Optimized Implicit Finite-Difference Migration for VTI Media

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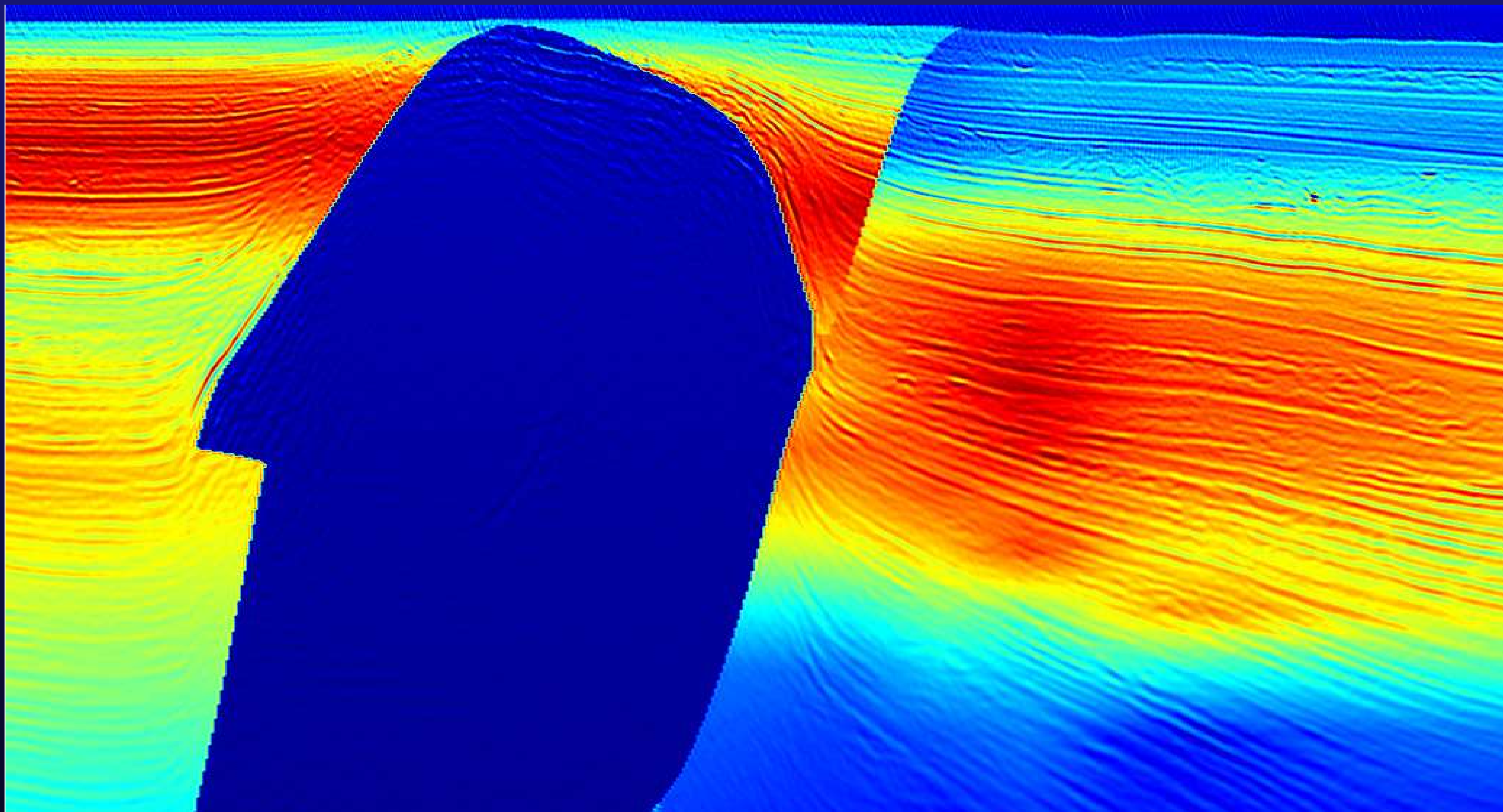
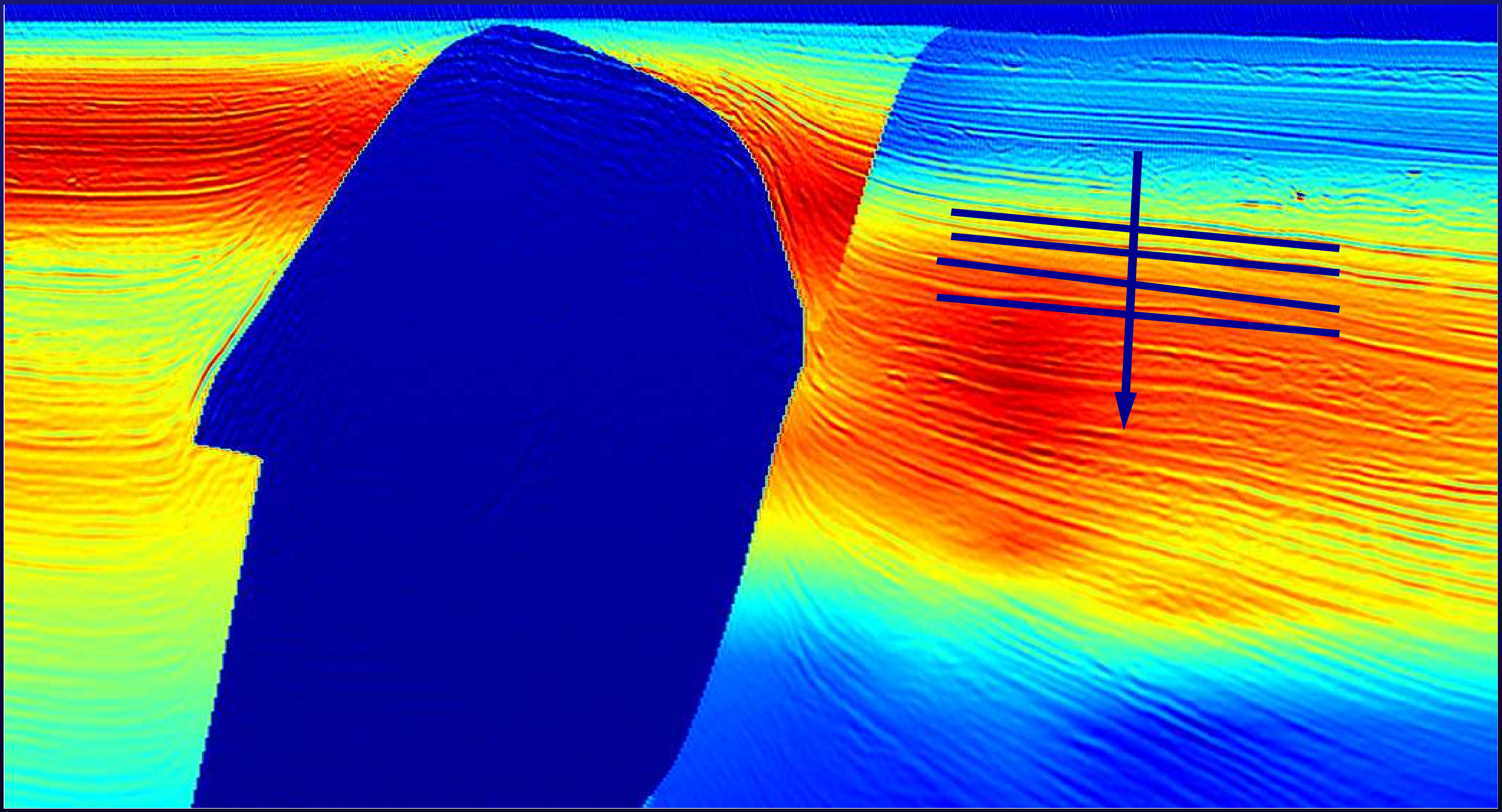
SEP124: pages 279-292

VTI media



VTI media



Dispersion relation for VTI media is complex

$$a_4 k_z^4 + a_2 k_z^2 + a_0 = 0$$

$$\begin{aligned}
a_4 &= (f-1), \\
a_2 &= 2[(f-1)(1+\varepsilon) - f(\varepsilon-\delta)](k_x^2+k_y^2) + \left(\frac{\omega}{v_{p0}}\right)^2 (2-f), \\
a_0 &= [(f-1)(1+2\varepsilon)](k_x^2+k_y^2)^2 + \left(\frac{\omega}{v_{p0}}\right)^2 (2-f+2\varepsilon)(k_x^2+k_y^2) - \frac{\omega^4}{v_{p0}^4}.
\end{aligned}$$

- ε and δ: Thomsen anisotropy parameters
- $f = 1 - \frac{v_s}{v_p}$
- v_{p0}: vertical Quasi-P wave velocity

Dispersion relation for VTI media is complex

If $V_s << V_p$ ($f \approx 1$), then k_z can be solved analytically as follows:

$$k_z \approx \frac{\omega}{v}\sqrt{\frac{1-(1+2\varepsilon)\frac{v^2}{\omega^2}k_x^2}{1-2(\varepsilon-\delta)\frac{v^2}{\omega^2}k_x^2}}.$$

Outline

- Implicit migration methods for isotropic media
- Optimized one-way wave-equation operator VTI
- Table-driven implicit finite-difference method
- Numerical examples

Dispersion relation for isotropic media

$$k_z^2 + k_x^2 = \frac{\omega^2}{v^2}$$

k_z can be solved analytically as follows:

$$\frac{k_z}{\omega/v} = \sqrt{1 - \left(\frac{k_z}{\omega/v}\right)^2}$$

Approximation by Taylor series analysis

$$\sqrt{1-S^2} = 1 - \frac{1}{2}S^2 - \frac{1}{8}S^4 + \cdots$$

15° equation:

$$\frac{k_z}{\omega/v} \approx 1 - \frac{1}{2}\left(\frac{k_x}{\omega/v}\right)^2$$

45° equation:

$$\frac{k_z}{\omega/v} \approx 1 - \frac{1}{2}\left(\frac{k_x}{\omega/v}\right)^2 - \frac{1}{8}\left(\frac{k_x}{\omega/v}\right)^4 \approx 1 + \frac{-\frac{1}{2}\frac{k_x^2}{(\omega/v)^2}}{1 - \frac{1}{4}\frac{k_x^2}{(\omega/v)^2}}$$

Padé approximation: function fitting

If the function $S_z(S_r) \in C^{n+m}$, then $S_z(S_r)$ can be approximated by a rational function

$$S_z(S_r) = \frac{\sum_{i=1}^{n} a_i S_r^i}{1 + \sum_{i=1}^{n} b_i S_r^i}$$

a_i and b_i can be solved by the Least-squares optimization:

$$\min \int_0^{\sin(\theta)} \left(\sum_{i=1}^{n} a_i (S_r)^i - S_z(S_r)(1 + \sum_{i=1}^{n} b_i (S_r)^i) \right)^2 dS_r.$$

Approximation for square-root operator

Lee and Suh (1985) suggests:

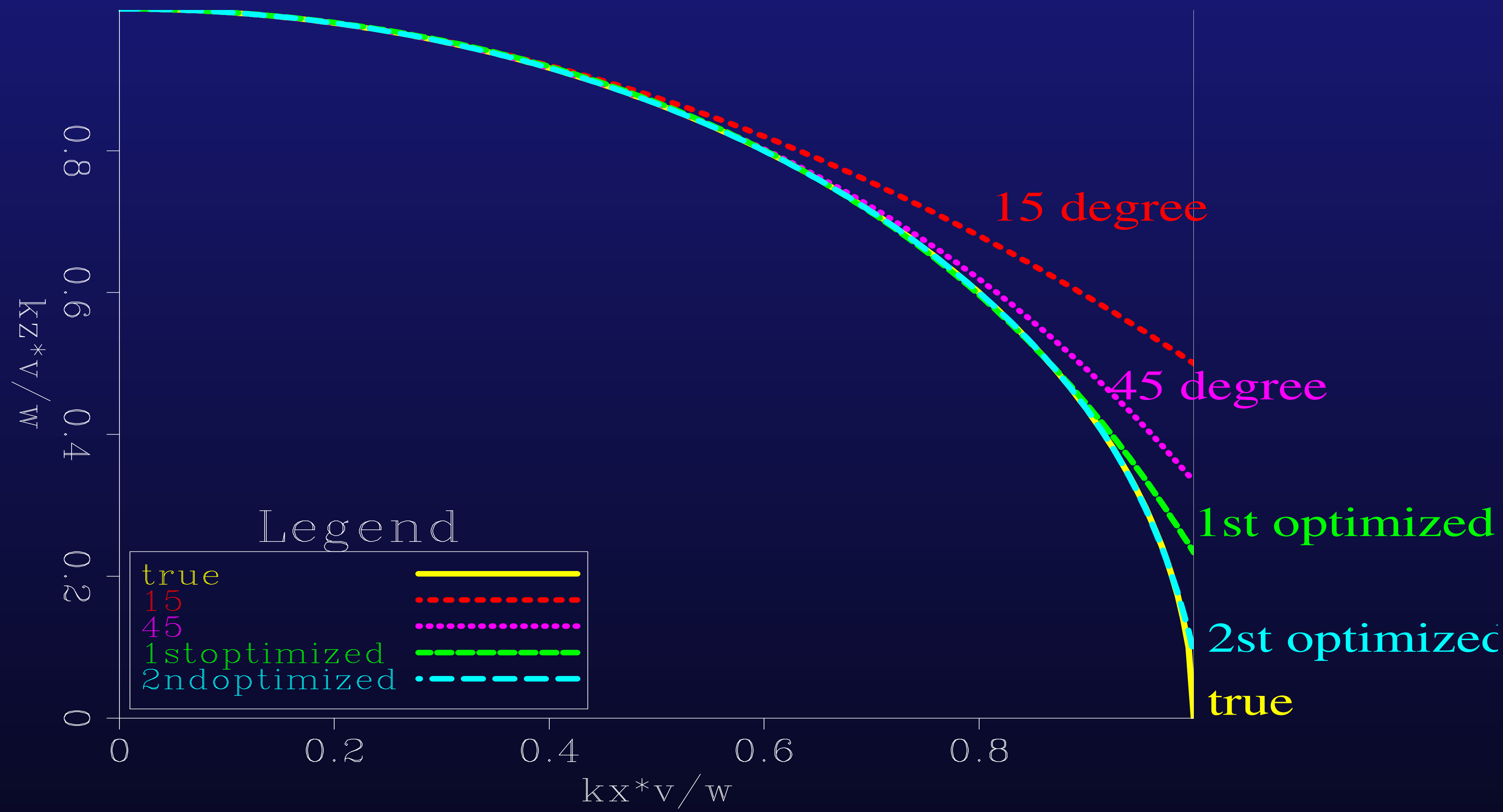
The first order:

$$\sqrt{1-S_r^2} \approx 1 - \frac{0.4782 S_r^2}{1 - 0.3764 S_r^2}$$

The second order:

$$\sqrt{1-S_r^2} \approx 1 - \frac{0.4976 S_r^2 - 0.4086 S_r^4}{1 - 1.0967 S_r^2 + 0.1946 S_r^4}$$

Comparison of dispersion relation



Advantage of LS optimization

- More accurate scheme with the same computation cost.
- An analytical expression between k_z and k_x is not required.

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Taylor series analysis for VTI media

Under the weak anisotropy assumption, Ristow and Ruhl (1991) obtained the approximation as follow:

$$\frac{k_z}{\omega/v_p} \approx 1 - \frac{\alpha_1 \frac{k_x^2}{(\omega/v_p)^2}}{1 - \beta_1 \frac{k_x^2}{(\omega/v_p)^2}}$$

$$\alpha_1 = 0.5(1+2\delta) \qquad \beta_1 = \frac{2(\varepsilon - \delta)}{1+2\delta} + 0.25(1+2\delta)$$

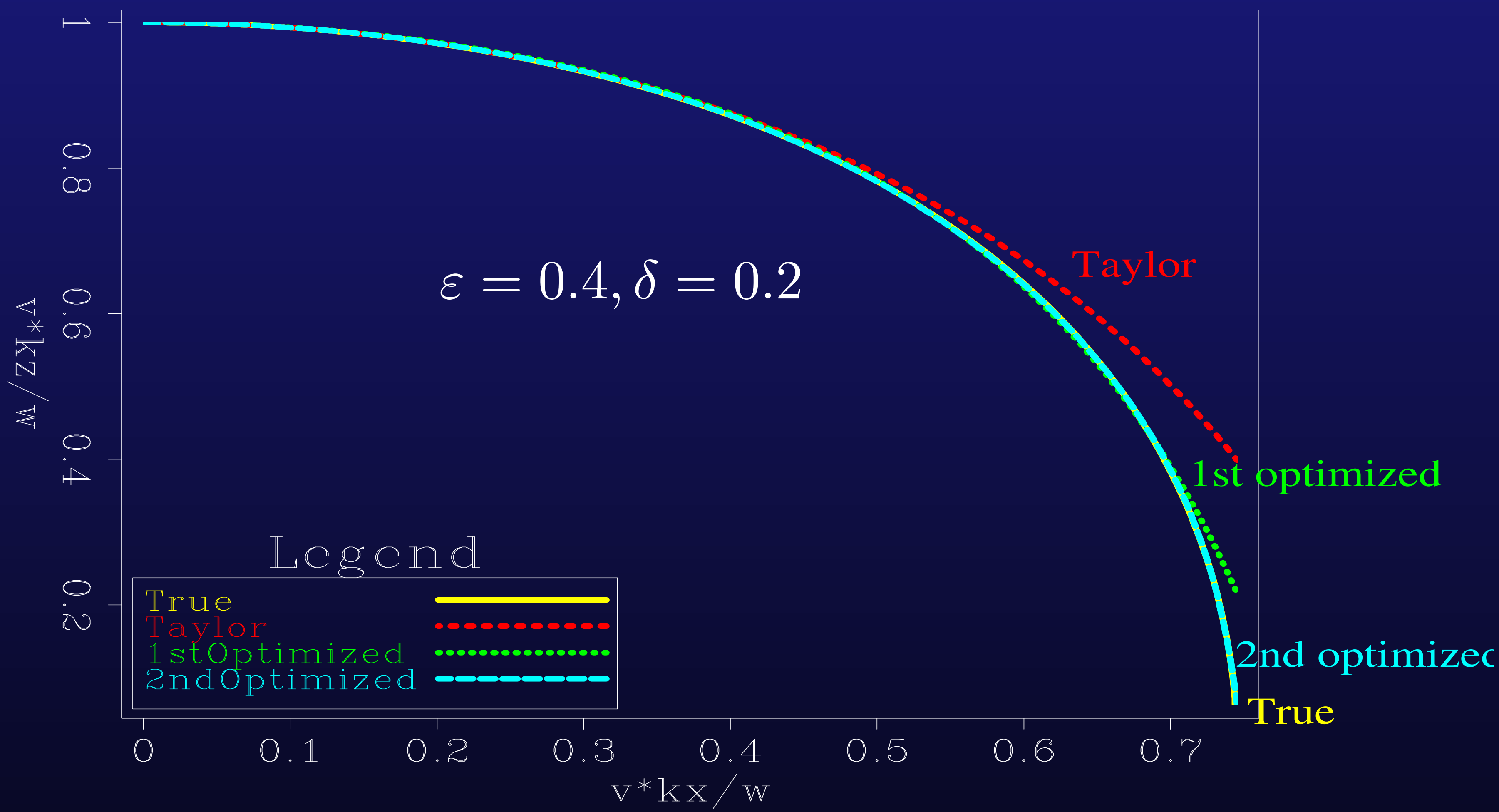
LS optimization for VTI media

Given ε, δ and $S_r \in [0,\ \sin(\theta_{max})]$, calculate a_i and b_i such that

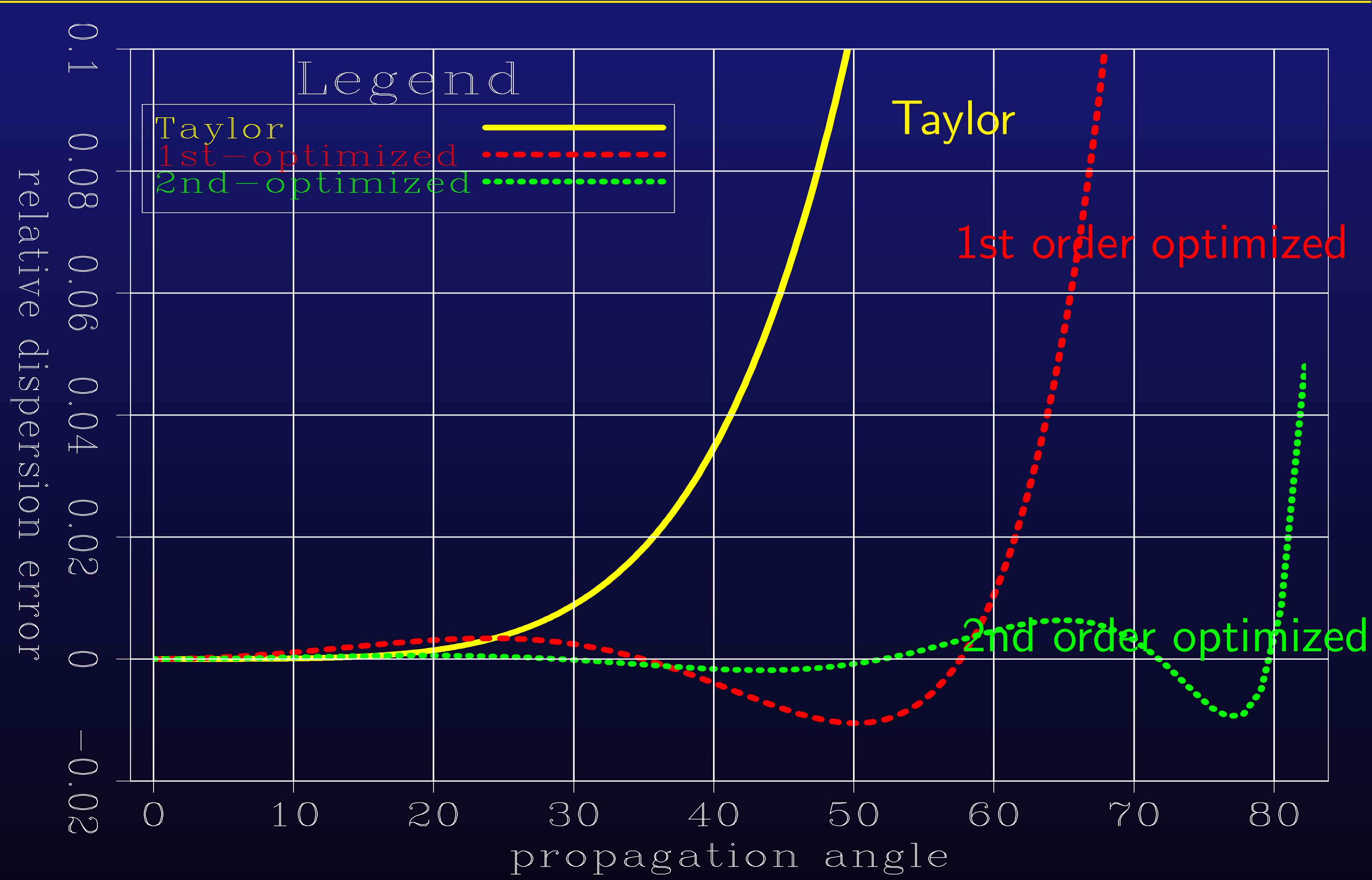
$$\sqrt{\frac{1-2(\varepsilon-\delta)S_r^2}{1-(1+2\varepsilon)S_r^2}} \approx \frac{\sum_{i=1}^{n} a_i S_r^i}{1+\sum_{i=1}^{n} b_i S_r^i}$$

- a_i and b_i are functions of ε and δ.
- If ε and δ vary laterally, a_i and b_i vary laterally.

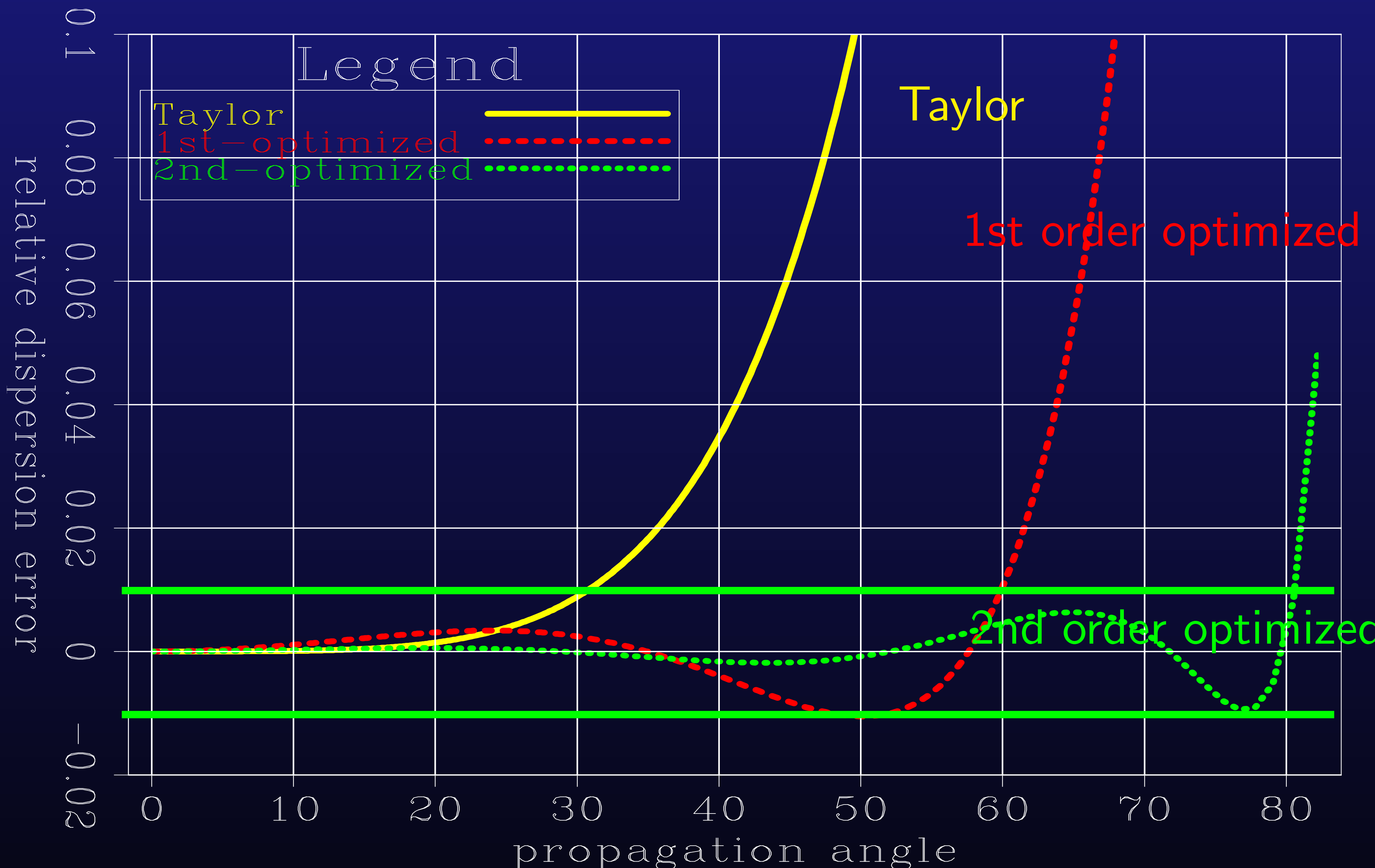
Dispersion relation comparison



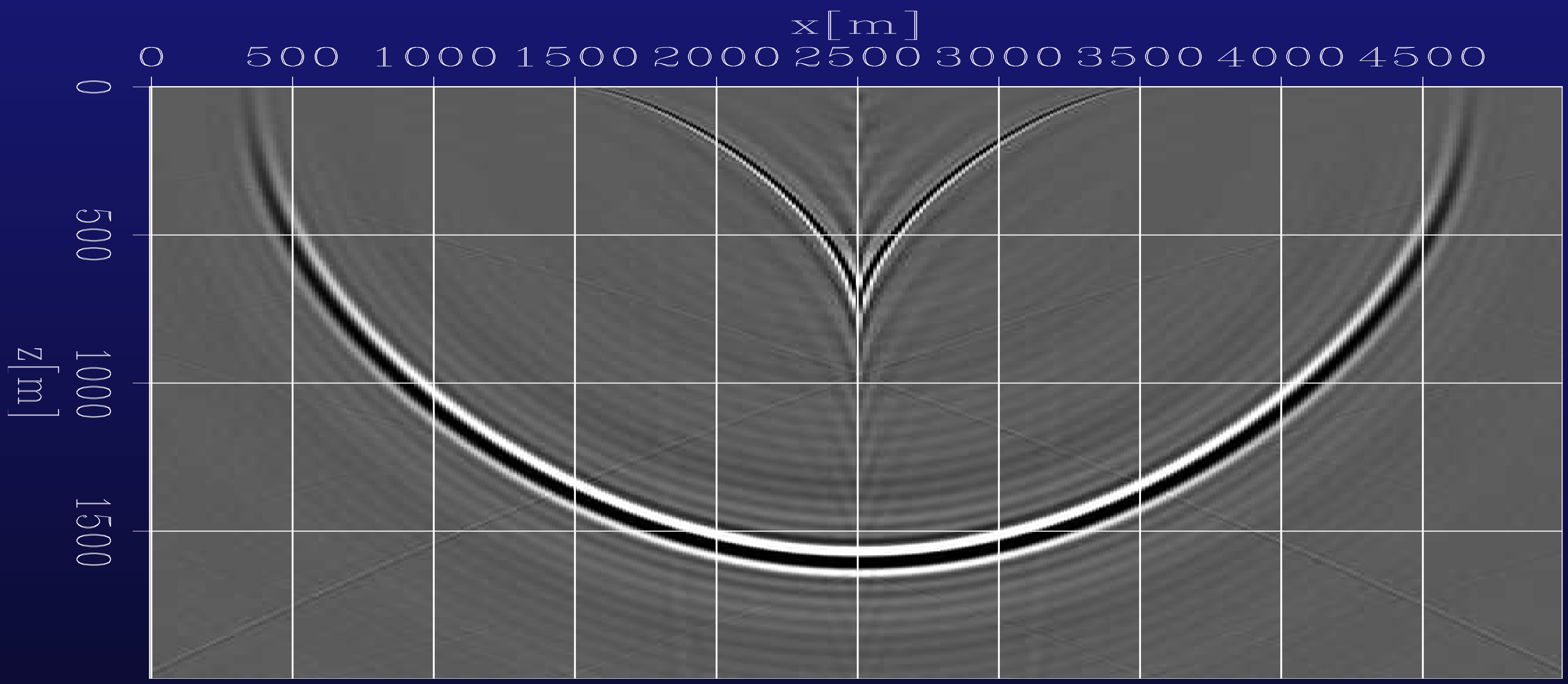
Relative dispersion error



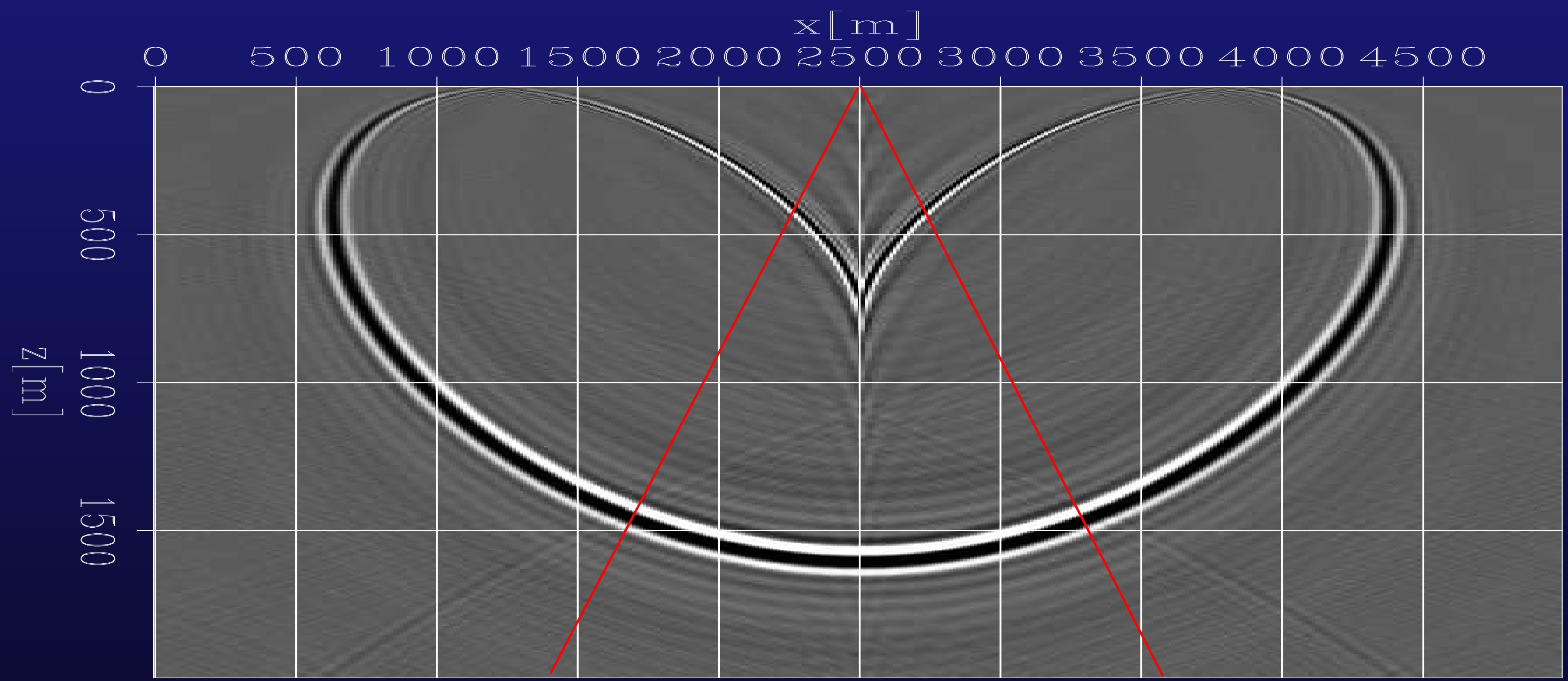
Relative dispersion error



Impulse response: Phase shift method



Impulse response: Taylor analysis



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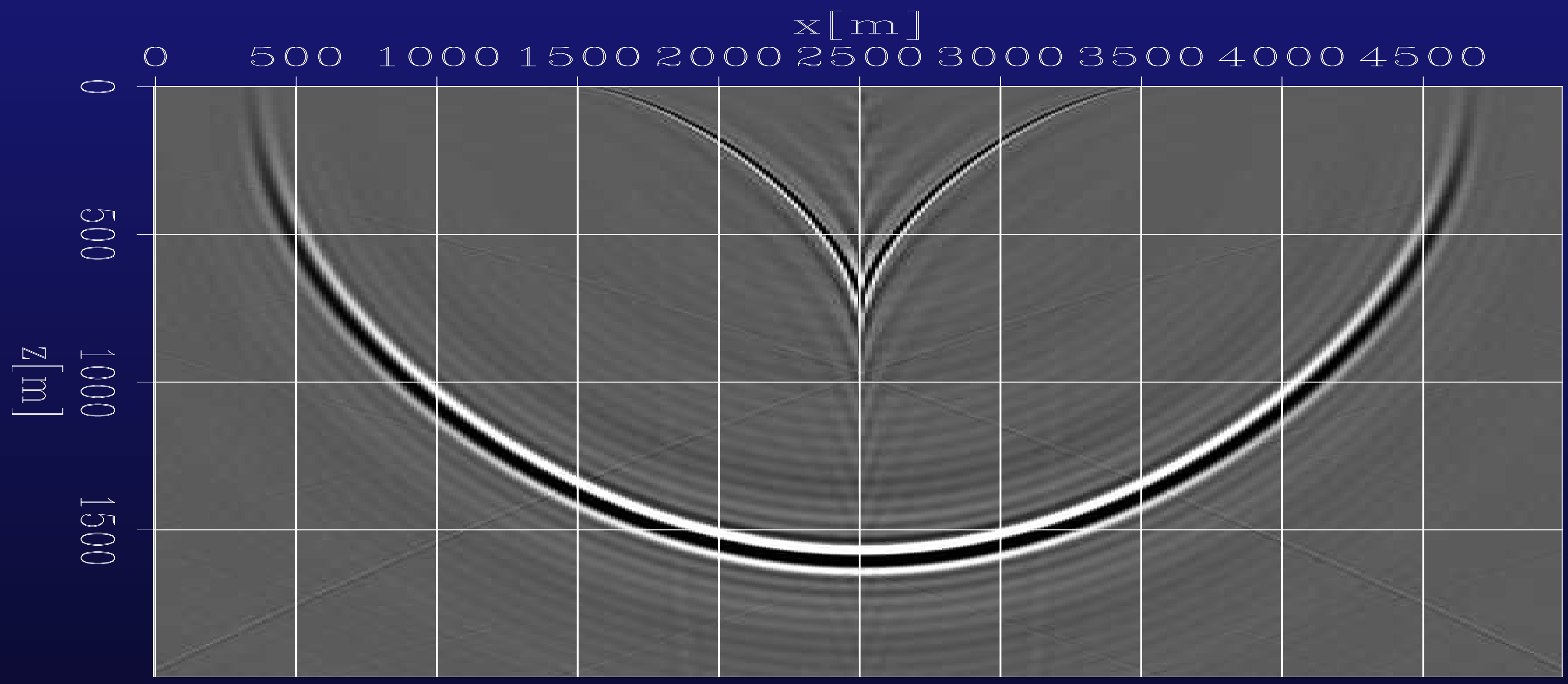
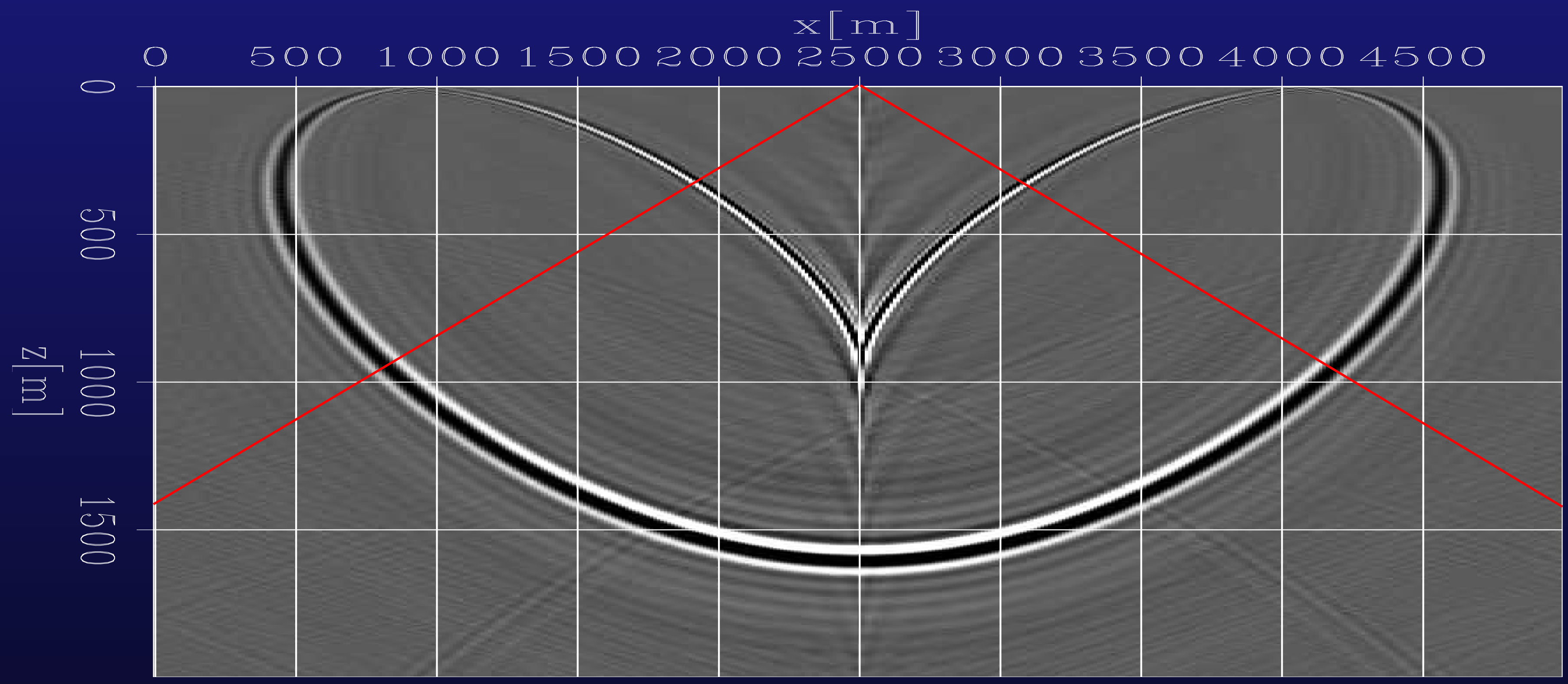
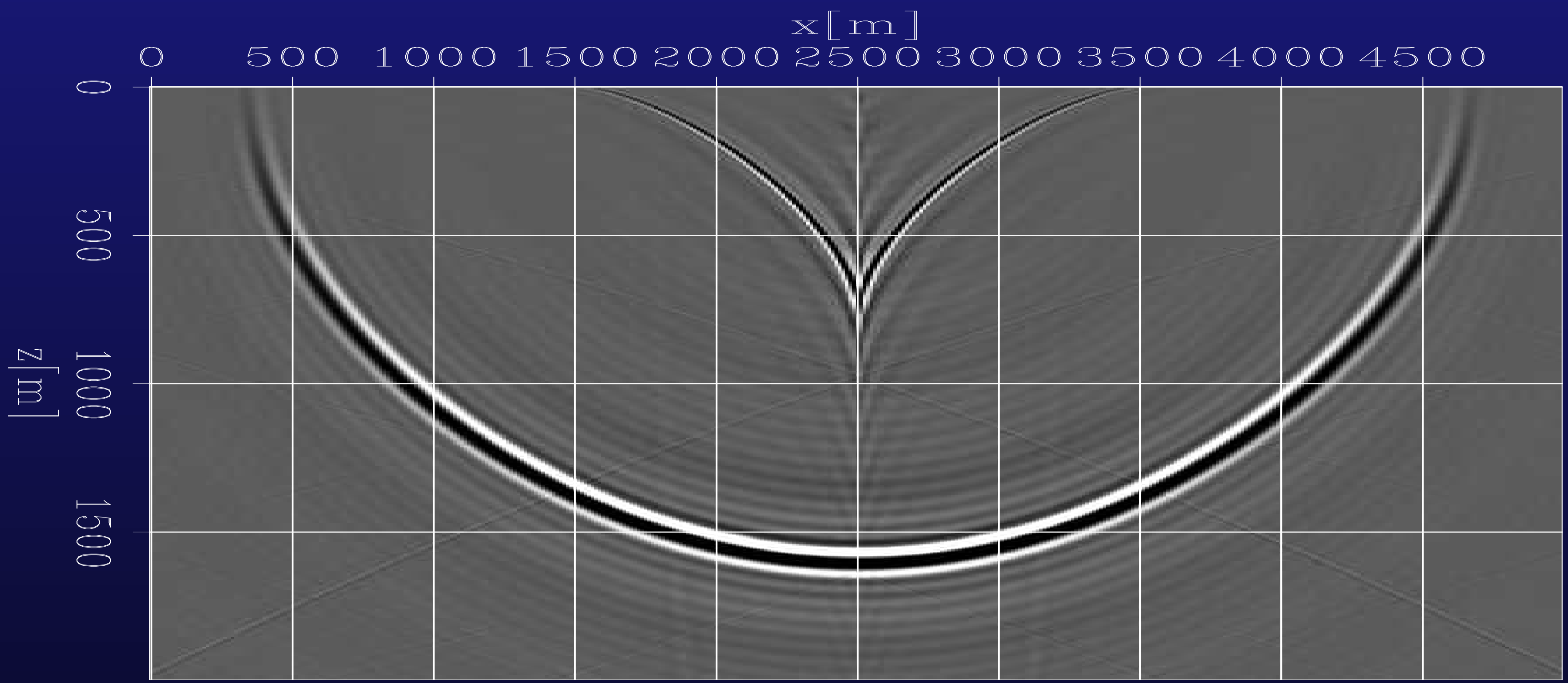


Impulse response: first order optimization



Impulse response: Phase shift method



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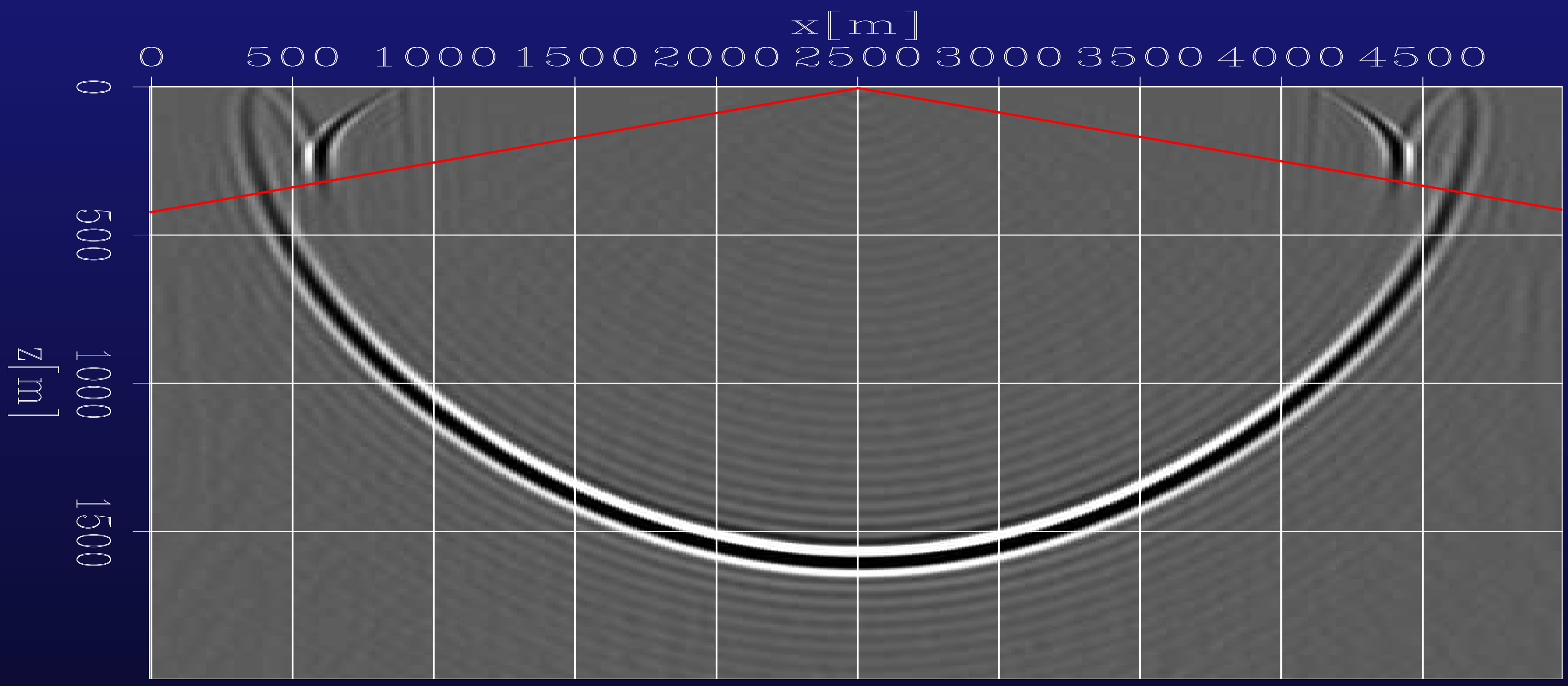
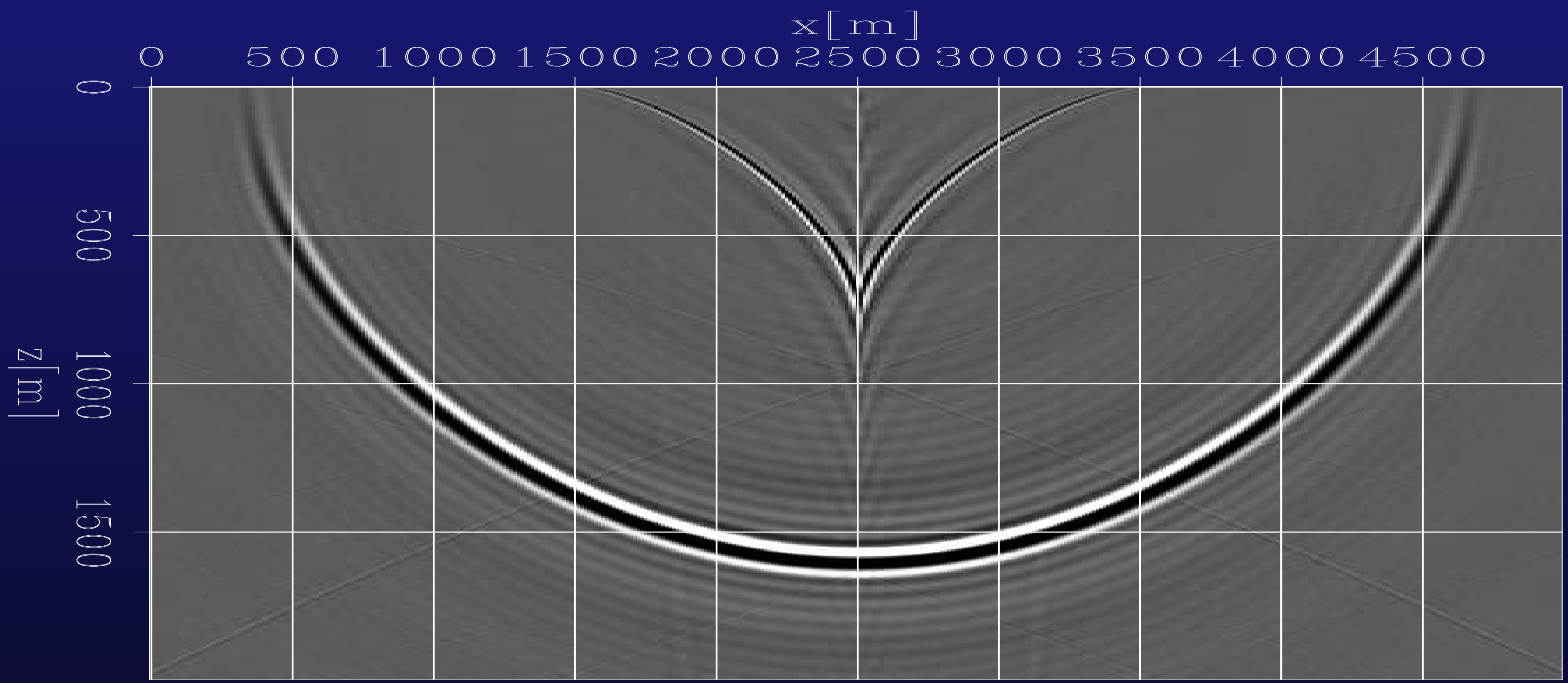


Impulse response: Phase shift method



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Finite-difference for isotropic media

	x_1	x_2		x_{nx-1}	x_{nx}
z_i	$\varepsilon_1=0.0$ $\delta_1=0.0$	$\varepsilon_2=0.0$ $\delta_2=0.0$	$\cdots\cdots$	$\varepsilon_{nx-1}=0.0$ $\delta_{nx-1}=0.0$	$\varepsilon_{nx}=0.0$ $\delta_{nx}=0.0$

	x_1	x_2		x_{nx-1}	x_{nx}
z_i	$\alpha_1=0.5$ $\beta_1=0.25$	$\alpha_1=0.5$ $\beta_1=0.25$	$\cdots\cdots$	$\alpha_1=0.5$ $\beta_1=0.25$	$\alpha_1=0.5$ $\beta_1=0.25$

$$\frac{\partial P}{\partial z} = i\frac{\omega}{v(x)} \cdot \frac{\alpha_1 \left(\frac{v(x)}{\omega}\right)^2 \frac{\partial^2}{\partial x^2}}{1 + \beta_1 \left(\frac{v(x)}{\omega}\right)^2 \frac{\partial^2}{\partial x^2}} P$$

Laterally varying VTI media

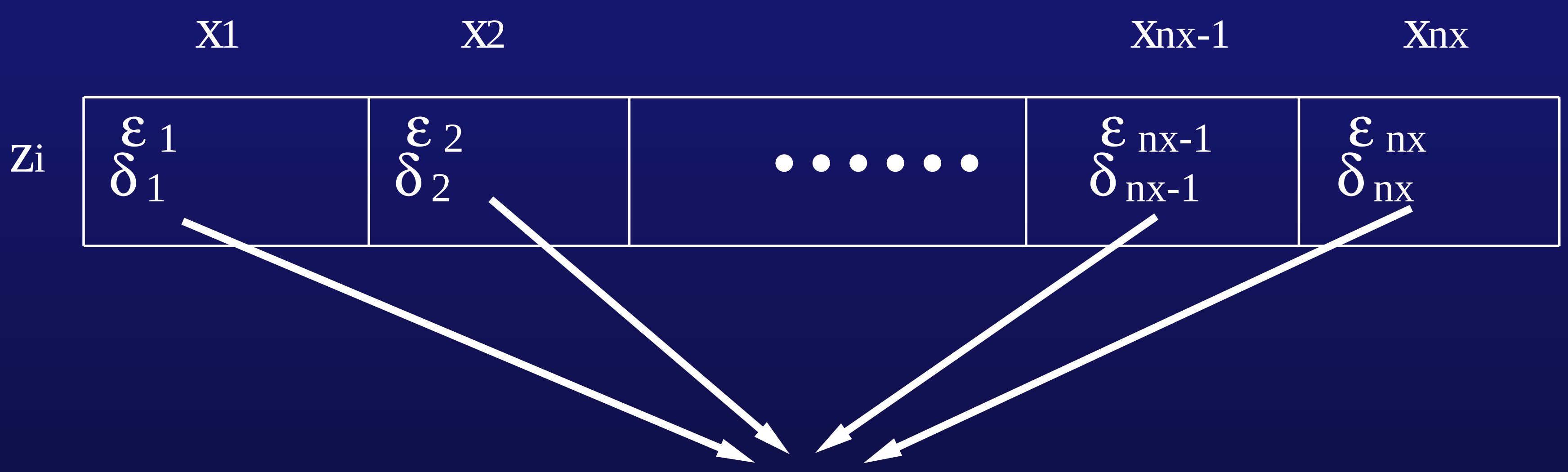
$$\frac{\partial P}{\partial z} = i\frac{\omega}{v(x)} \cdot \frac{\alpha_1(x)\left(\frac{v(x)}{\omega}\right)^2 \frac{\partial^2}{\partial x^2}}{1 + \beta_1(x)\left(\frac{v(x)}{\omega}\right)^2 \frac{\partial^2}{\partial x^2}} P$$

α_1 and β_1 vary laterally.

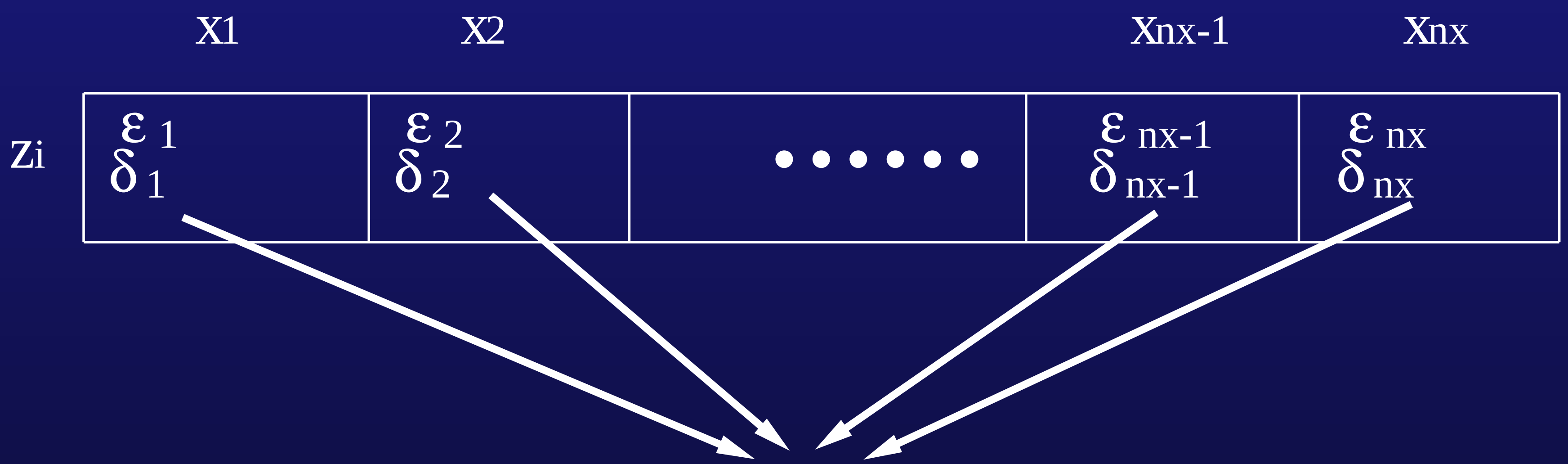
Finite-difference (Taylor series) for VTI

	x1	x2		xnx-1	xnx
zi	ε_1 δ_1	ε_2 δ_2	• • • • • •	ε_{nx-1} δ_{nx-1}	ε_{nx} δ_{nx}

Finite-difference (Taylor series) for VTI

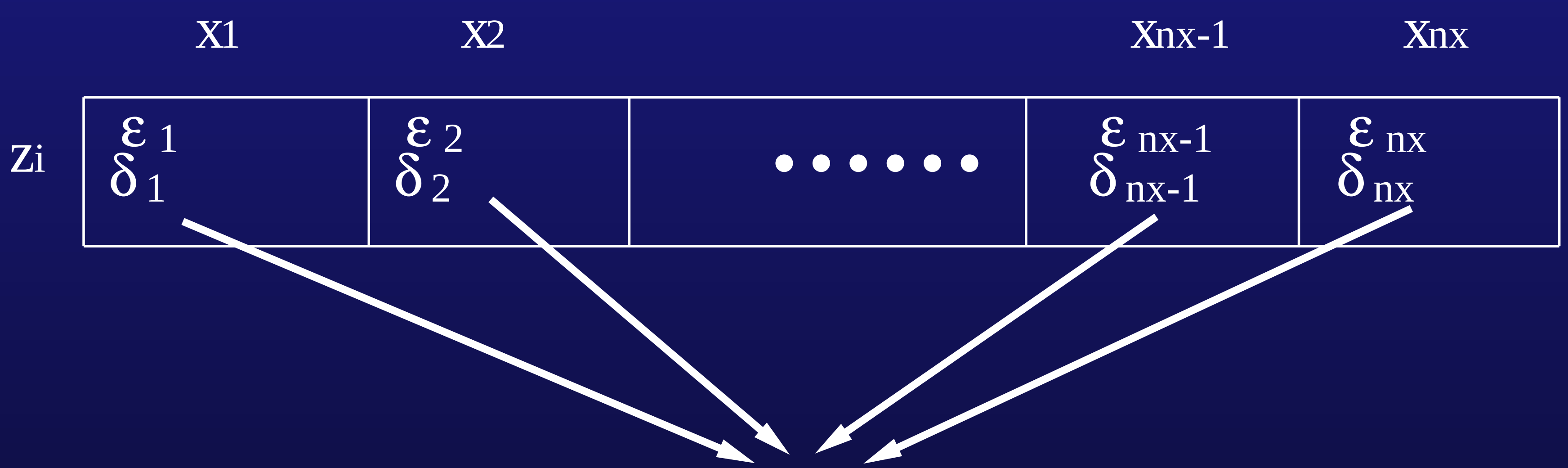


Finite-difference (Taylor series) for VTI



$$\alpha_1 = 0.5(1+2\delta)$$
$$\beta_1 = \frac{2(\varepsilon-\delta)}{1+2\delta} + 0.25(1+2\delta)$$

Finite-difference (Taylor series) for VTI



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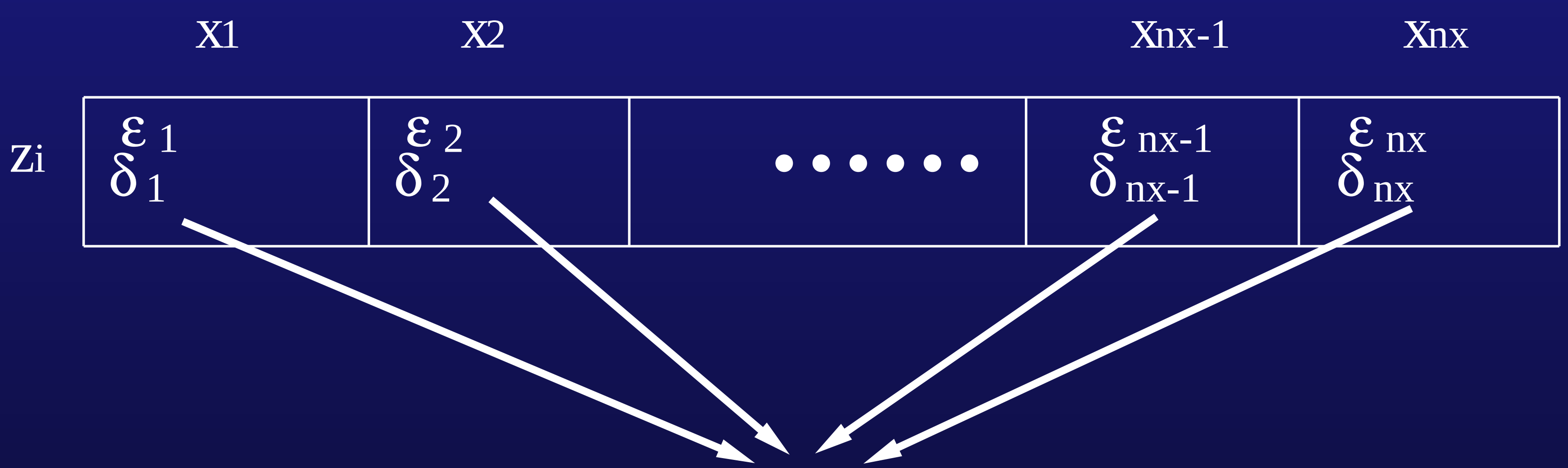
Finite-difference (Taylor series) for VTI



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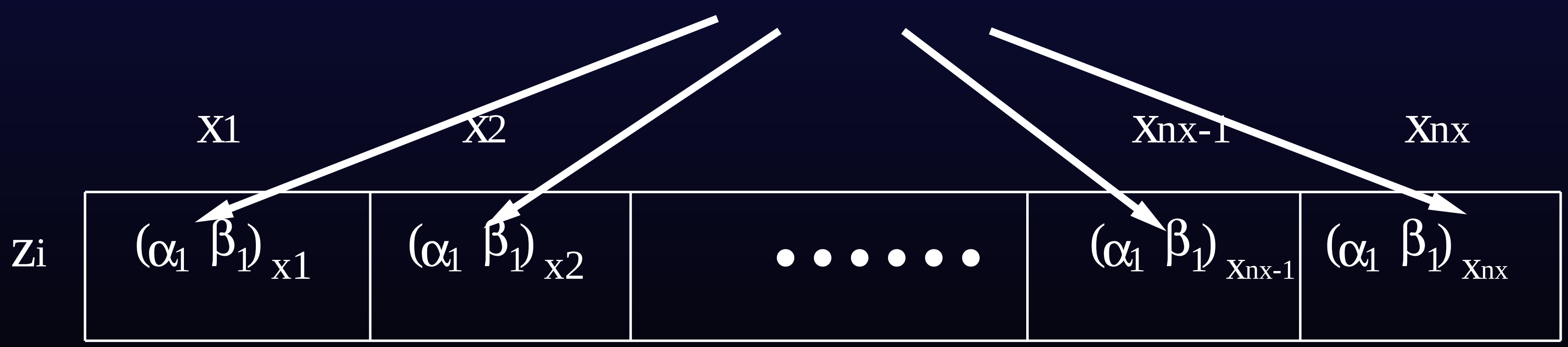


Table driven implicit finite-difference

	X1	X2		Xnx-1	Xnx
Zi	ε_1 δ_1	ε_2 δ_2	• • • • • •	ε_{nx-1} δ_{nx-1}	ε_{nx} δ_{nx}

Table driven implicit finite-difference

	X1	X2		Xnx-1	Xnx
Zi	ε_1 δ_1	ε_2 δ_2	• • • • • •	ε_{nx-1} δ_{nx-1}	ε_{nx} δ_{nx}

	ε_{min}	$\varepsilon_{min}+d\varepsilon$			ε_{max}
δ_{min}					
$\delta_{min}+d\delta$			α_1 β_1		
δ_{max}					

Table driven implicit finite-difference

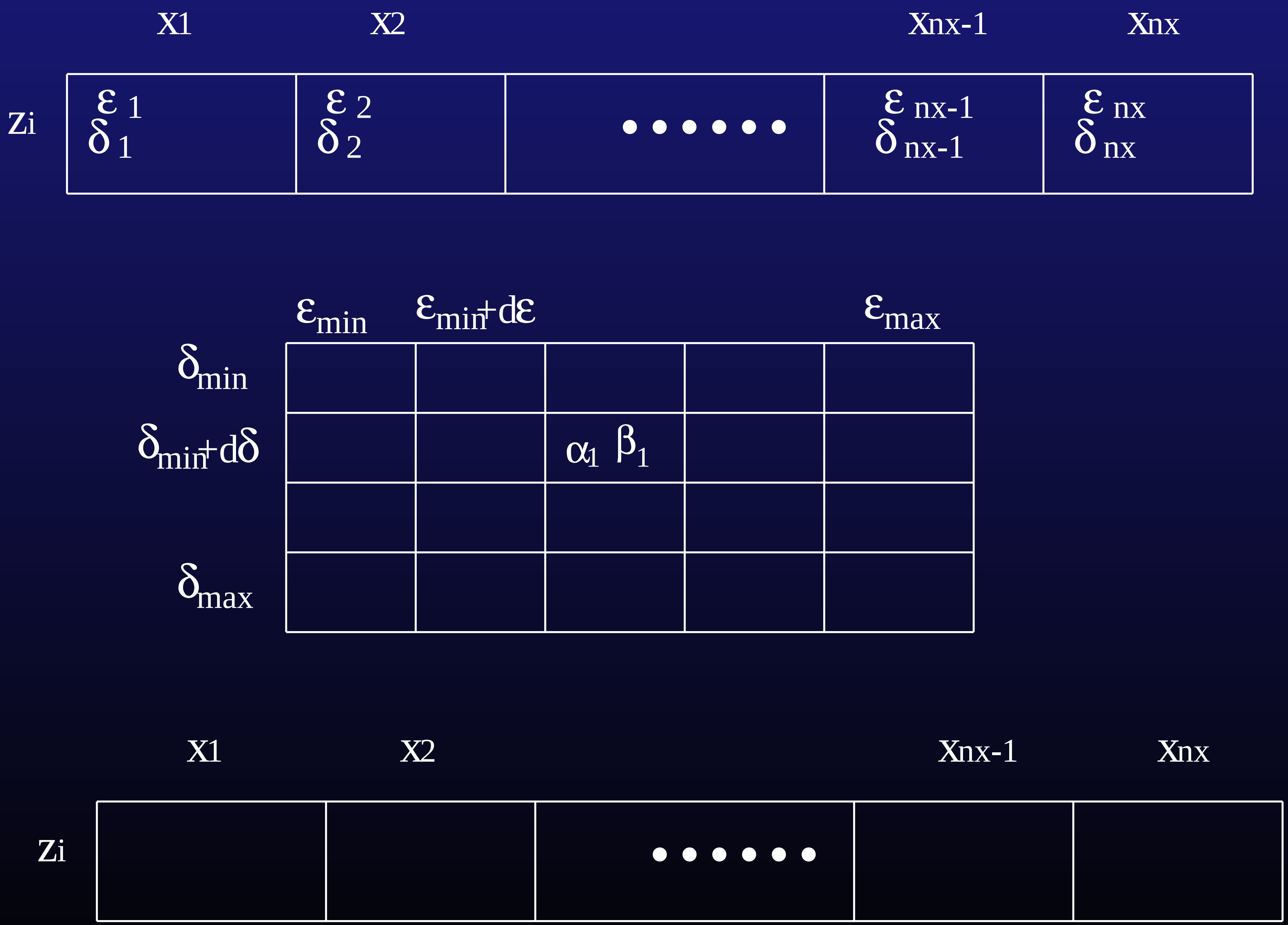


Table driven implicit finite-difference

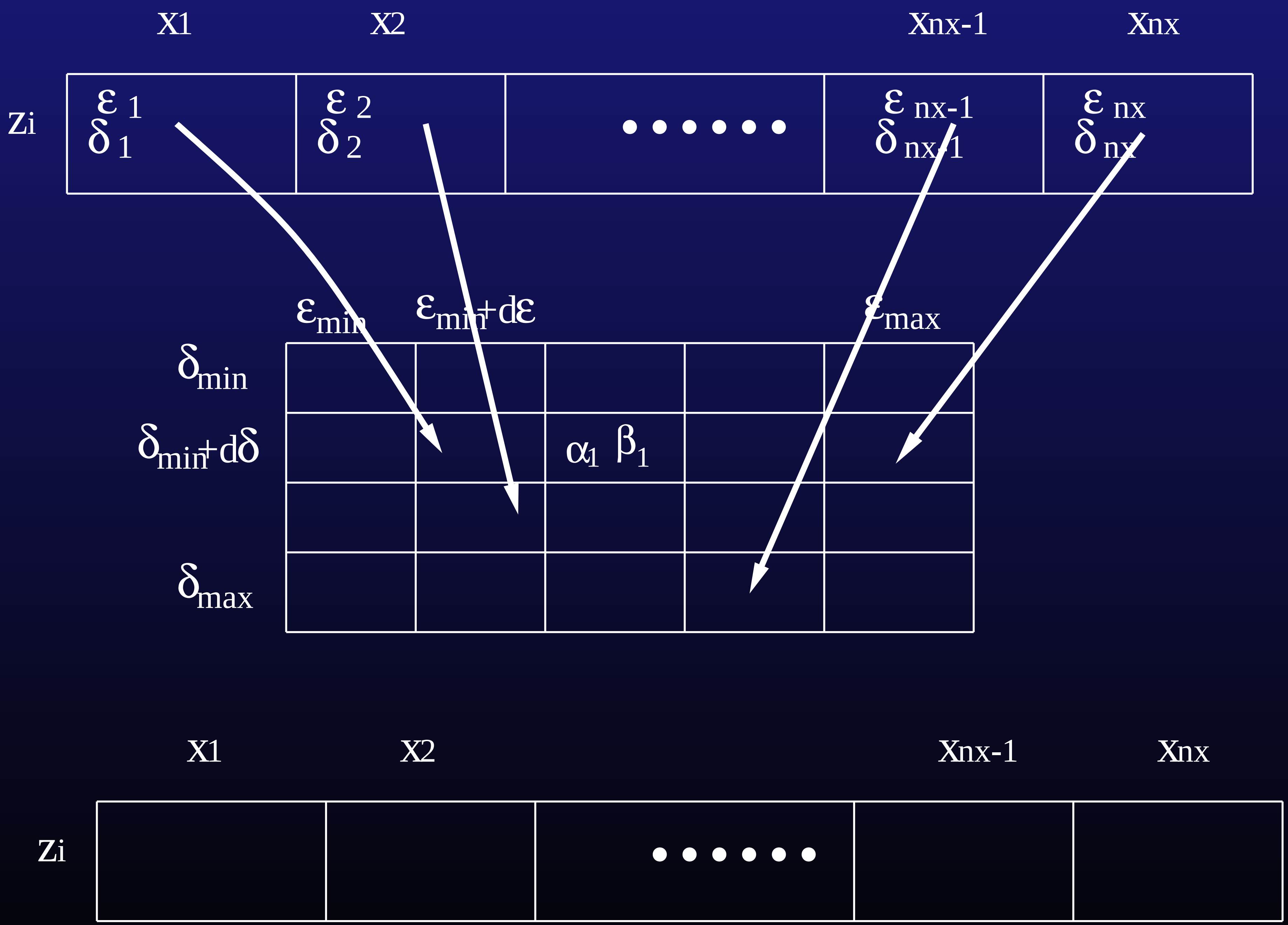
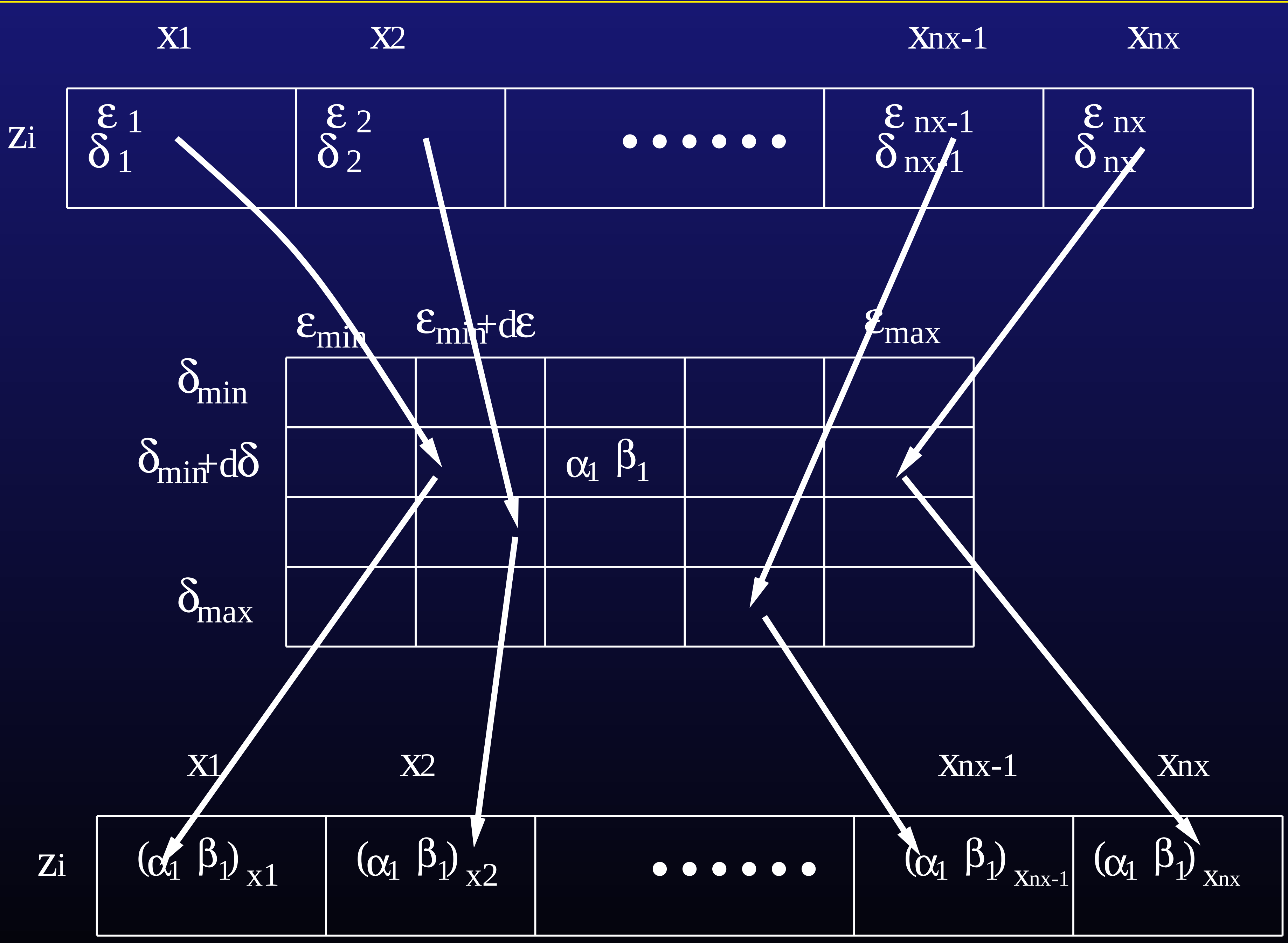


Table driven implicit finite-difference



Computation cost

Cost of Implicit finite-difference
for VTI media

=

Cost of Implicit finite-difference for isotropic media	$+$	table searching

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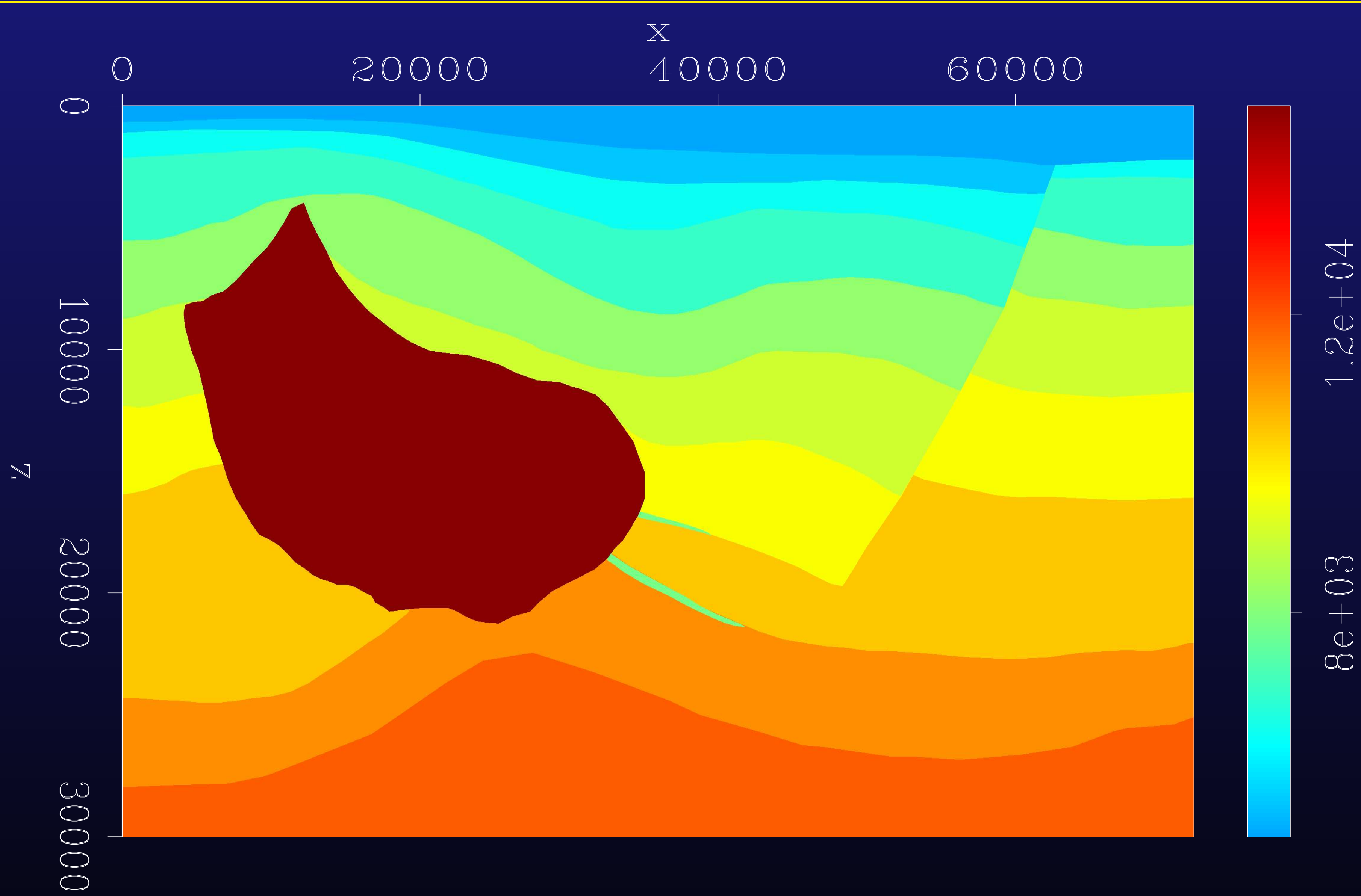
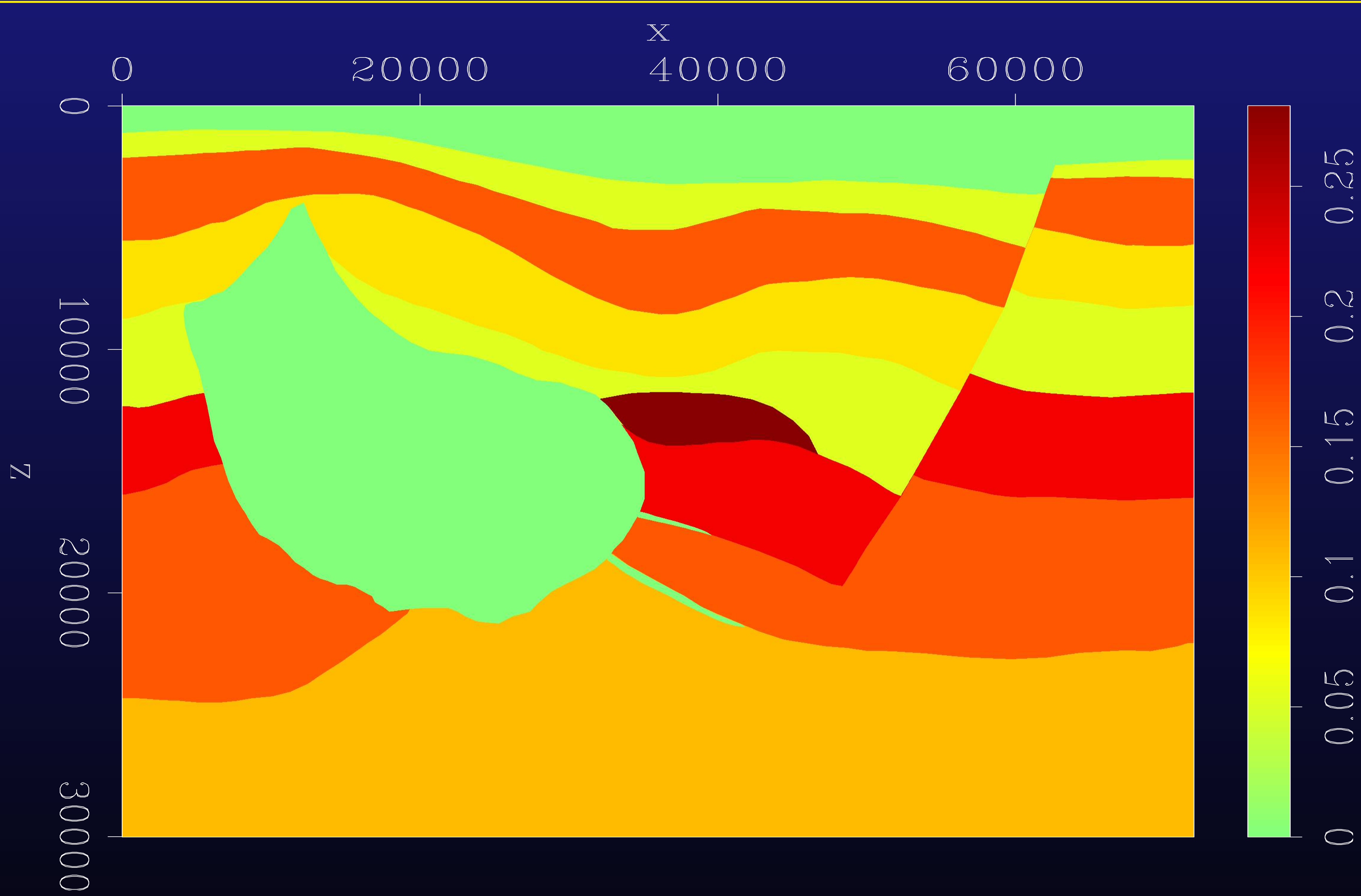
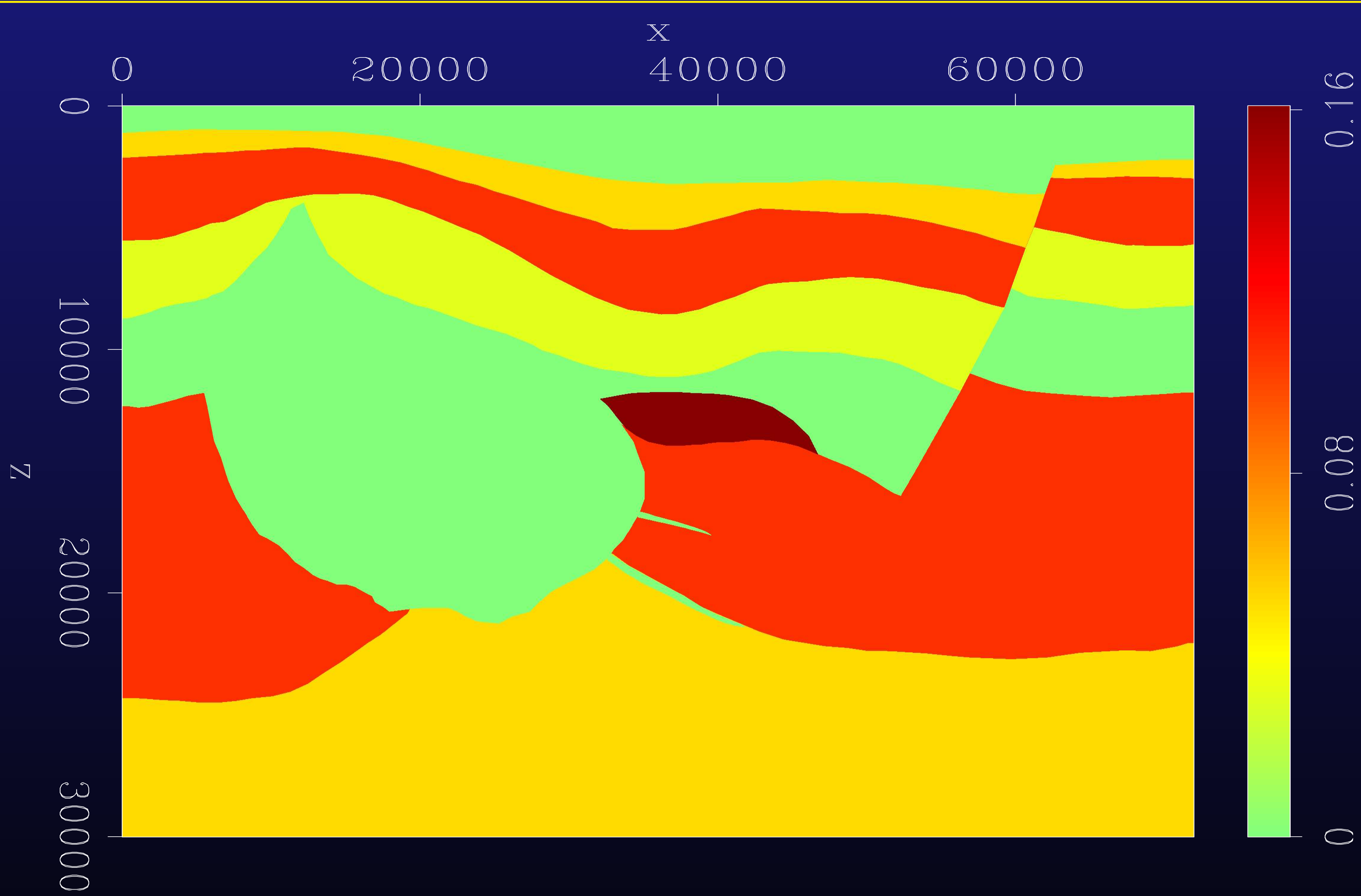
Vertical velocity



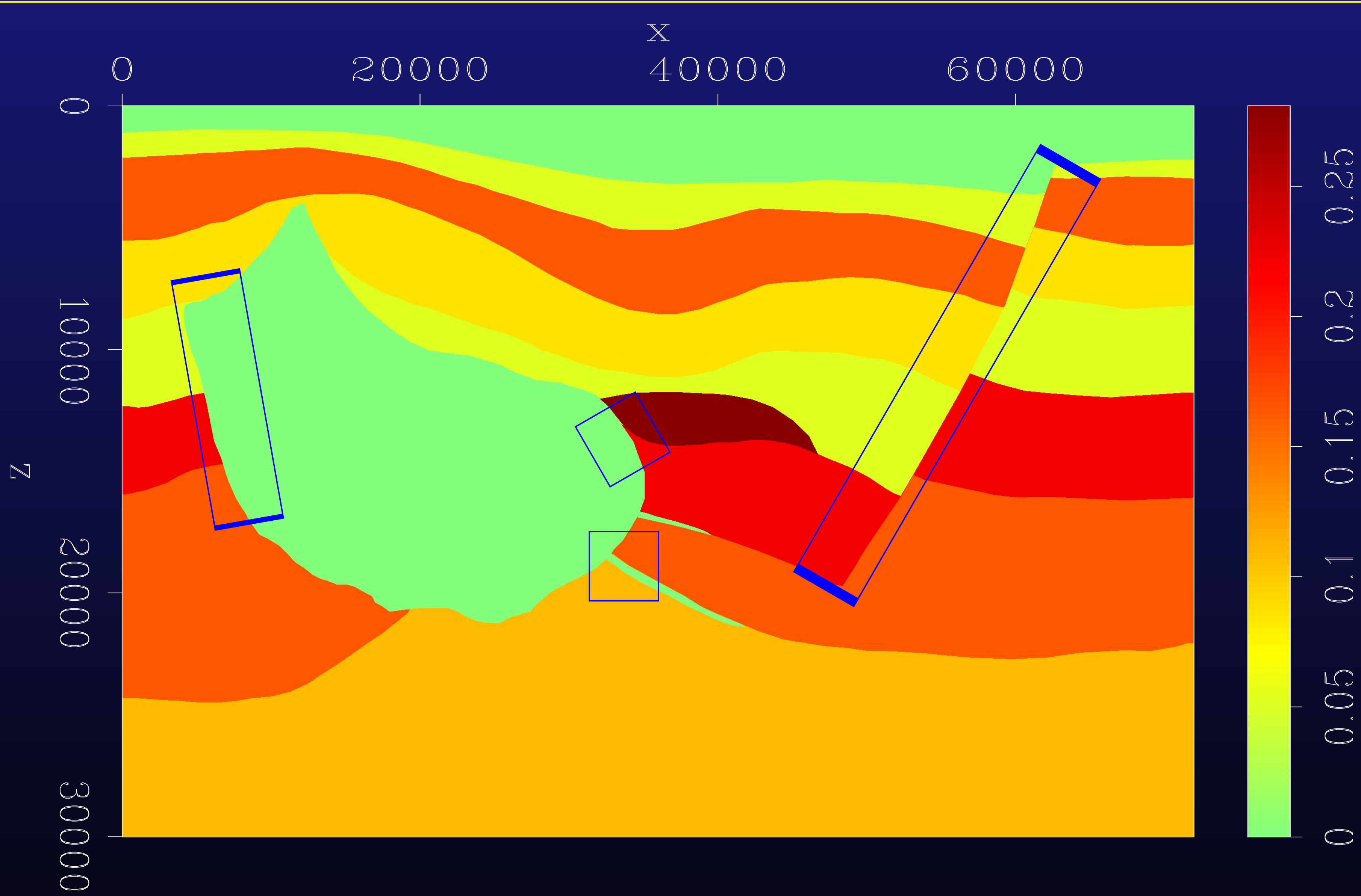
Anisotropy parameter ε



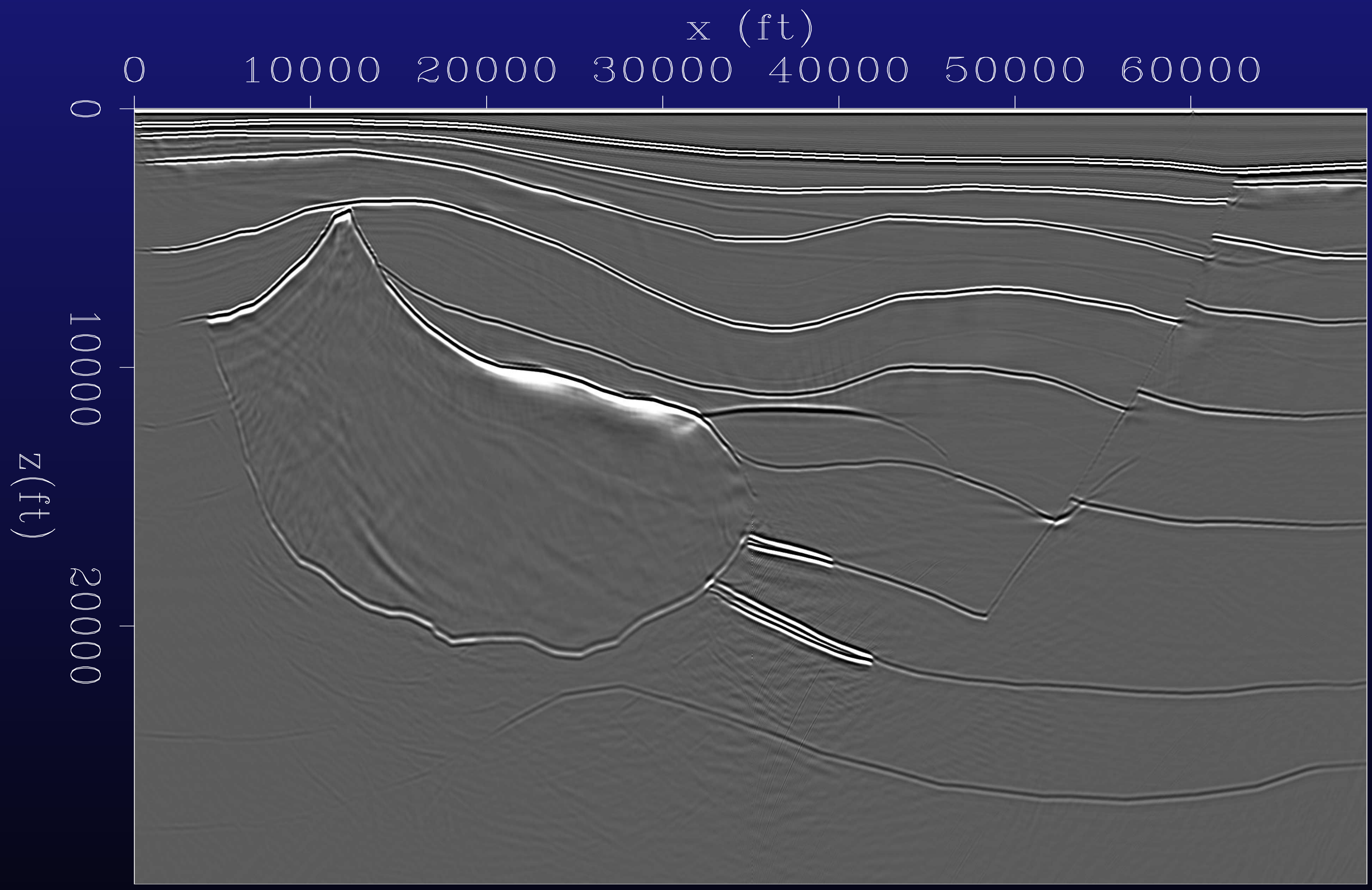
Anisotropy parameter δ



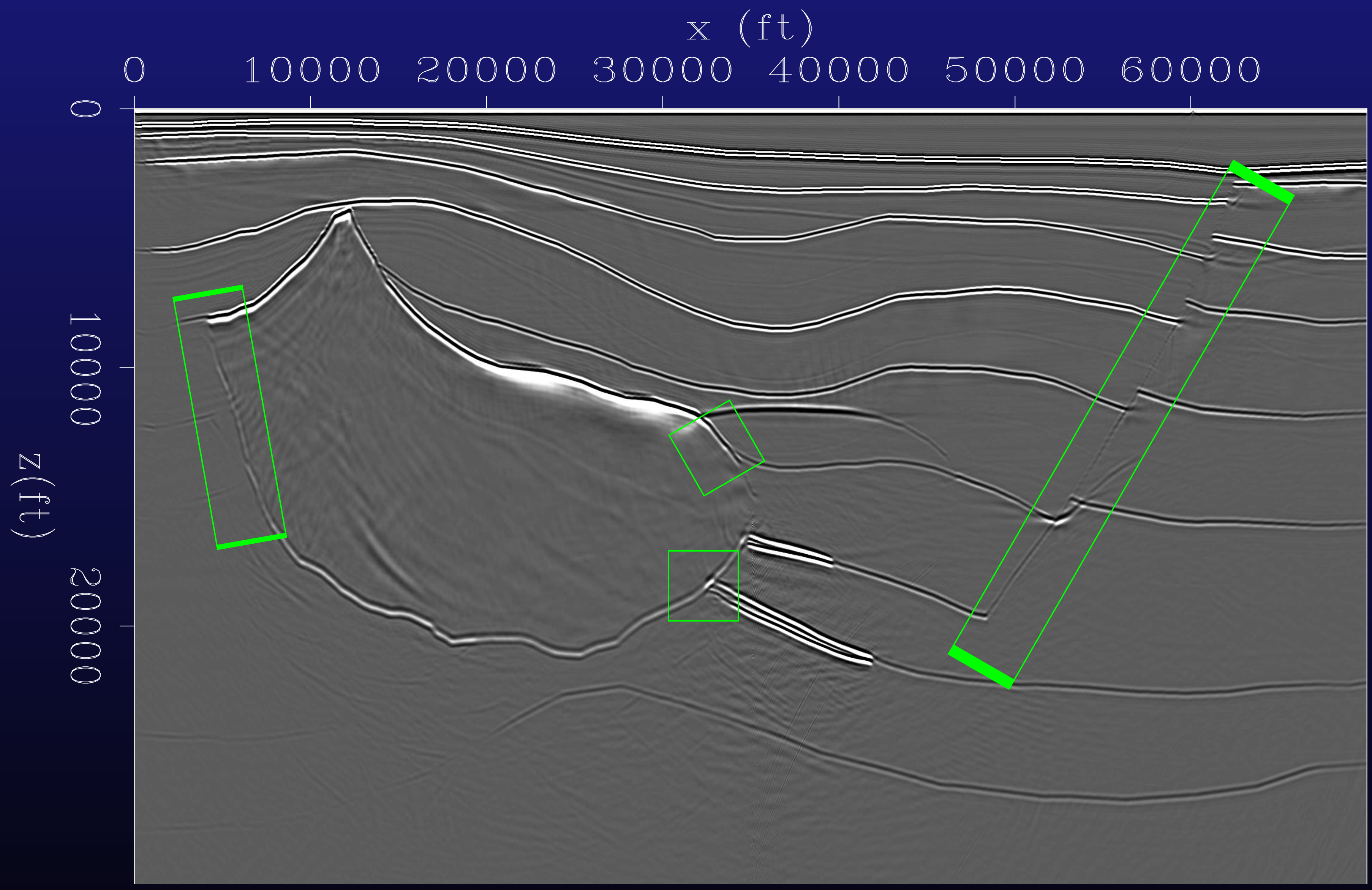
Challenges of imaging



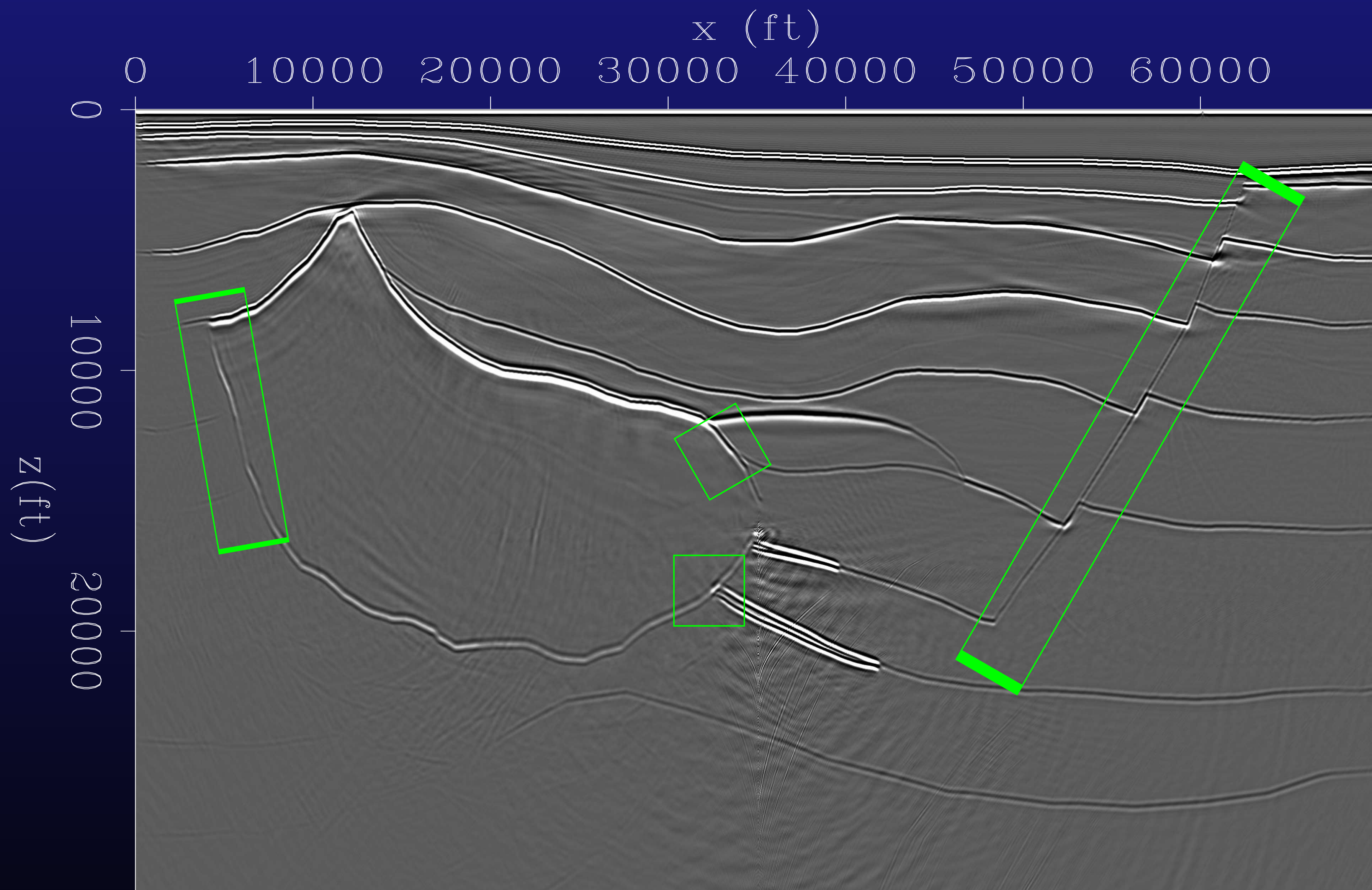
The first order optimized finite-difference



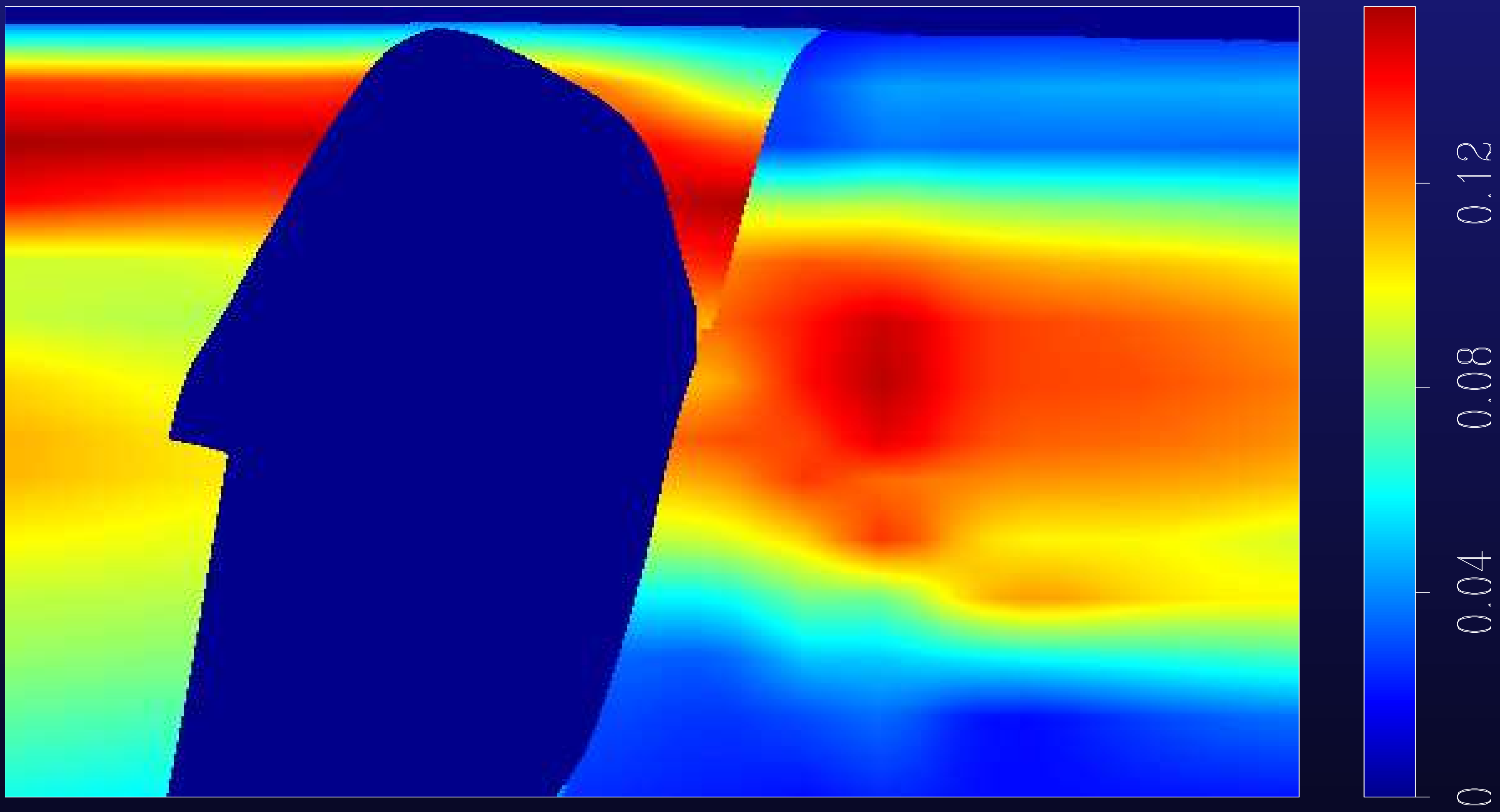
The First order optimized finite-difference



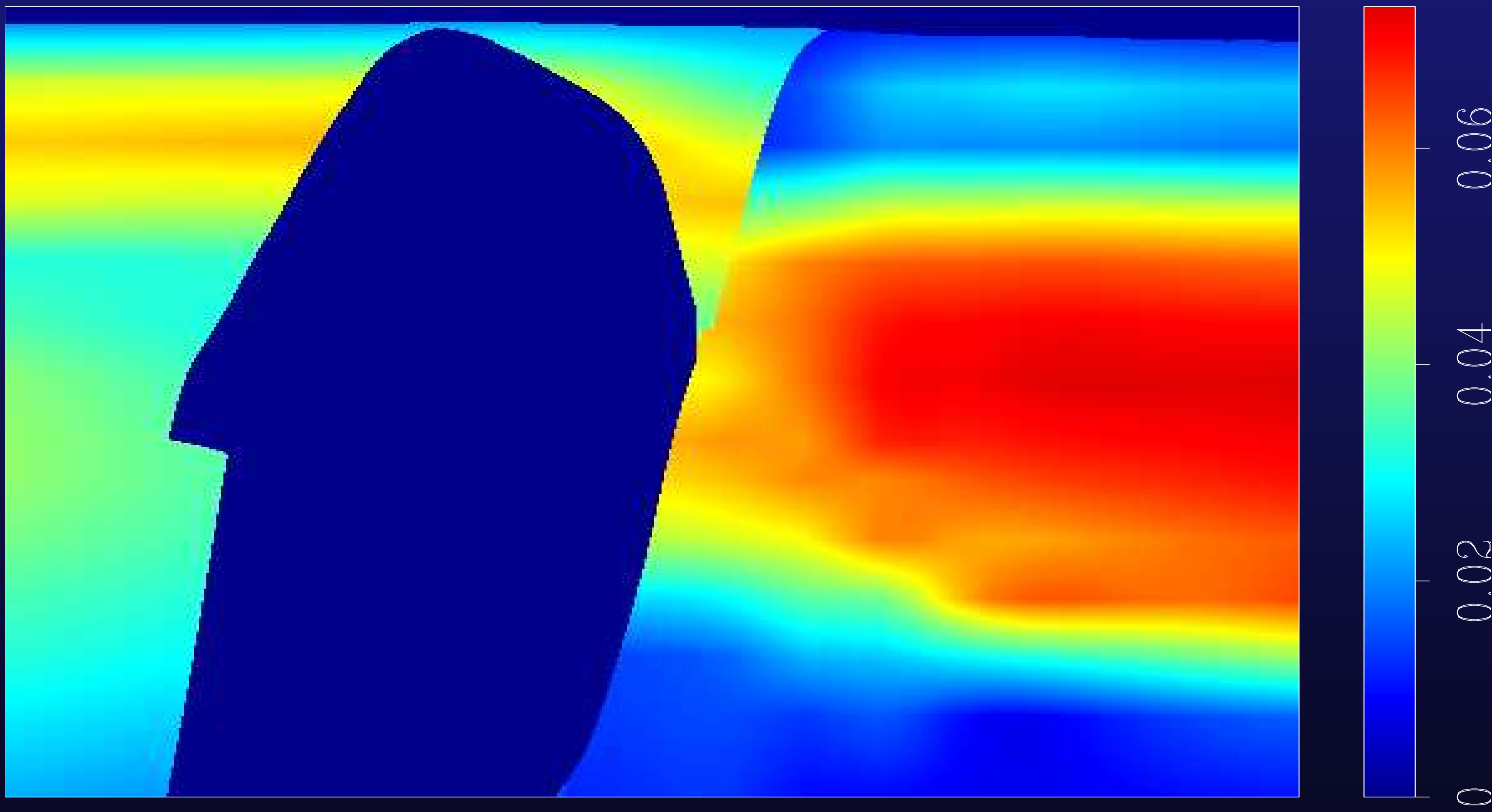
The 2nd order optimized finite-difference



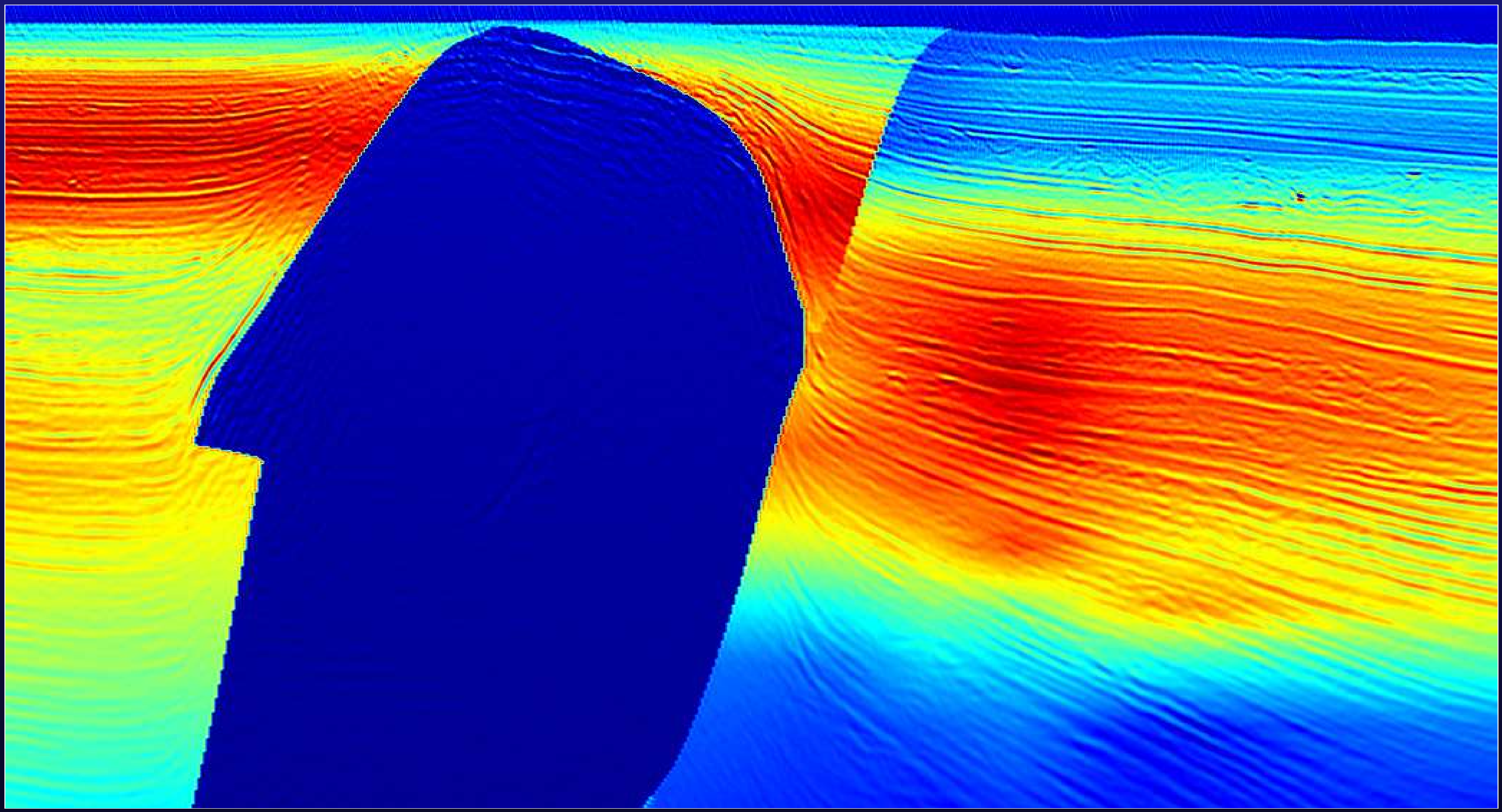
Anisotropy parameter ε



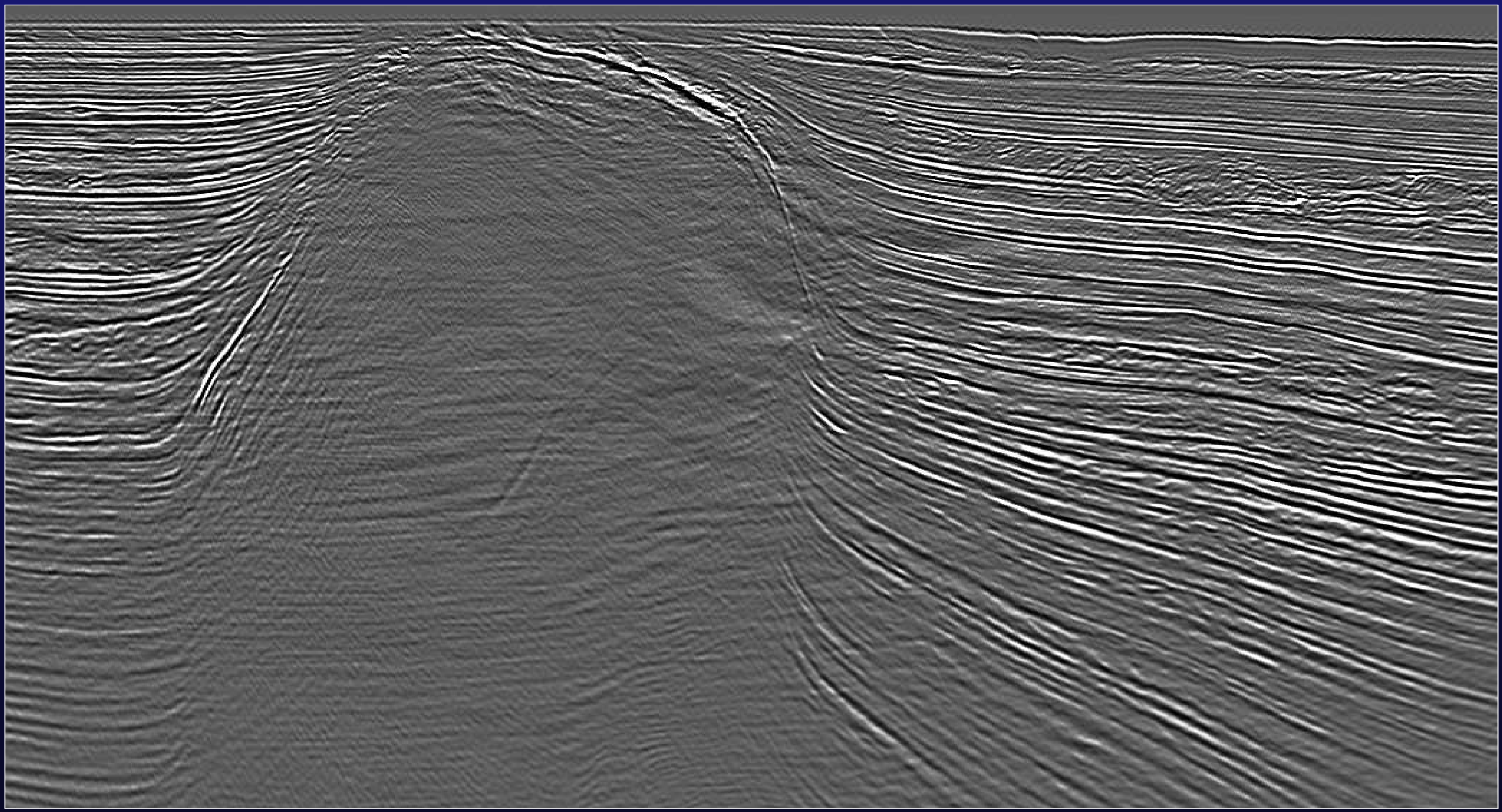
Anisotropy parameter δ



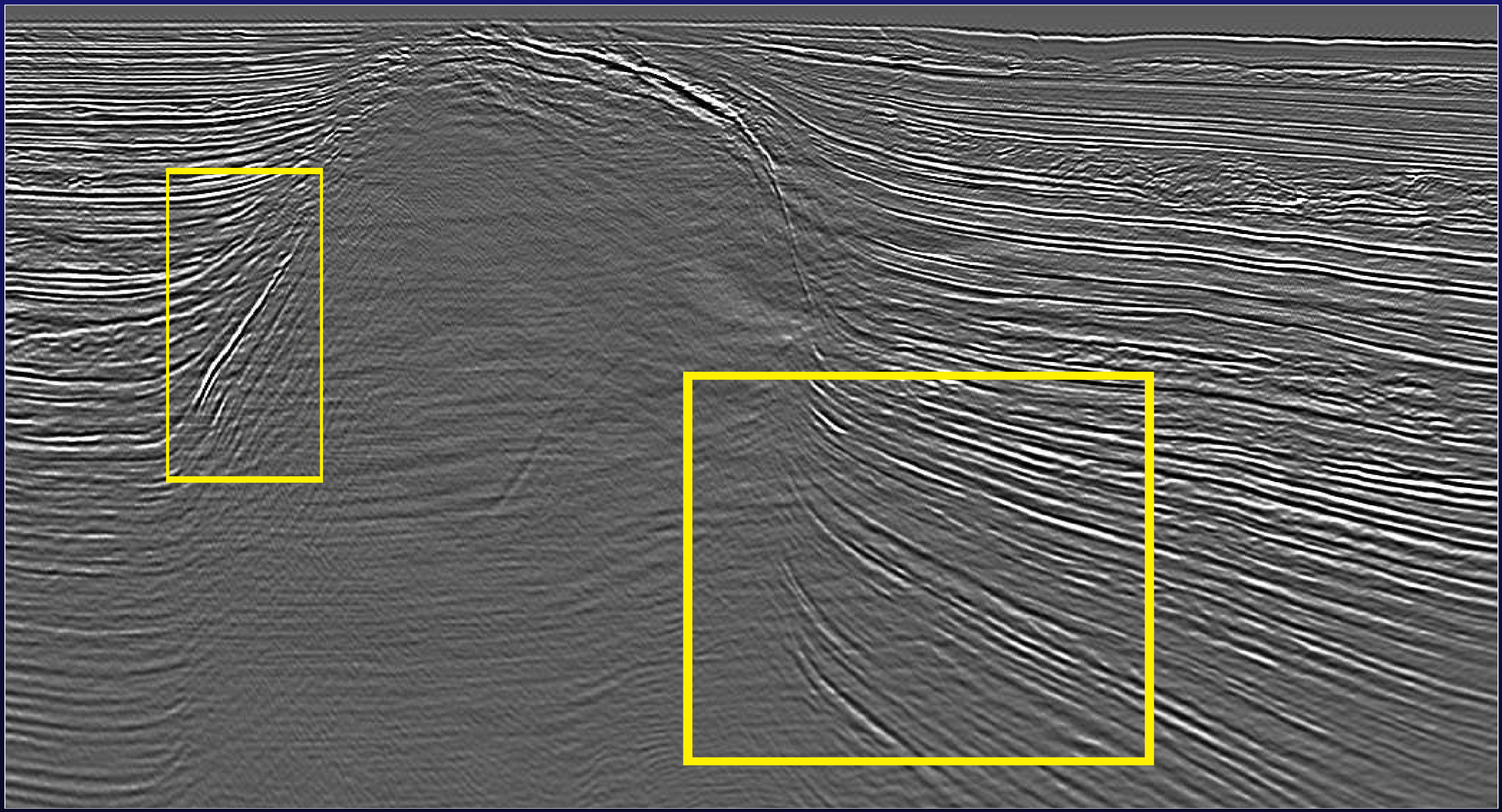
Anisotropy parameter with image



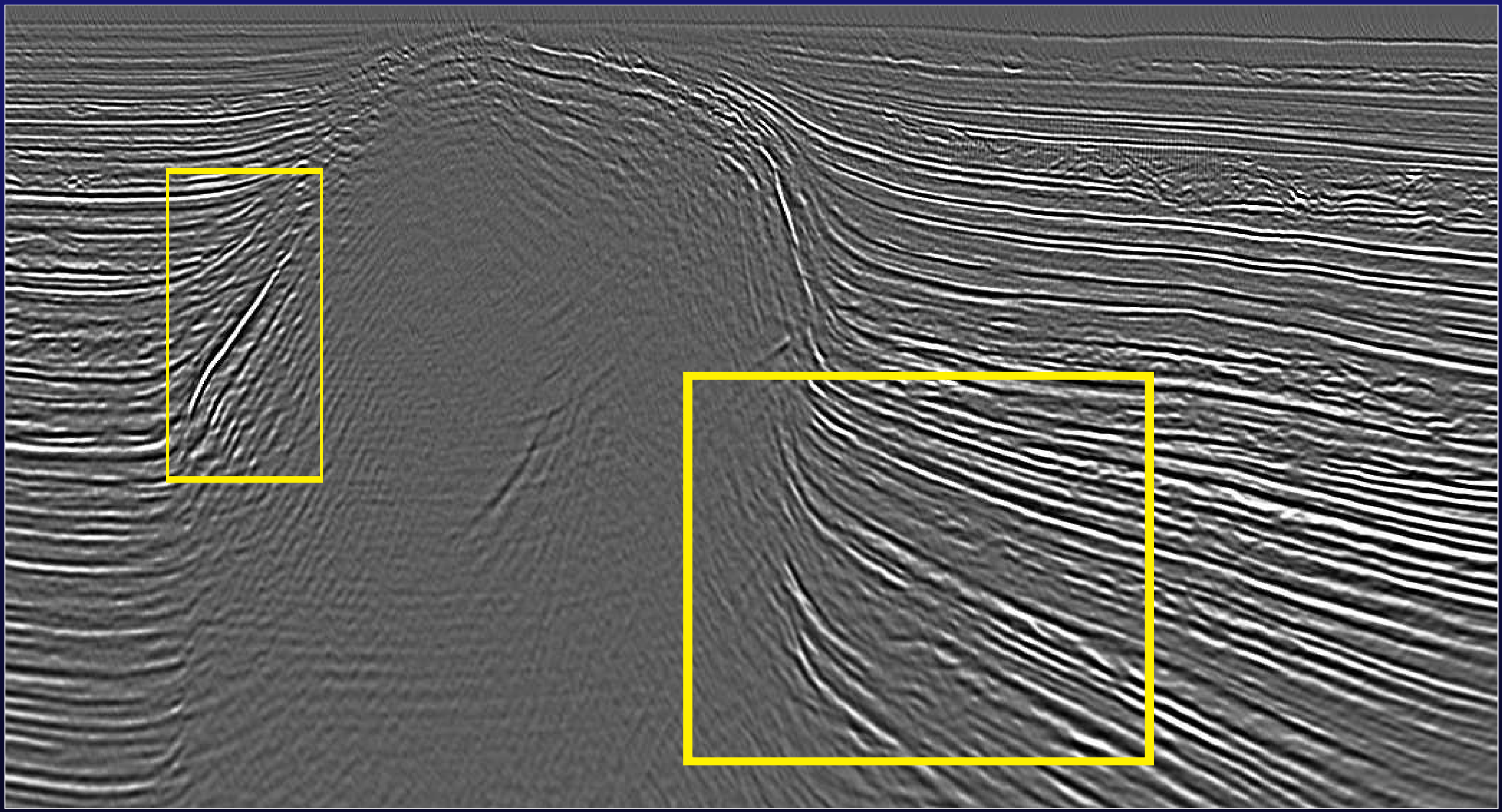
Optimized implicit finite-difference



Optimized implicit finite-difference



Plane-wave migration in tilted coordinates



Conclusions

- By optimization, we design implicit finite-difference scheme for VTI media. The first order finite-difference is accurate to 60° and the second order finite-difference is accurate to 80°.

- The computation cost for the implicit finite-difference scheme for VTI media is almost same as isotropic media.

- The scheme works in laterally varying media and it is stable.

Acknowledgments

- Amerada Hess and ExxonMobil for making the synthetic and real data available.
- Faqi Liu from Amerada Hess for useful discussion.