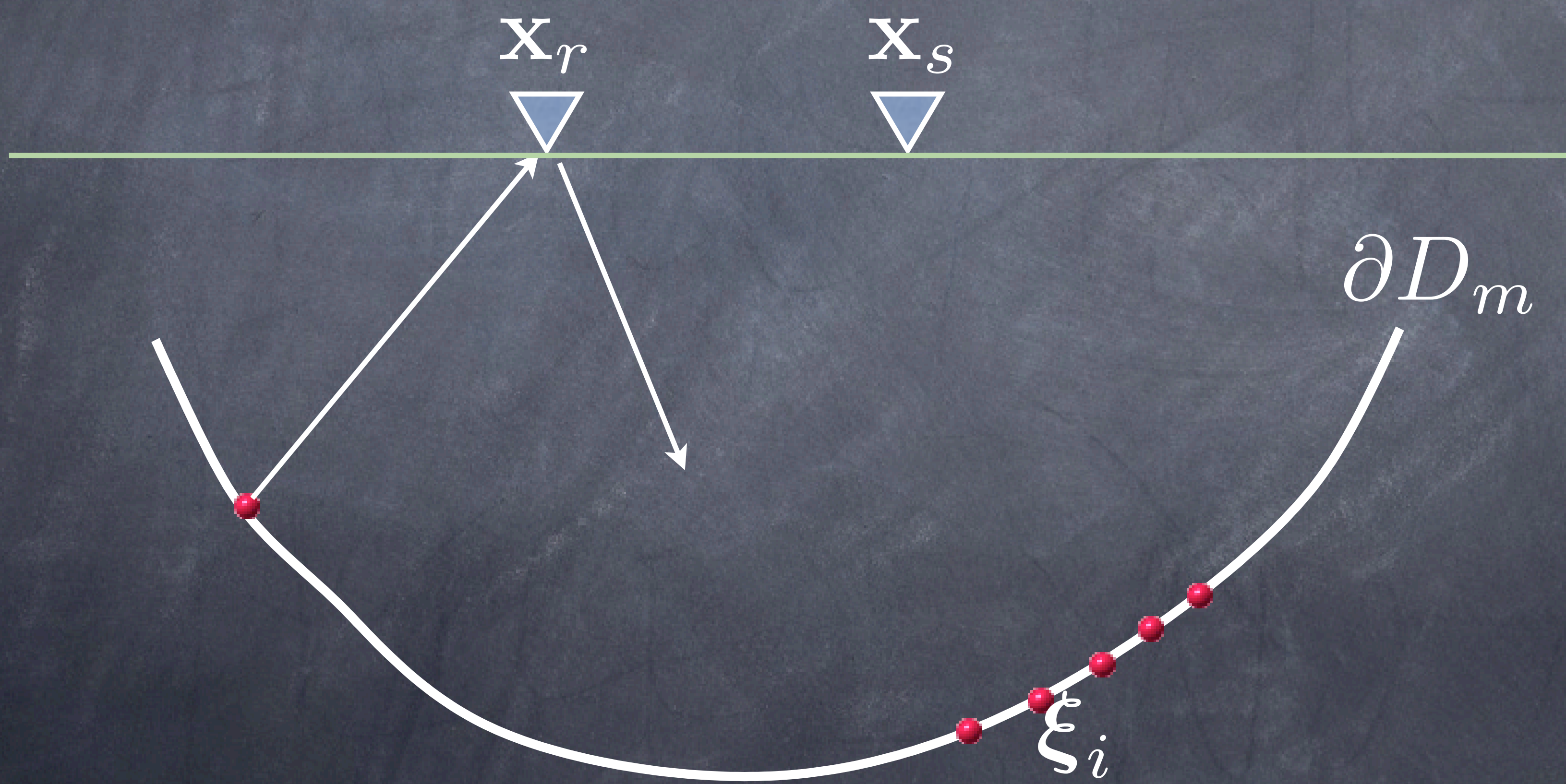
$$2\Re[R(\mathbf{x}_r, \mathbf{x}_s, \omega)] = \delta(\mathbf{x}_s - \mathbf{x}_r) - \int_{\partial D_m} T(\mathbf{x}_r, \boldsymbol{\xi}, \omega) T^*(\mathbf{x}_s, \boldsymbol{\xi}, \omega)\, \mathrm{d}^2\boldsymbol{\xi}$$



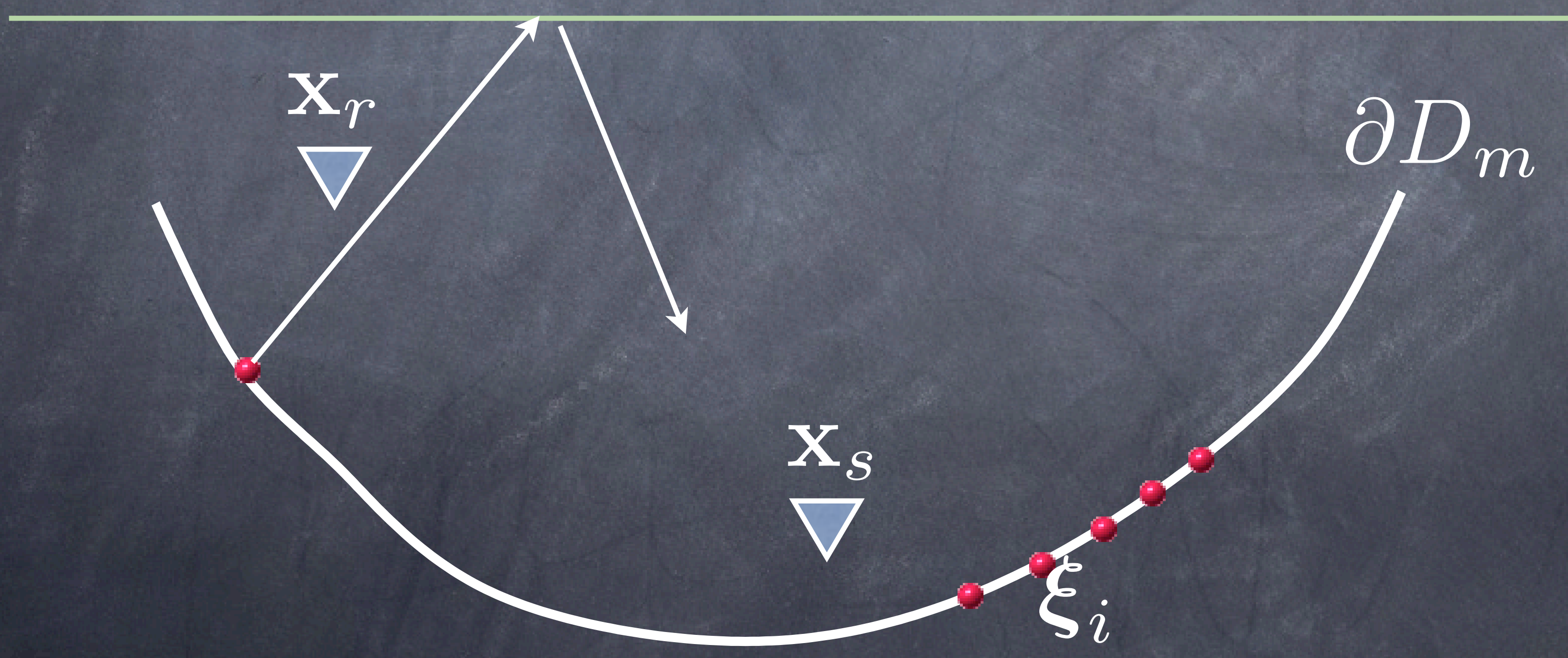
$$2\Re[R(\mathbf{x}_r, \mathbf{x}_s, \omega)] = \delta(\mathbf{x}_s - \mathbf{x}_r) - \int_{\partial D_m} T(\mathbf{x}_r, \boldsymbol{\xi}, \omega) T^*(\mathbf{x}_s, \boldsymbol{\xi}, \omega)\, \mathrm{d}^2\boldsymbol{\xi}$$



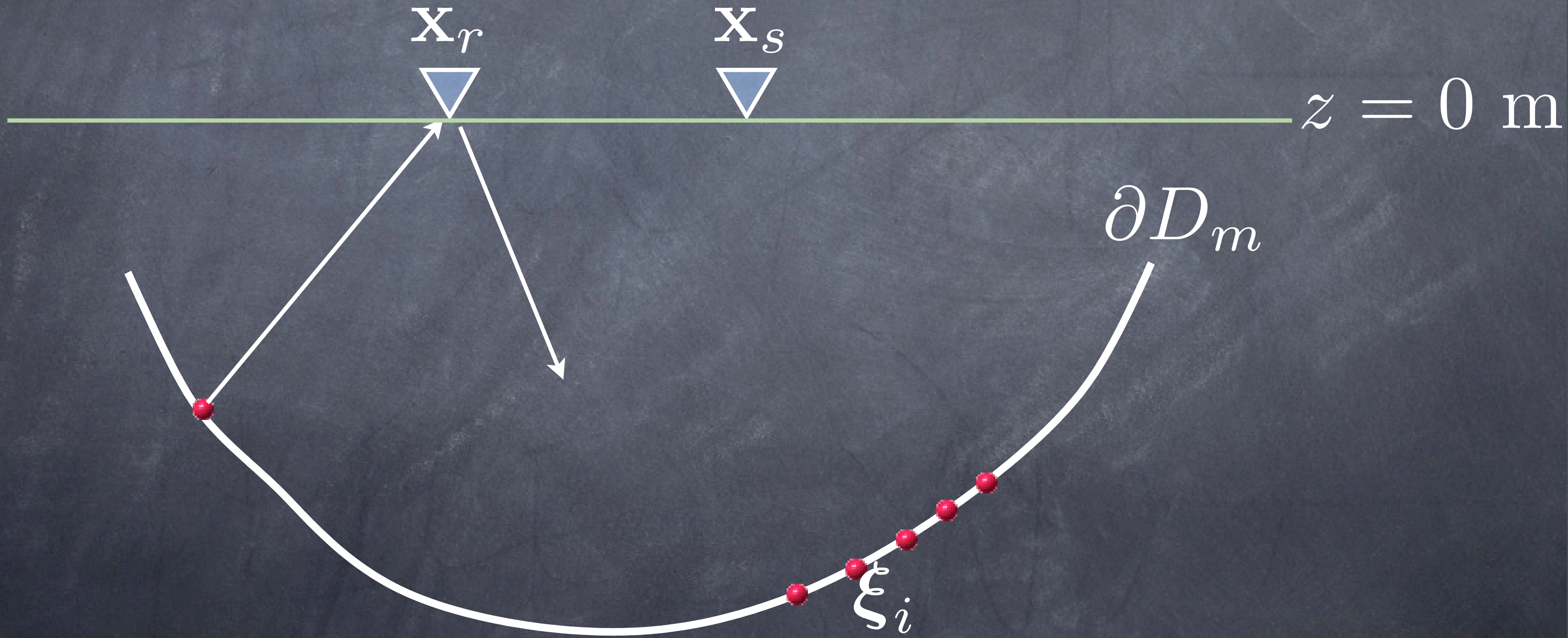
$$2\Re[R(\mathbf{x}_r, \mathbf{x}_s, \omega)] = \delta(\mathbf{x}_s - \mathbf{x}_r) - \int_{\partial D_m} T(\mathbf{x}_r, \boldsymbol{\xi}, \omega) T^*(\mathbf{x}_s, \boldsymbol{\xi}, \omega)\, \mathrm{d}^2\boldsymbol{\xi}$$



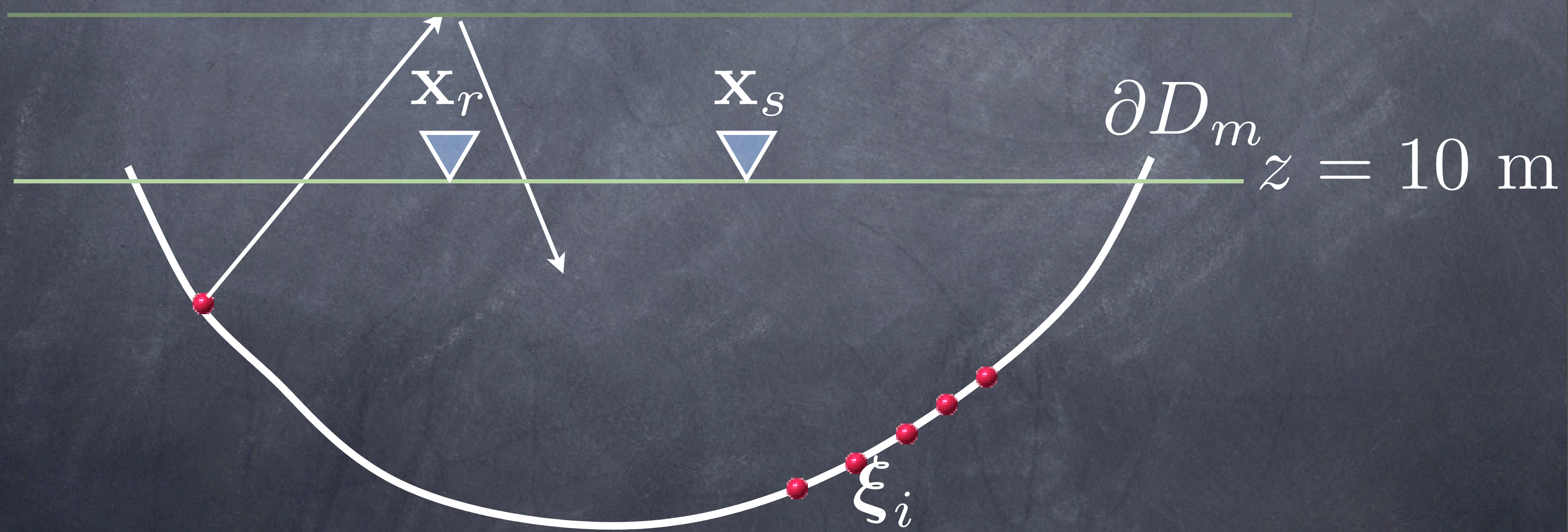
$$2\Re[R(\mathbf{x}_r, \mathbf{x}_s, \omega)] = \delta(\mathbf{x}_s - \mathbf{x}_r) - \int_{\partial D_m} T(\mathbf{x}_r, \boldsymbol{\xi}, \omega) T^*(\mathbf{x}_s, \boldsymbol{\xi}, \omega)\, \mathrm{d}^2\boldsymbol{\xi}$$



$$2\Re[R_0(\mathbf{x}_r, \mathbf{x}_s, \omega)] = \delta(\mathbf{x}_s - \mathbf{x}_r) - \int_{\partial D_m} \mathrm{e}^{+\phi_0} T(\mathbf{x}_r, \boldsymbol{\xi}, \omega) \mathrm{e}^{-\phi_0} T^*(\mathbf{x}_s, \boldsymbol{\xi}, \omega)\, \mathrm{d}^2\boldsymbol{\xi}$$



$$2\Re[R_{10}(\mathbf{x}_r, \mathbf{x}_s, \omega)] = \delta(\mathbf{x}_s - \mathbf{x}_r) - \int_{\partial D_m} \mathrm{e}^{+\phi_{10}} T(\mathbf{x}_r, \boldsymbol{\xi}, \omega) \mathrm{e}^{-\phi_{10}} T^*(\mathbf{x}_s, \boldsymbol{\xi}, \omega) \, \mathrm{d}^2\boldsymbol{\xi}$$



$$
\begin{array}{ccc}
T_{z=0}(\mathbf{x}_r;\xi,\omega) & \otimes & T_{z=0}(\mathbf{x}_r;\xi,\omega) \\
\Big| & & \Big| \\
\mathrm{e}^{-\phi_{10}} & & \mathrm{e}^{+\phi_{10}} \\
\downarrow & & \downarrow \\
T^{-}_{z=10}(\mathbf{x}_r;\xi,\omega) & \otimes & T^{+}_{z=10}(\mathbf{x}_r;\xi,\omega)
\end{array}
$$

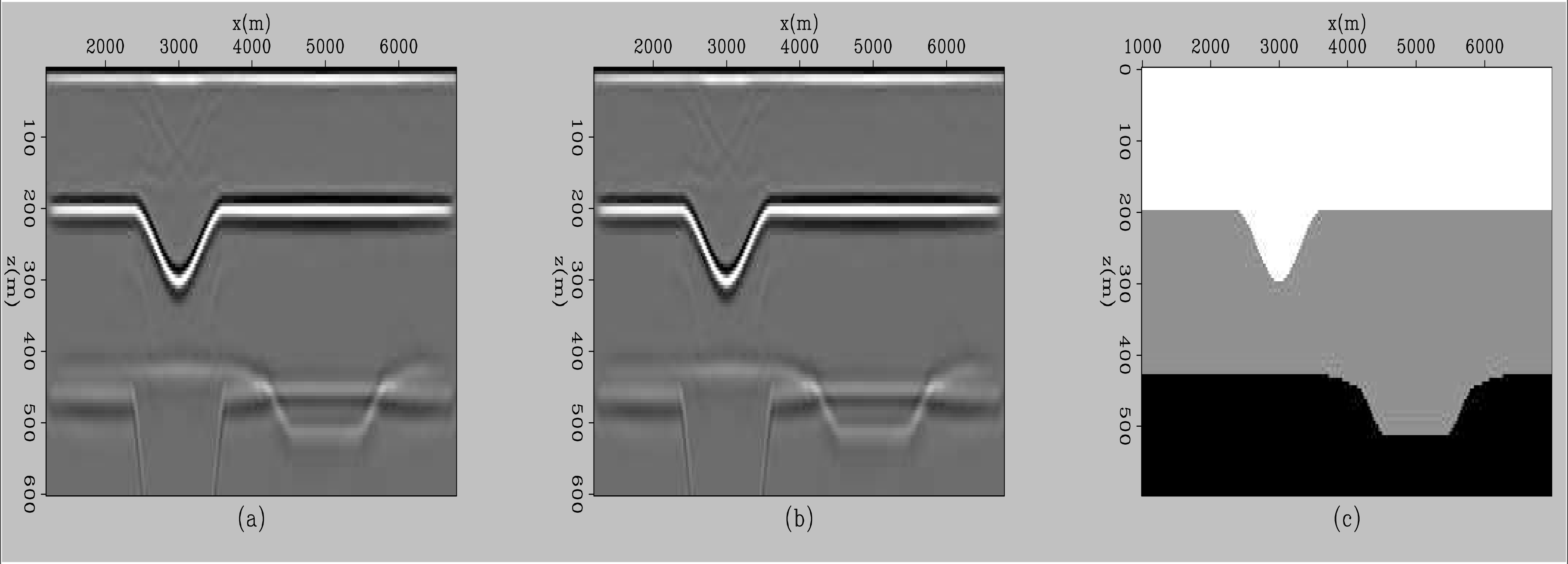
$$\underbrace{\sum_{\mathbf{x}_s}\sum_{\omega}\left[U_{z=0}(\mathbf{x}_r;\mathbf{x}_s,\omega)\otimes D_{z=0}(\mathbf{x}_r;\mathbf{x}_s,\omega)\right]}_{}\;=\;\sum_{\boldsymbol{\xi}}\sum_{\omega}\left[T_{z=0}(\mathbf{x}_r;\boldsymbol{\xi},\omega)\otimes T_{z=0}(\mathbf{x}_r;\boldsymbol{\xi},\omega)\right]$$

shot-profile migration | transmission imaging

$$\Big\downarrow \mathrm{e}^{-\phi} \qquad \Big\downarrow \mathrm{e}^{+\phi} \qquad\qquad \Big\downarrow \mathrm{e}^{-\phi} \qquad \Big\downarrow \mathrm{e}^{+\phi}$$

$$\sum_{\mathbf{x}_s}\sum_{\omega}\left[U_{z=1}(\mathbf{x}_r;\mathbf{x}_s,\omega)\otimes D_{z=1}(\mathbf{x}_r;\mathbf{x}_s,\omega)\right]=\sum_{\boldsymbol{\xi}}\sum_{\omega}\left[T^{-}_{z=1}(\mathbf{x}_r;\boldsymbol{\xi},\omega)\otimes T^{+}_{z=1}(\mathbf{x}_r;\boldsymbol{\xi},\omega)\right]$$

Direct migration

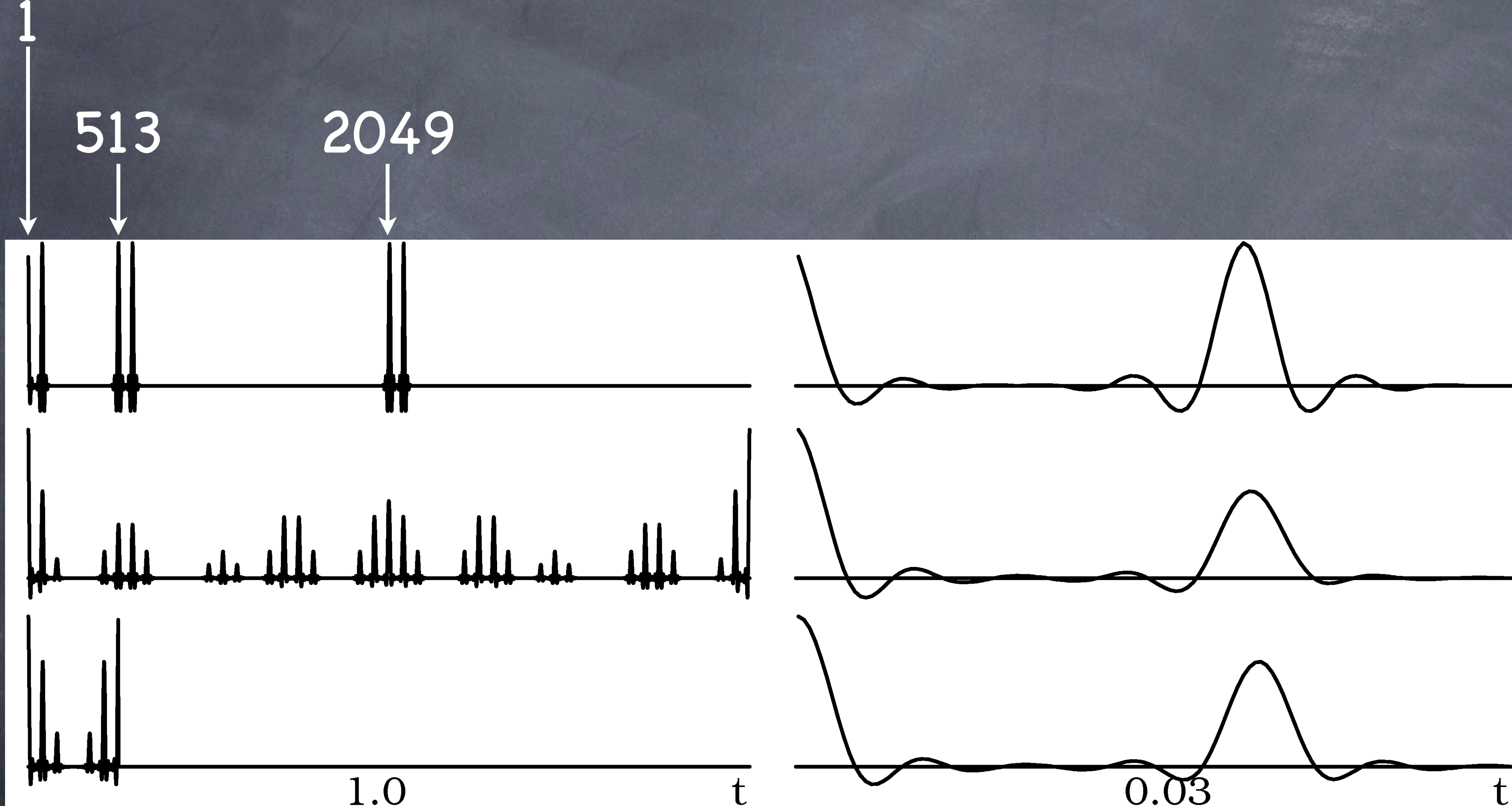


truly passive data

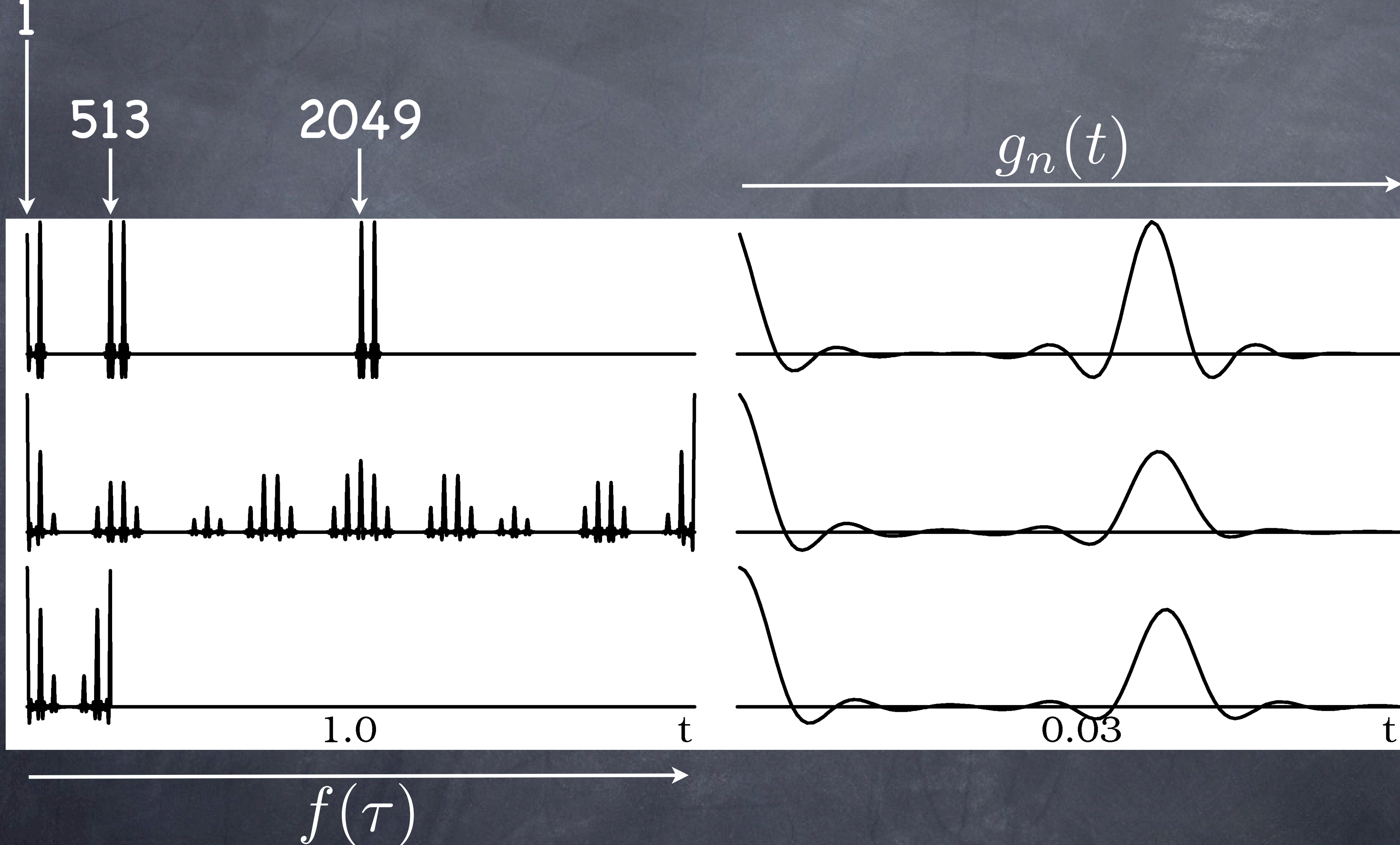
$$T_f(\mathbf{x}_r, \tau) \neq T(\mathbf{x}_r, \boldsymbol{\xi}, t)$$

$$T_f(\mathbf{x}_r, \tau) = T\left(\mathbf{x}_r, n_{\boldsymbol{\xi}} * \max(t) + ??\right)$$

$$T_f(\mathbf{x}_r, \tau) = ?$$



$$F(\omega_k) = \frac{1}{\sqrt{n_\tau}} \sum_{n=0}^{n_\tau - 1} f(n\Delta\tau) \mathrm{e}^{-in\Delta\tau \frac{k2\pi}{n_\tau \Delta\tau}}$$


1
513
2049
$g_n(t)$
1.0
t
0.03
t
$f(\tau)$

$$F(\omega_k) = \frac{1}{\sqrt{n_\tau}} \sum_{n=0}^{n_\tau - 1} f(n\Delta\tau) \mathrm{e}^{-in\Delta\tau \frac{k2\pi}{n_\tau \Delta\tau}}$$

$$= \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathrm{DFT}[g_n(t)]|_\omega$$

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$\varpi = \omega$

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$$= \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathrm{DFT}[g_n(t)]|_\omega$$

ΣG is subsampled F

$\varpi = \omega$

$$F(\omega_k) = \frac{1}{\sqrt{n_\tau}} \sum_{n=0}^{n_\tau - 1} f(n\Delta\tau) \mathrm{e}^{-in\Delta\tau \frac{k2\pi}{n_\tau \Delta\tau}}$$

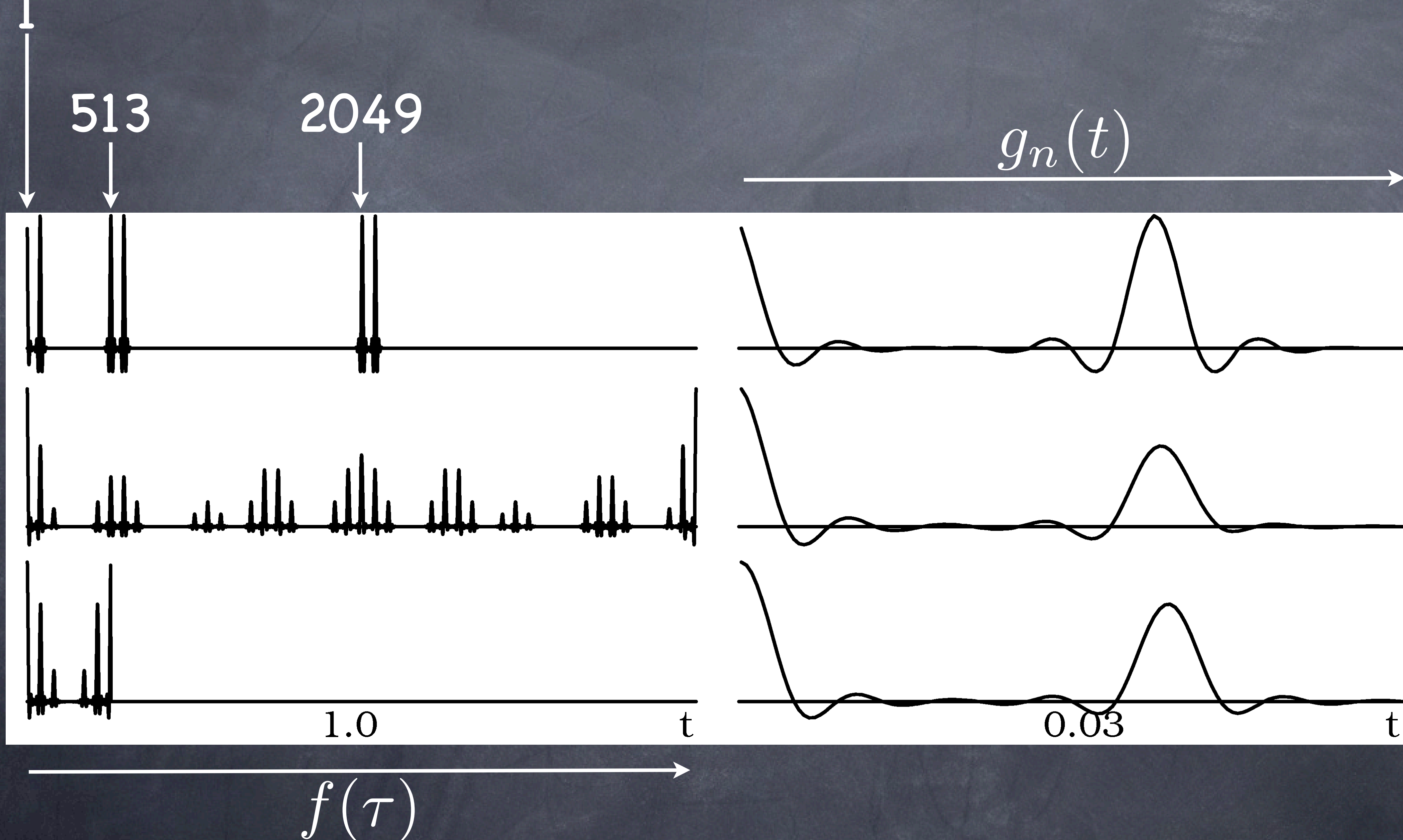
$$= \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathrm{DFT}[g_n(t)]|_\omega$$

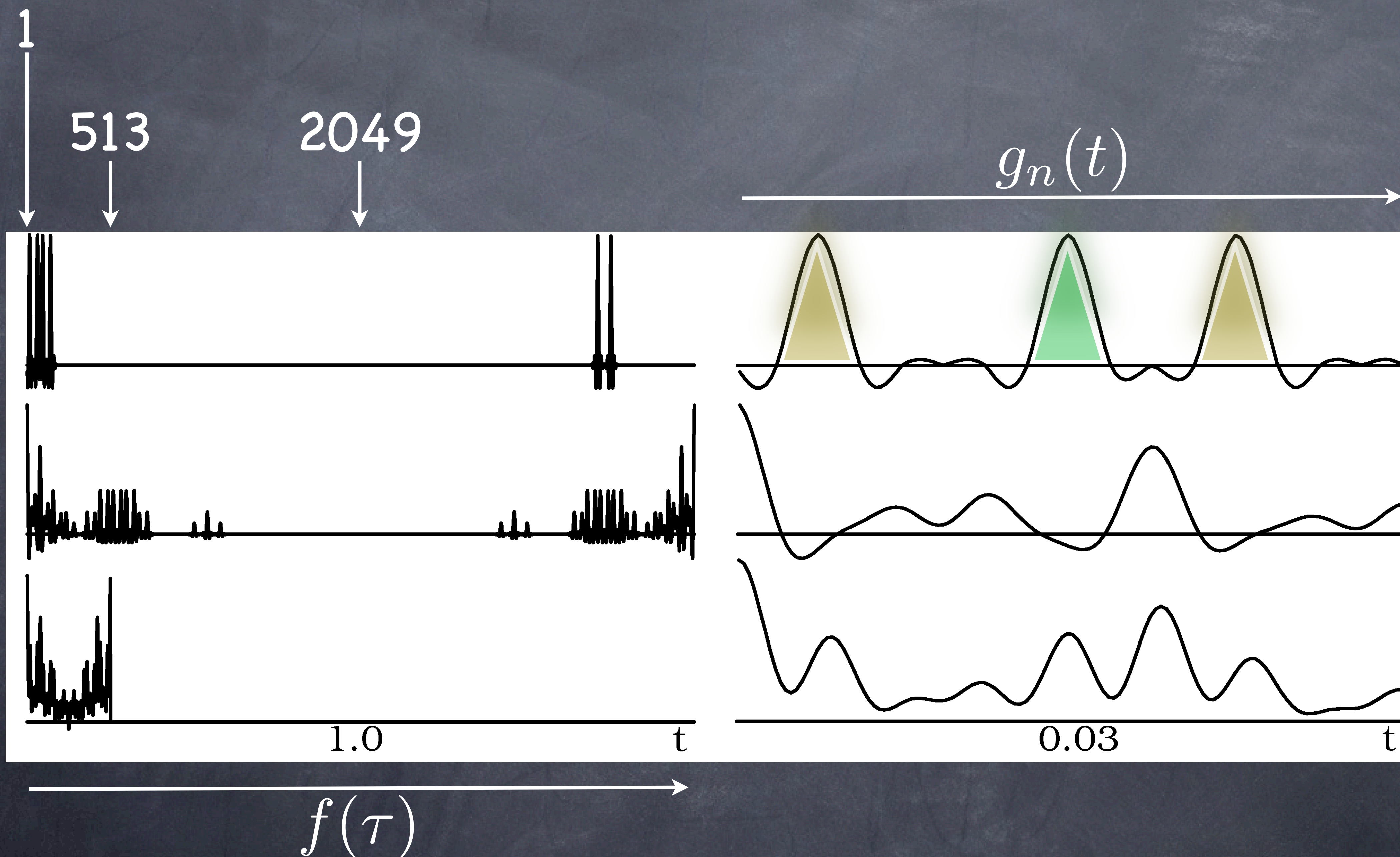
ΣG is subsampled F

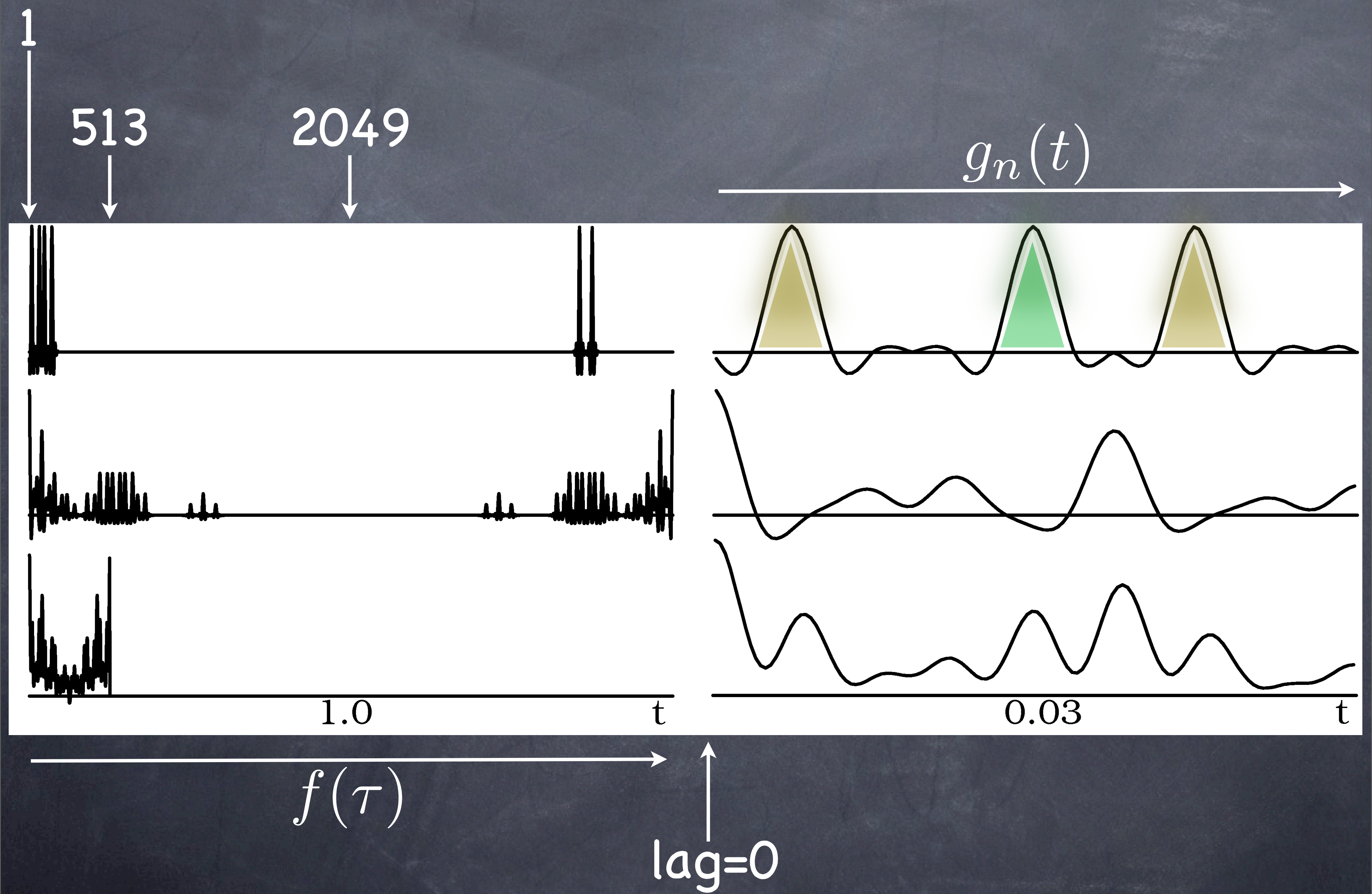
$$= \frac{1}{\sqrt{N}} \mathrm{DFT}[\sum_{n=1}^{N} g_n(t)]|_\omega$$

F is aliased g's

$$\varpi = \omega$$


1
513
2049
$g_n(t)$
1.0
t
0.03
t
$f(\tau)$

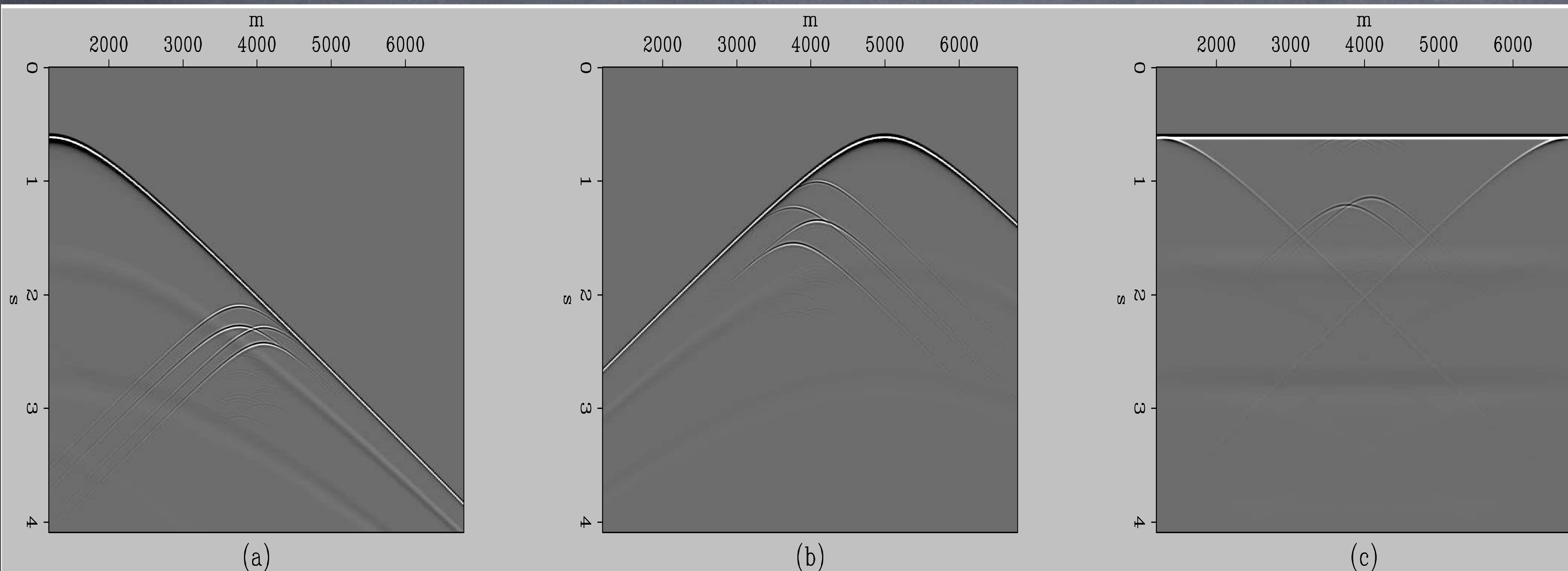

1
513
2049
$g_n(t)$
1.0
t
0.03
t
$f(\tau)$

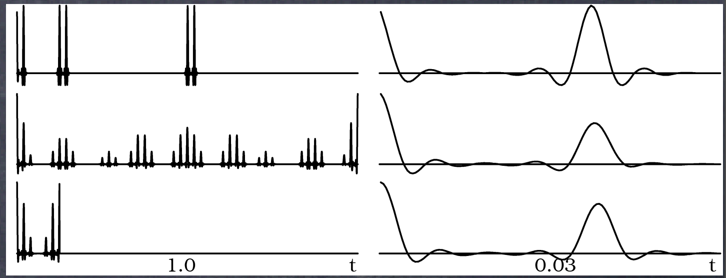


1
513
2049
$g_n(t)$
1.0
t
0.03
t
$f(\tau)$
lag=0

$$F|\varpi = \frac{1}{\sqrt{N}}\sum_{n=1}^{N} \mathrm{DFT}[g_n(t)]|_\omega$$

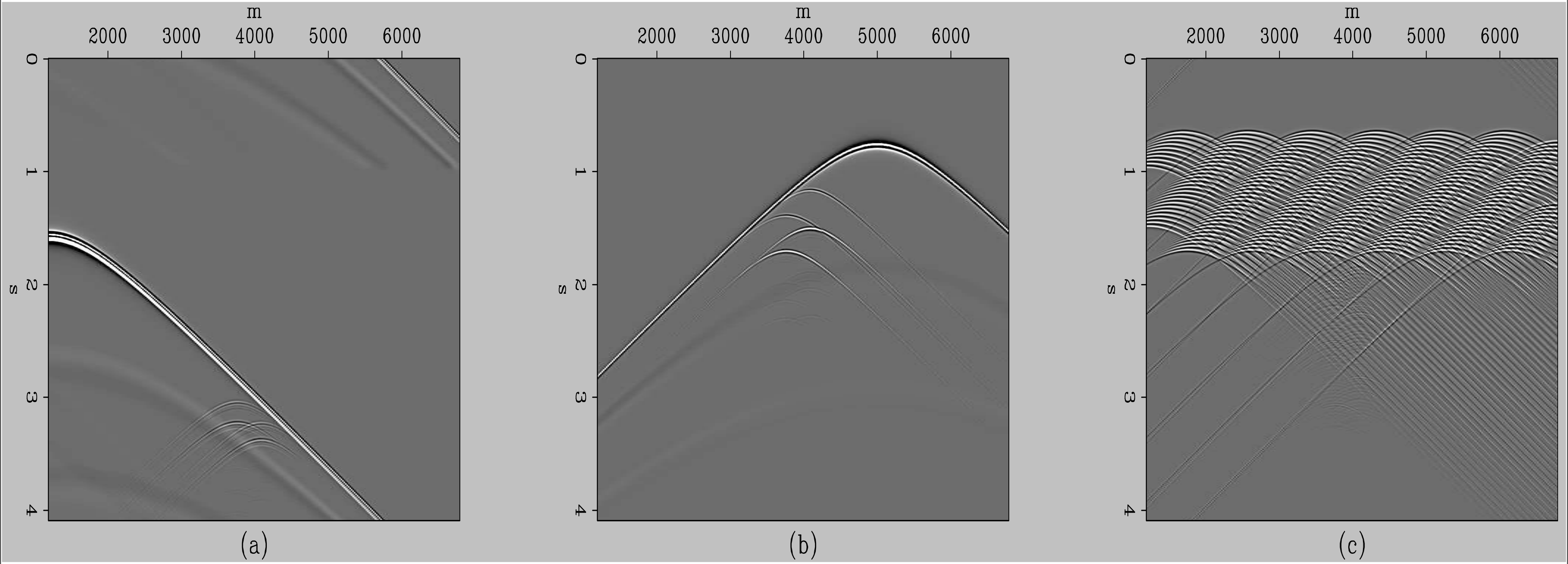
π(f) is aliased g's

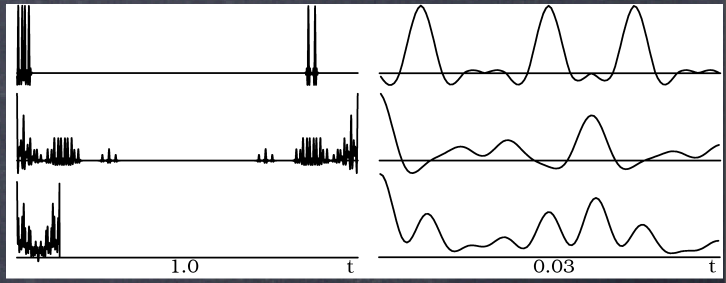




$$F|\varpi = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \mathrm{DFT}[g_n(t)]|_\omega$$

π(f) is aliased g's





correlating passive data

What is $T_f T_f^*$?

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$$\tilde{R} = T_f T_f^* \neq R$$

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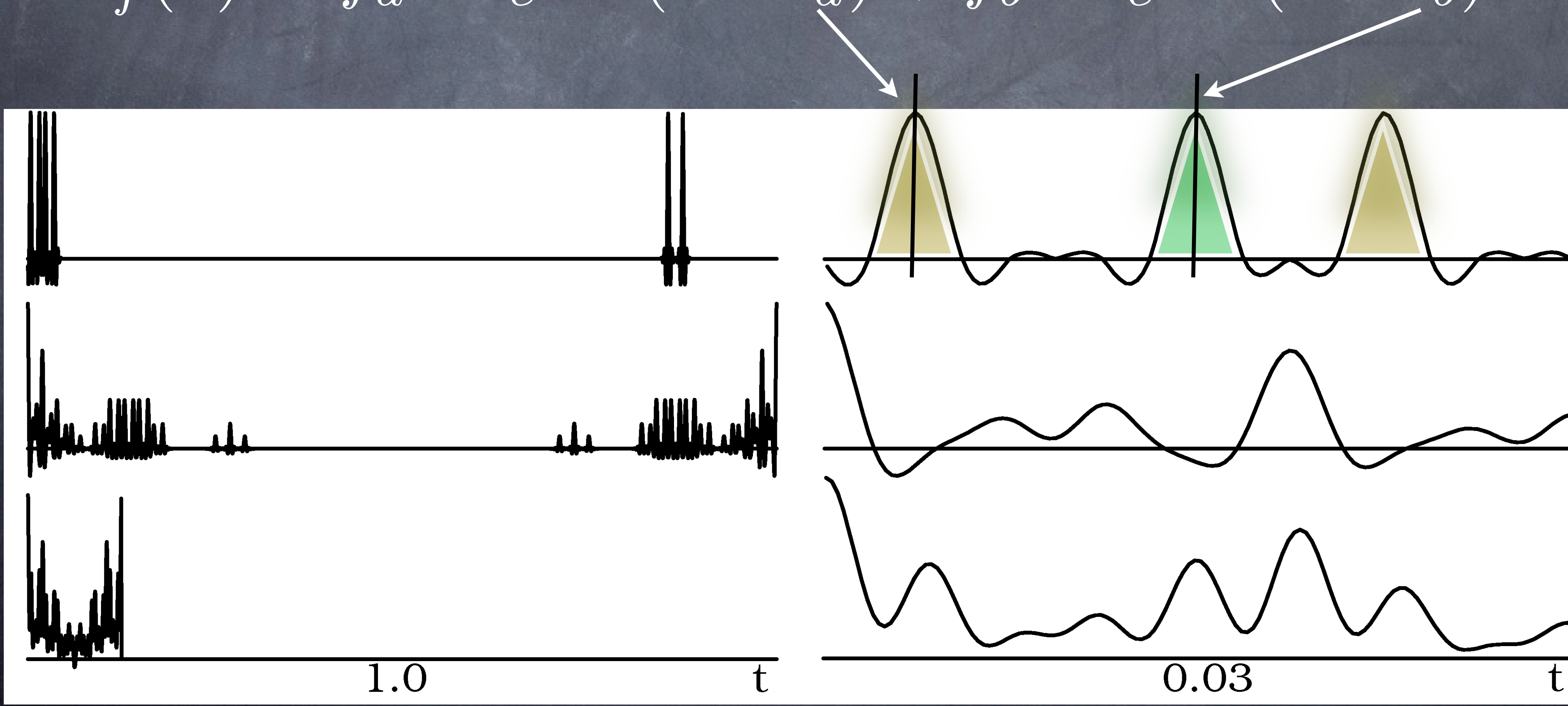
$$\tilde{R} = T_f T_f^* \neq R$$

$$T_f(\tau) = f_a * I_e * \delta(t - \tau_a) + f_b * I_e * \delta(t - \tau_b)$$

What is $T_f T_f^*$?

$$\tilde{R} = T_f T_f^* \neq R$$

$$T_f(\tau) = f_a * I_e * \delta(t - \tau_a) + f_b * I_e * \delta(t - \tau_b)$$



What is $T_f T_f^*$?

$$\tilde{R} = T_f T_f^* \neq R$$

$$T_f(\tau) = f_a * I_e * \delta(t-\tau_a) + f_b * I_e * \delta(t-\tau_b)$$

$$T_f T_f^* = 2I_e^2 + f^{a,b} I_e^2 \mathrm{e}^{-i\omega(\tau_a-\tau_b)} + f^{a,b} I_e^2 \mathrm{e}^{-i\omega(\tau_b-\tau_a)}$$

What is $T_f T_f^*$?

$$\tilde{R} = T_f T_f^* \neq R$$

$$T_f(\tau) = f_a * I_e * \delta(t - \tau_a) + f_b * I_e * \delta(t - \tau_b)$$

$$T_f T_f^* = 2I_e^2 + f^{a,b} I_e^2 \mathrm{e}^{-i\omega(\tau_a - \tau_b)} + f^{a,b} I_e^2 \mathrm{e}^{-i\omega(\tau_b - \tau_a)}$$

not zero-phase

non-impulsive
& miss-located

What is $T_f T_f^*$?

$$\tilde{R} = T_f T_f^* \neq R$$

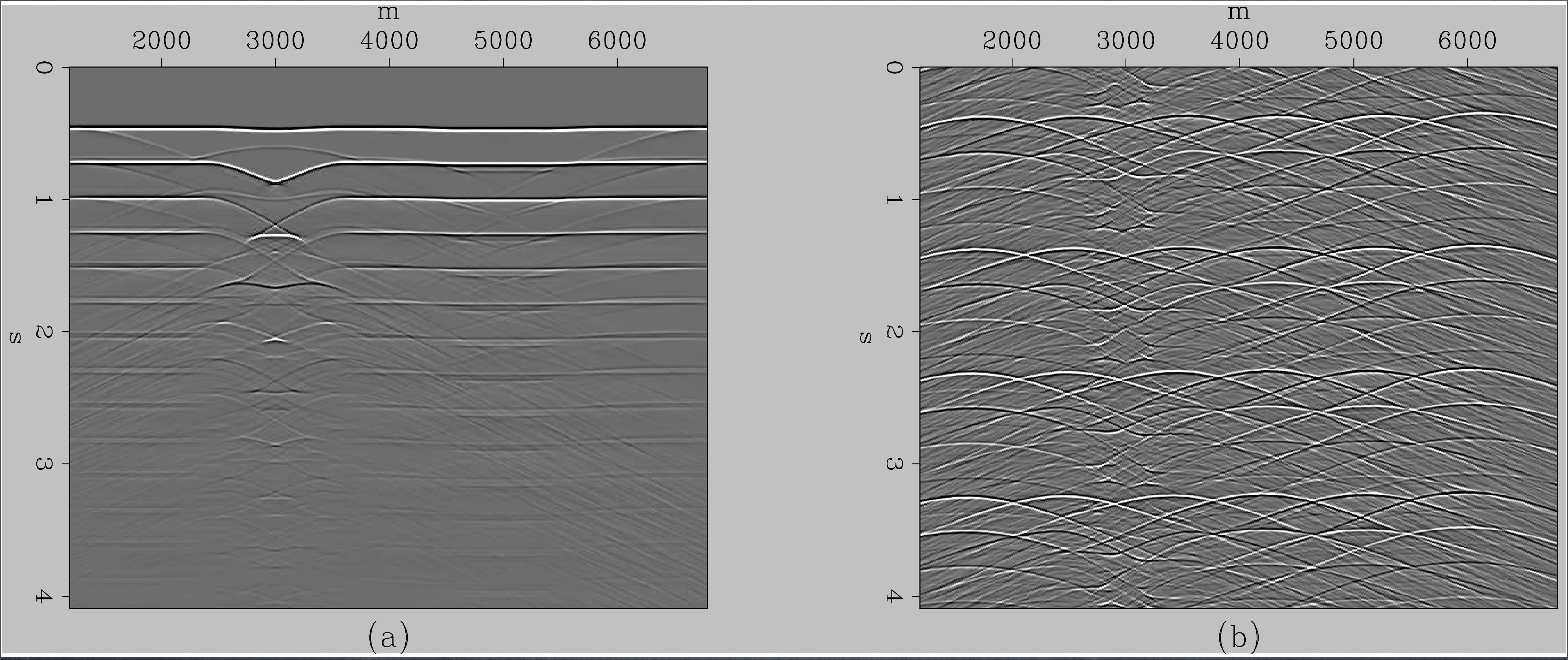
$$T_f(\tau) = f_a * I_e * \delta(t - \tau_a) + f_b * I_e * \delta(t - \tau_b)$$

$$T_f T_f^* = 2I_e^2 + f^{a,b} I_e^2 \mathrm{e}^{-i\omega(\tau_a - \tau_b)} + f^{a,b} I_e^2 \mathrm{e}^{-i\omega(\tau_b - \tau_a)}$$

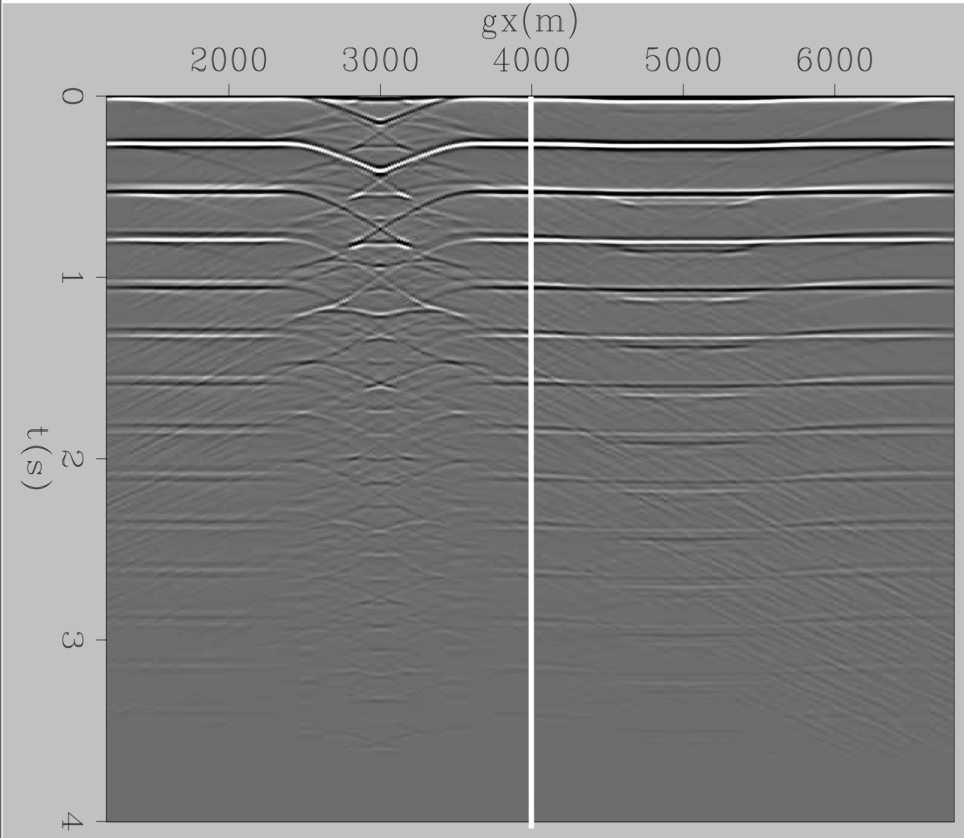
Random

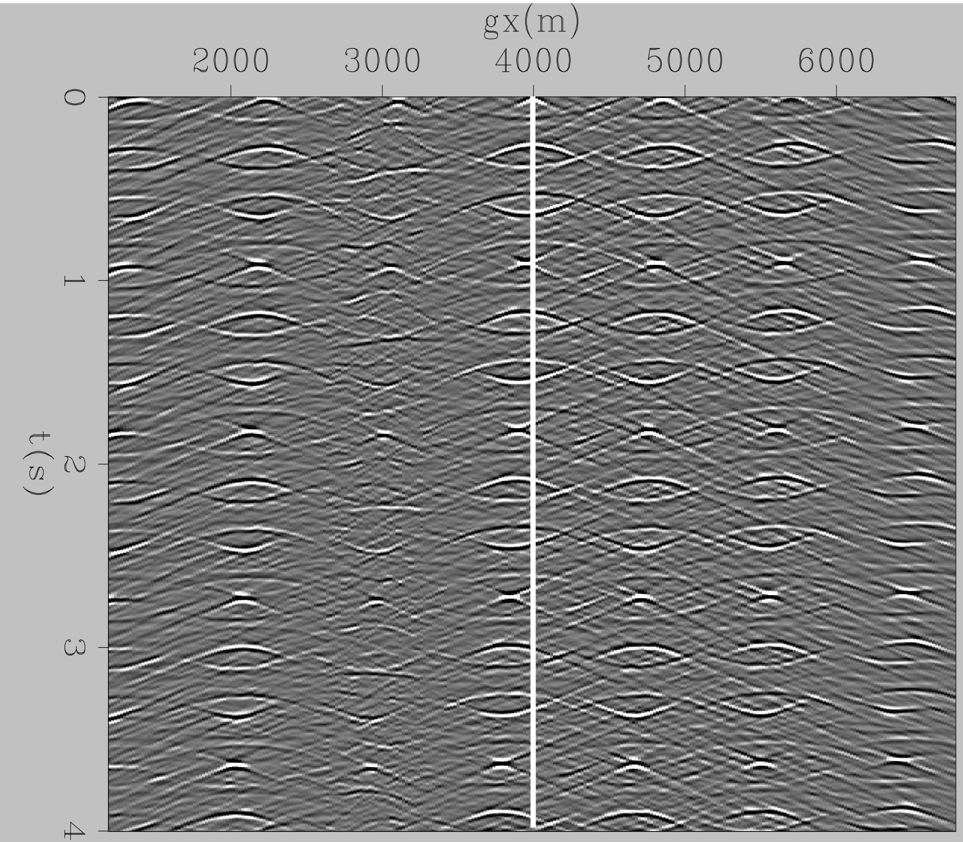
migrating passive data

multi-source wavefields

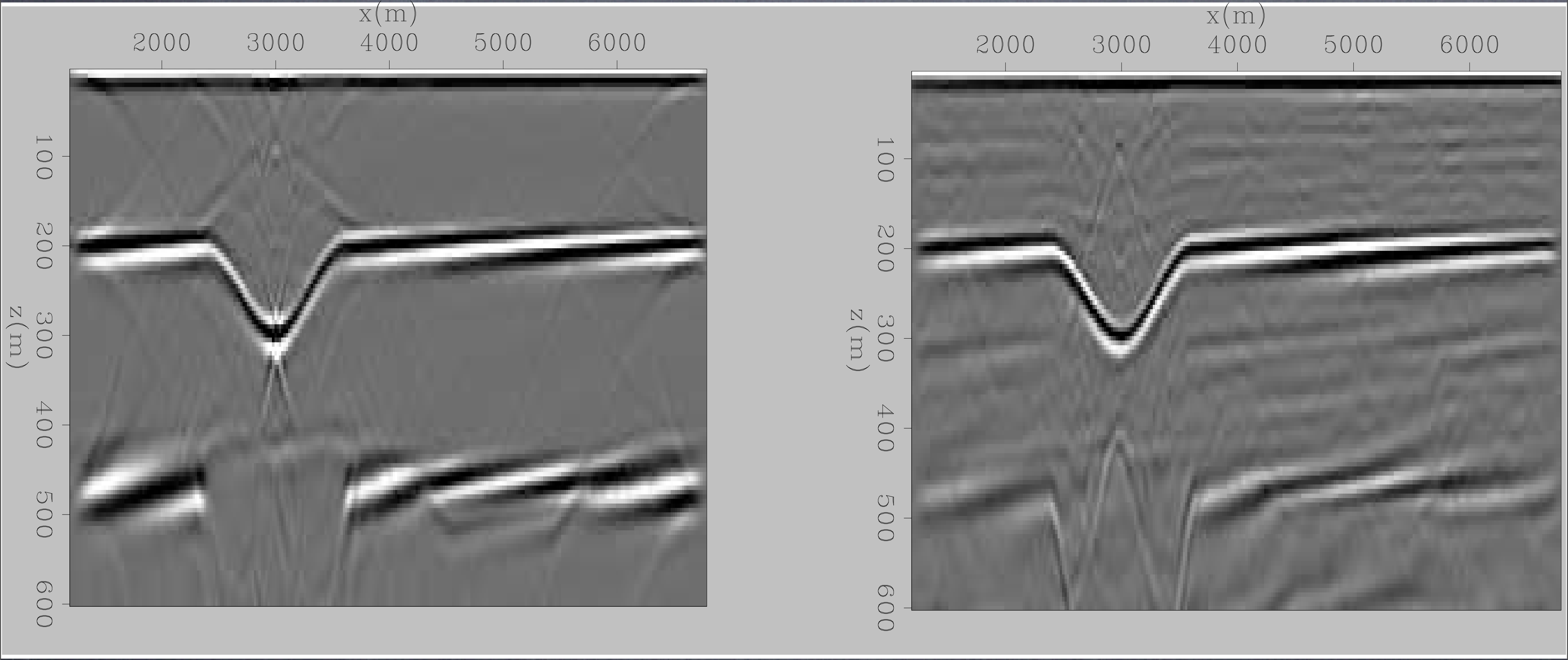


synthesized shotgathers

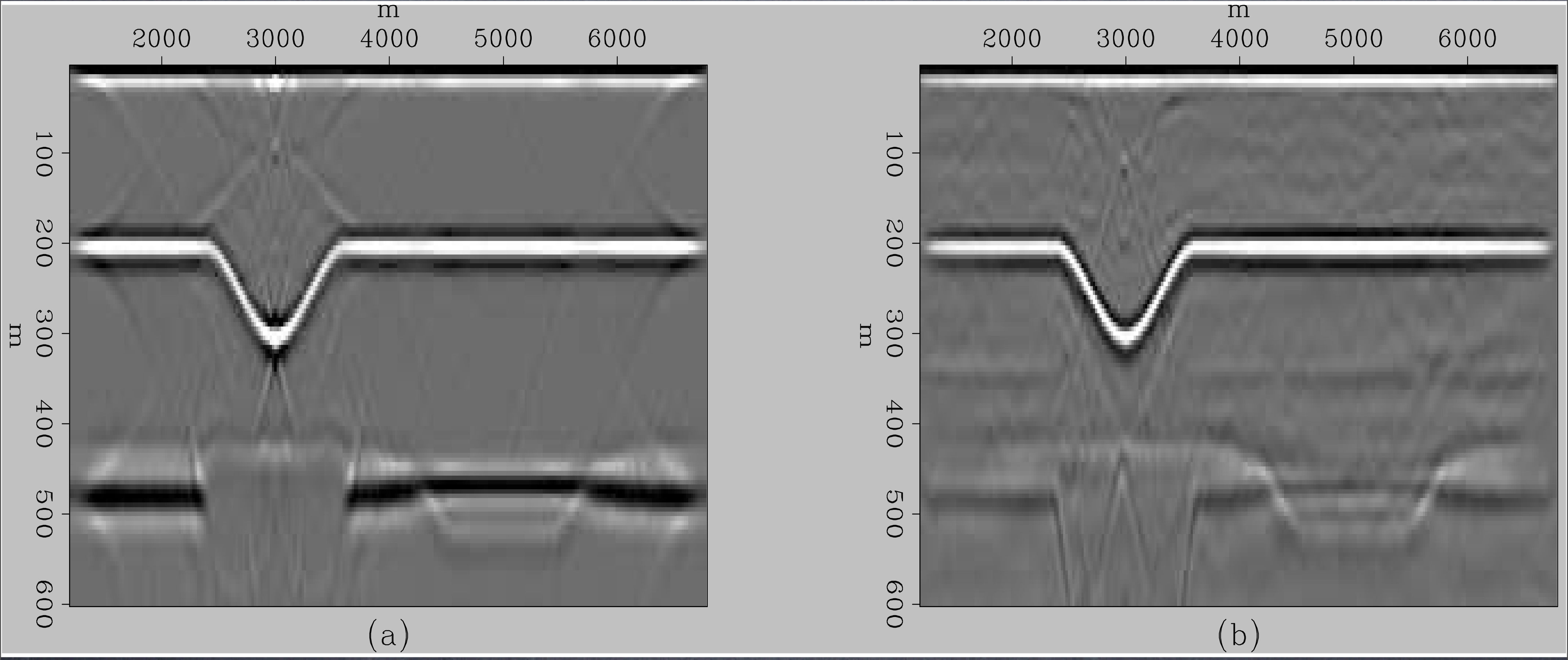




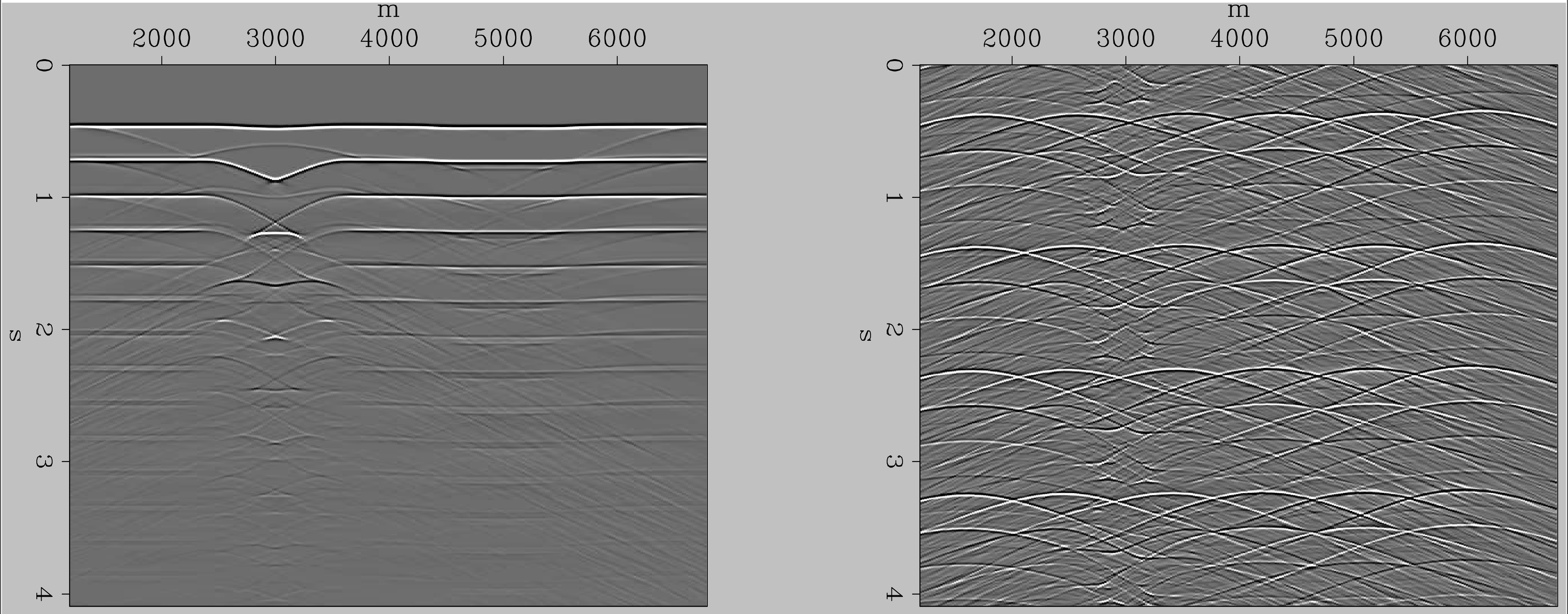
Xcorr, migrate

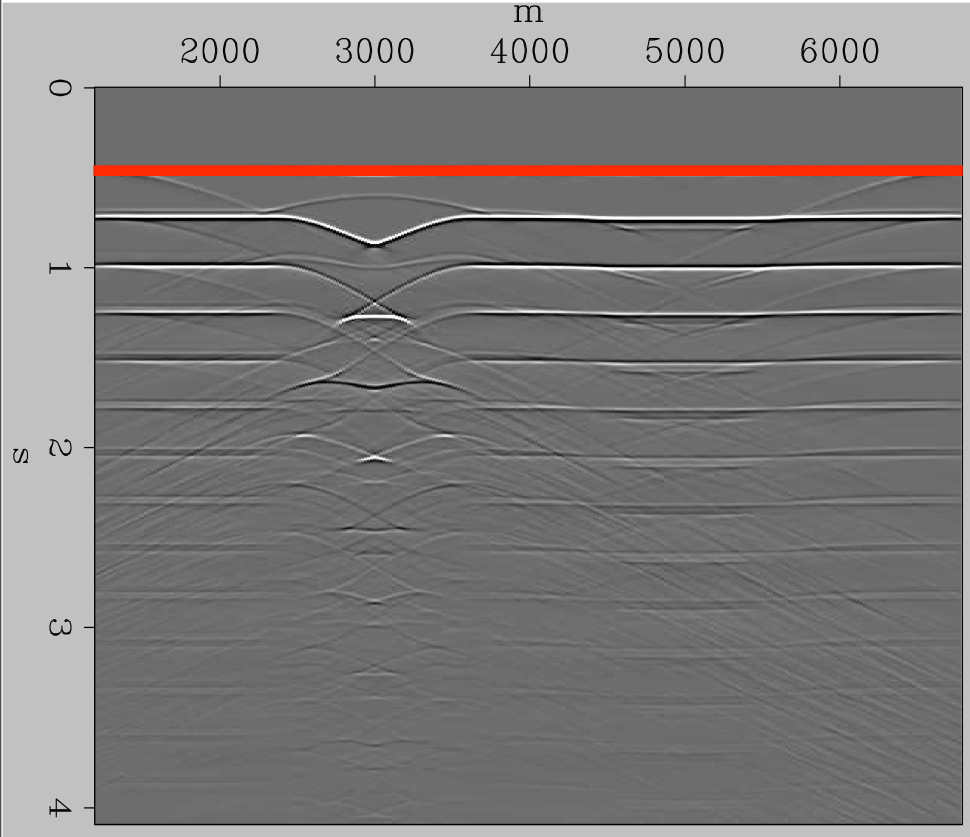


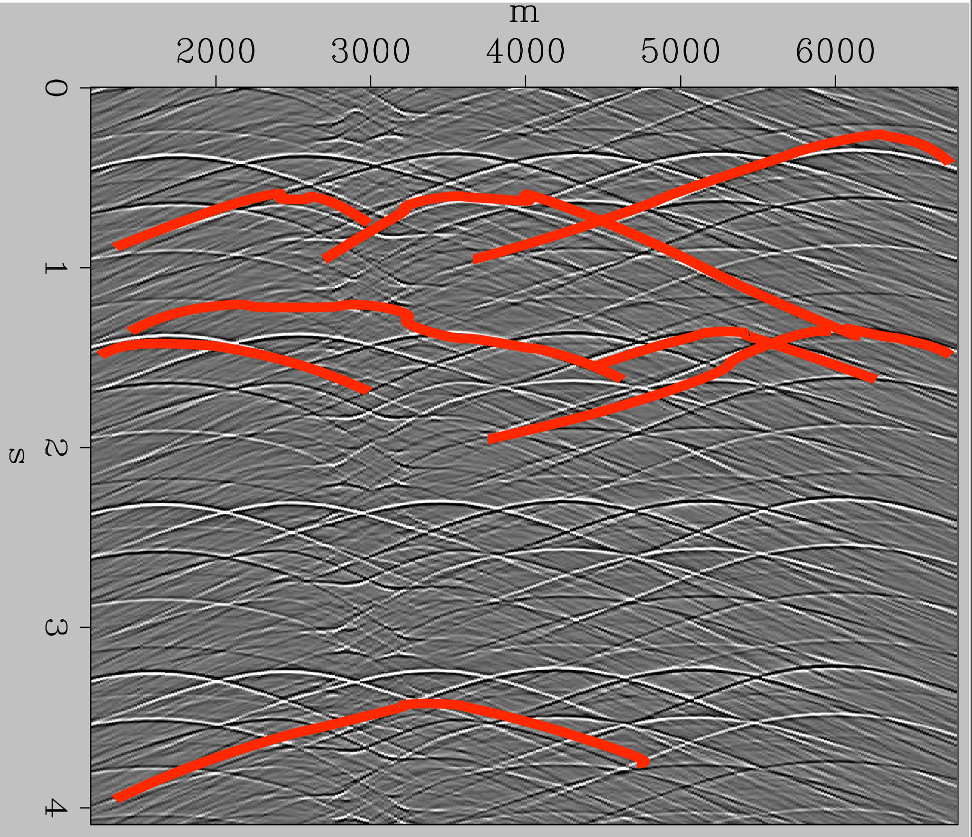
direct migration

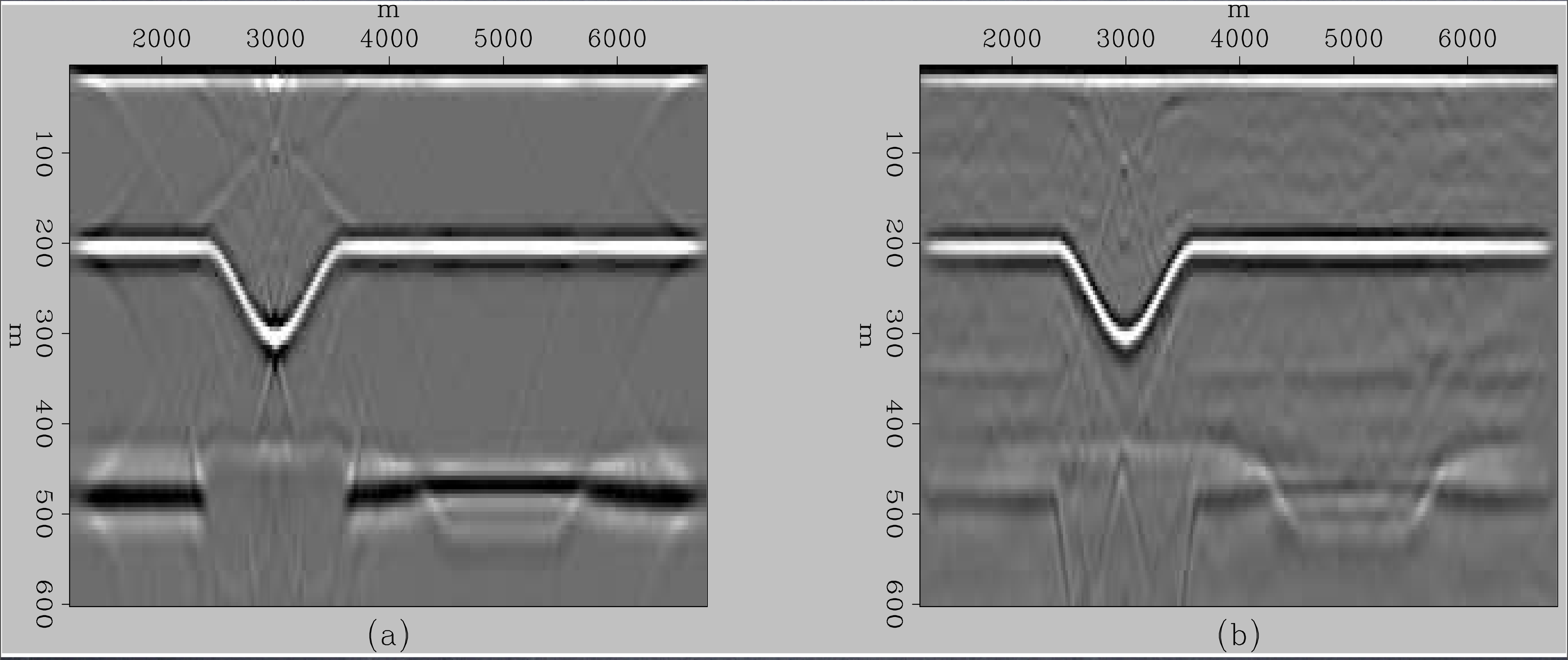


$$
\begin{array}{ccccccc}
\multicolumn{3}{c}{\text{plane-wave migration}} & & \multicolumn{3}{c}{\text{wavefront imaging}} \\
\hline
\sum_{\mathbf{x}_s} U_{z=0}(\mathbf{x}_r, \mathbf{x}_s, \omega) & \otimes & \sum_{\mathbf{x}_s} D_{z=0}(\mathbf{x}_r, \mathbf{x}_s, \omega) & = & T_{z=0}(\mathbf{x}_r, \omega) & \otimes & T_{z=0}(\mathbf{x}_r, \omega) \\
| & & | & & | & & | \\
\mathrm{e}^{-\phi} & & \mathrm{e}^{+\phi} & & \mathrm{e}^{-\phi} & & \mathrm{e}^{+\phi} \\
\downarrow & & \downarrow & & \downarrow & & \downarrow \\
U_{z=1}(\mathbf{x}_r, \omega) & \otimes & D_{z=1}(\mathbf{x}_r, \omega) & = & T^{-}_{z=1}(\mathbf{x}_r, \omega) & \otimes & T^{+}_{z=1}(\mathbf{x}_r, \omega)
\end{array}
$$

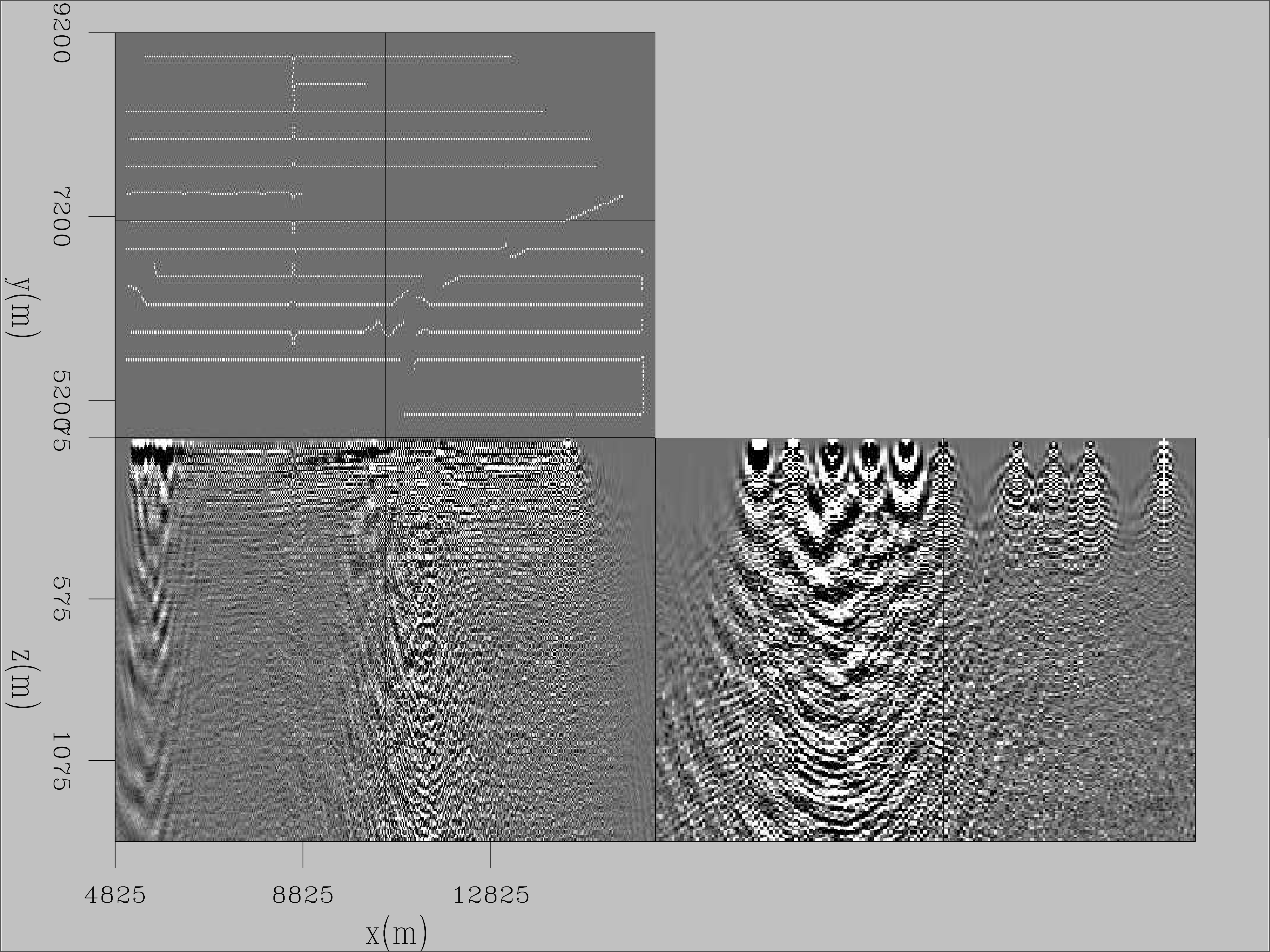

m
2000
3000
4000
5000
6000
0
1
2
3
4
s
m
2000
3000
4000
5000
6000
0
1
2
3
4
s
(a) (b)


m
2000
3000
4000
5000
6000
0
1
2
3
4
s
(a)


m
2000
3000
4000
5000
6000
0
1
2
3
4
s
(b)


m
2000
3000
4000
5000
6000
100
200
300
400
500
600
m
(a)
m
2000
3000
4000
5000
6000
100
200
300
400
500
600
m
(b)

- Transmission wavefields are the superposition of up- and down-going wavefields.
- The free surface is unimportant for processing transmission wavefields.
- Extrapolation operators care not for the peculiarities of a wavefield's initial conditions.
- Direct migration of transmission wavefields = datuming followed by imaging.
- Passive data = long records with undifferentiable source functions.
- Direct migration is the only passive data processing scheme that does not require random source functions.



9200
7200
5200
y(m)
75
575
1075
z(m)
4825
8825
12825
x(m)