Wave-equation Migration from Topography

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SEP117 page:27–42
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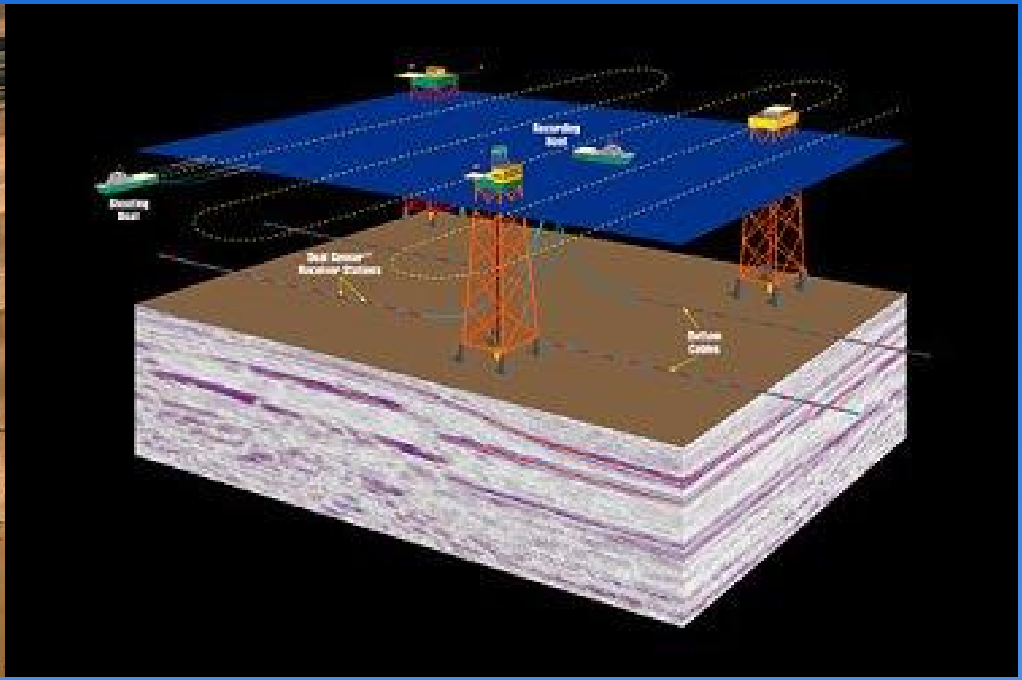
… since the mists of time

- Seismic data migration from acquisition topography can present a significant imaging challenge

Topography everywhere!



Exploration in Saudi Arabia

Seabed topography

Previous solutions

- Seismic data migration from acquisition topography can present a significant imaging challenge

- Previous processing solutions:
  - ⋆ Statics corrections
  - ⋆ Kirchhoff wavefield datuming (e.g., Berryhill, 1979)

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- Presence of strong lateral velocity contrast

  ⋆ Need wave-equation operators to handle multipathing

Recent Processing Advances

- Cartesian computational meshes for wavefield extrapolation is not ideal for data recorded on topographic surfaces

Cabling

Geophone cabling $\longrightarrow$ uniformly sample topography

Recent Processing Advances

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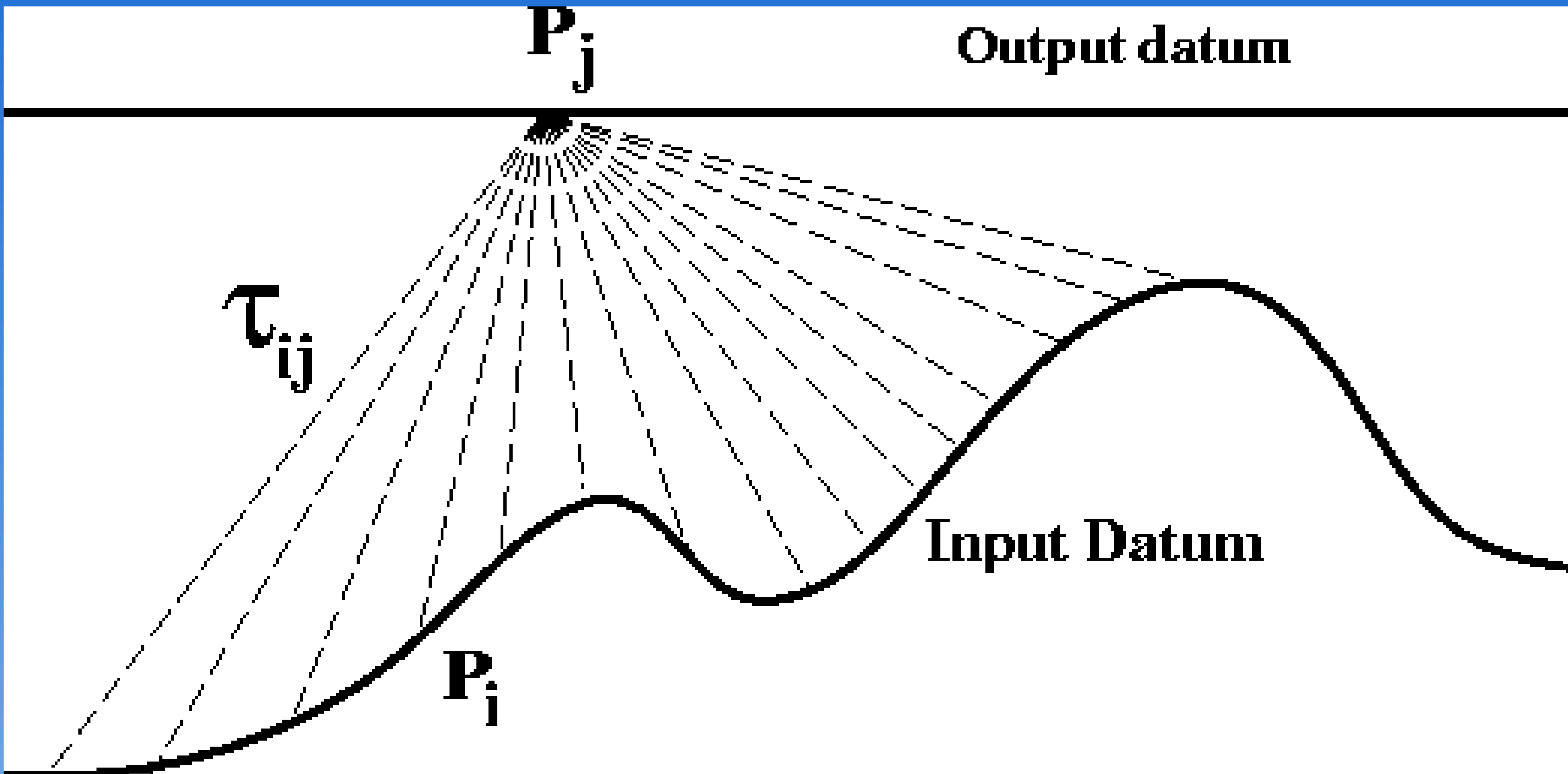
- Recent wave-equation processing solutions
  - ⋆ Flooding the topography (e.g., Bevc, 1997)
  - ⋆ Wavefield injection

Processing Solutions



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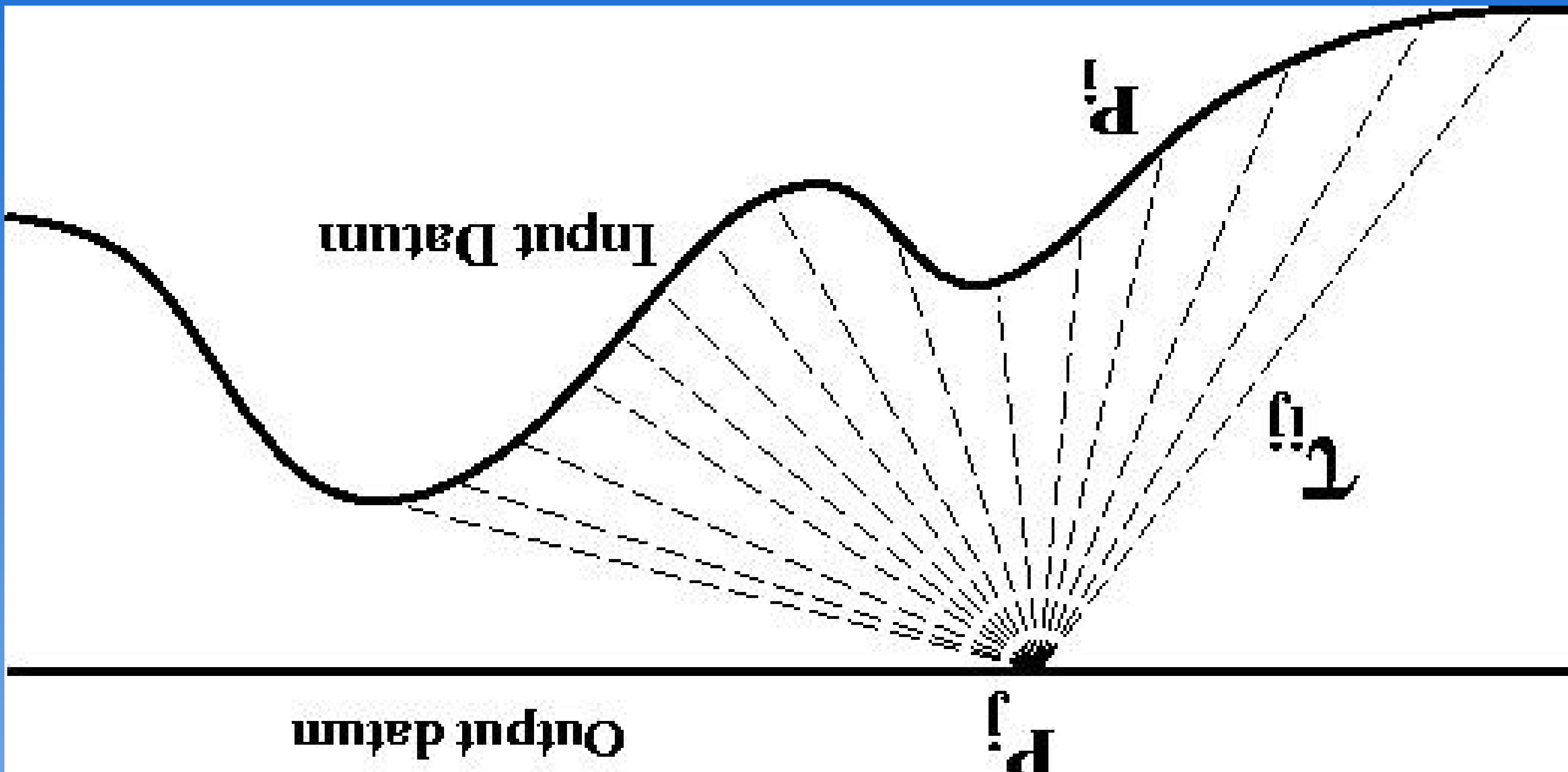
- Require significant preprocessing to conform with mesh
  - ⋆ How does this affect data fidelity?

Proposal

- Wave-equation migration from acquisition coordinates
  - ⋆ Honor data geometry

Proposal



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- What are the extrapolation equations appropriate for the topographic coordinate system?
  - ⋆ Requires: equations conformal with new geometry
  - ⋆ Riemannian wavefield extrapolation (RWE)

Agenda

- Conformal Mapping Overview
- RWE in 2-D
- Rocky Mountain Foothills example

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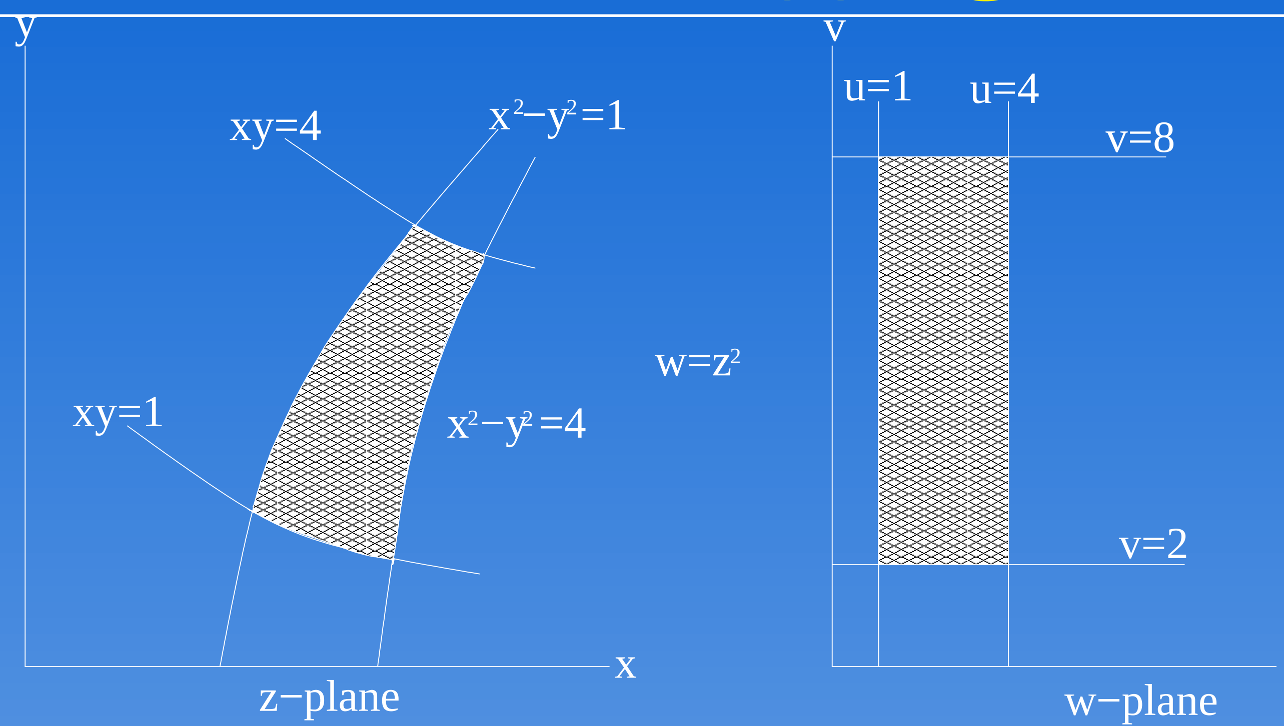
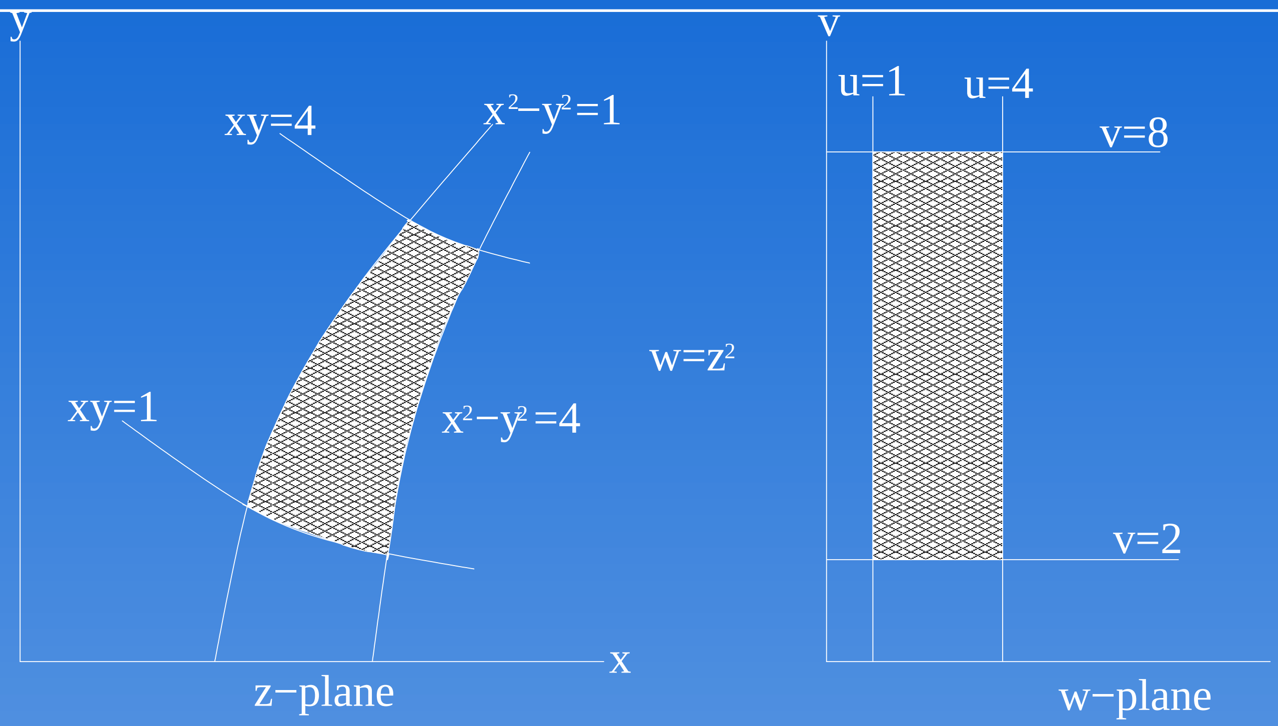
Conformal Mapping



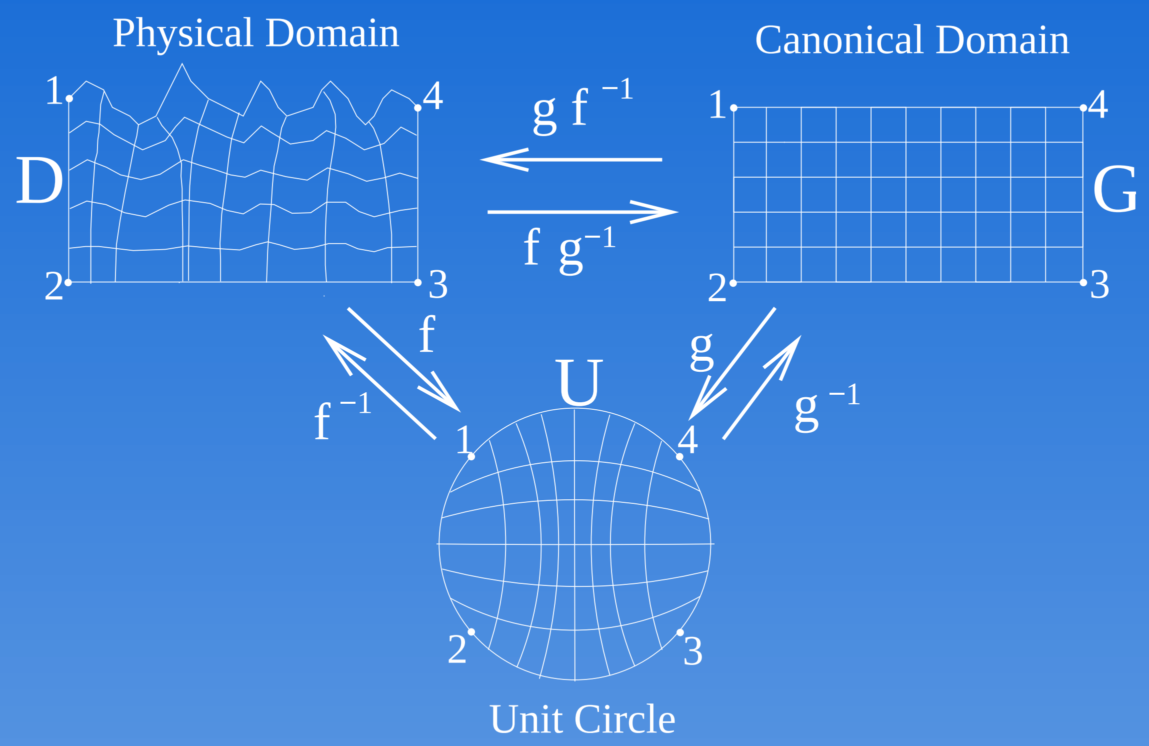
- Defines mapping of point sets between two different analytic domains in the complex plane
- $z = x + iy$ and $w = u(x, y) + iv(x, y)$ under $w = z^2$

Conservation of Angle



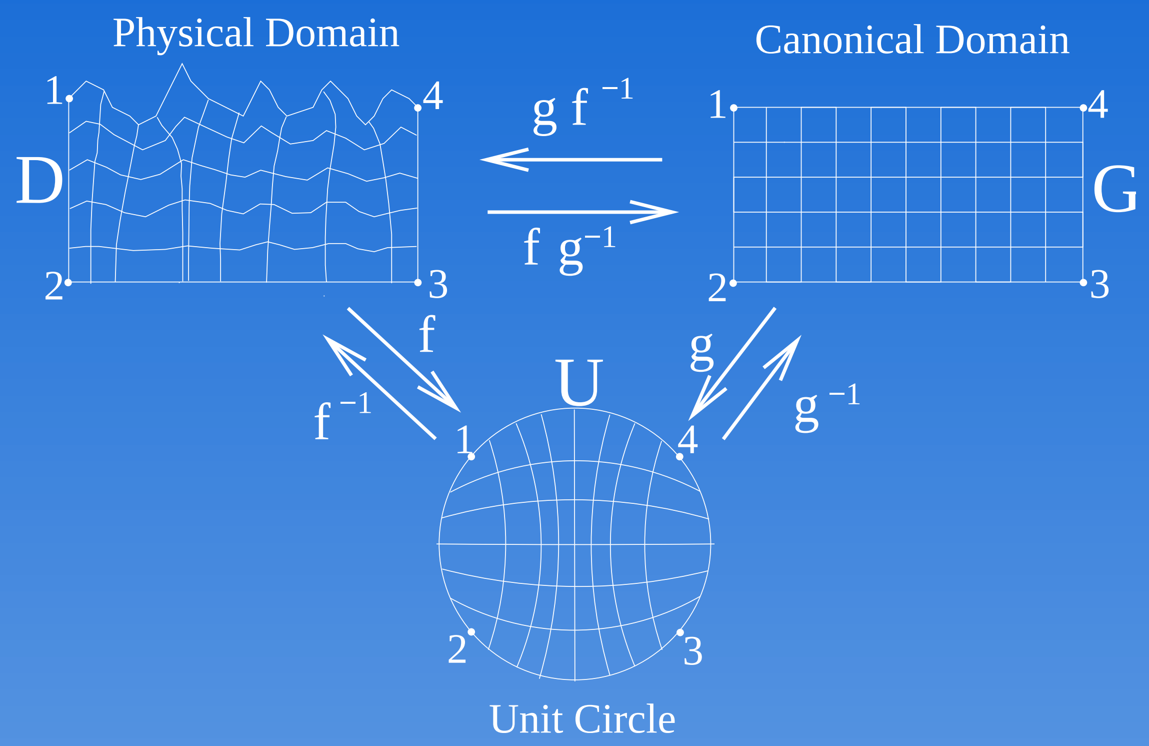
- Two continous arcs locally forming angle α_0 in the z-plane generate two continuous arcs separated by angle α_0 in the w-plane.

Riemann Mapping Theorem



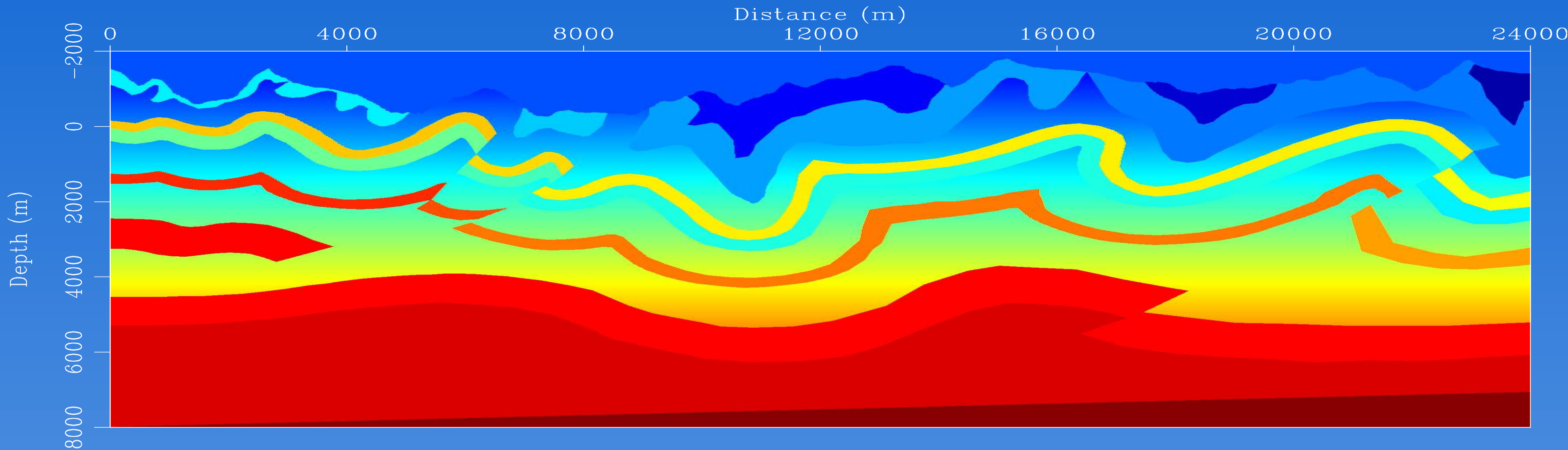
- Mappings $f : D \rightarrow U$ and $g : G \rightarrow U$ defined
- Composite mappings $f \cdot g^{-1} : D \rightarrow G$ and $g \cdot f^{-1} : G \rightarrow D$

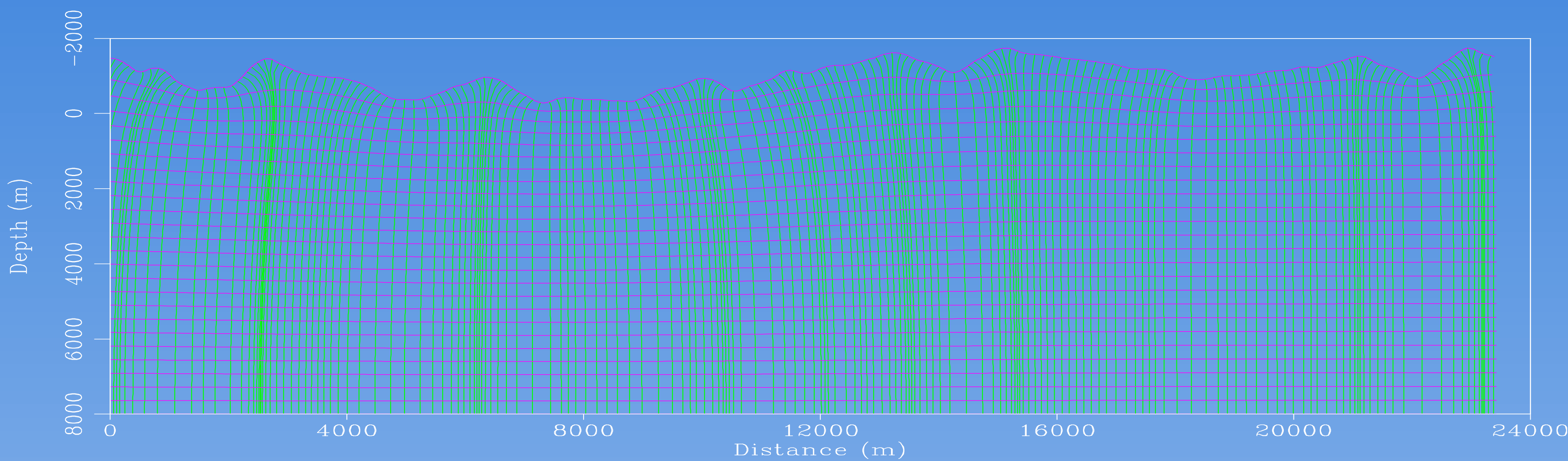
Riemann Mapping Theorem: Implications



- Topographic domain $\longrightarrow$ Cartesian through $w = f \cdot g^{-1}$
- $w \longrightarrow$ used to specify appropriate extrapolation equations

Riemann Mapping Theorem





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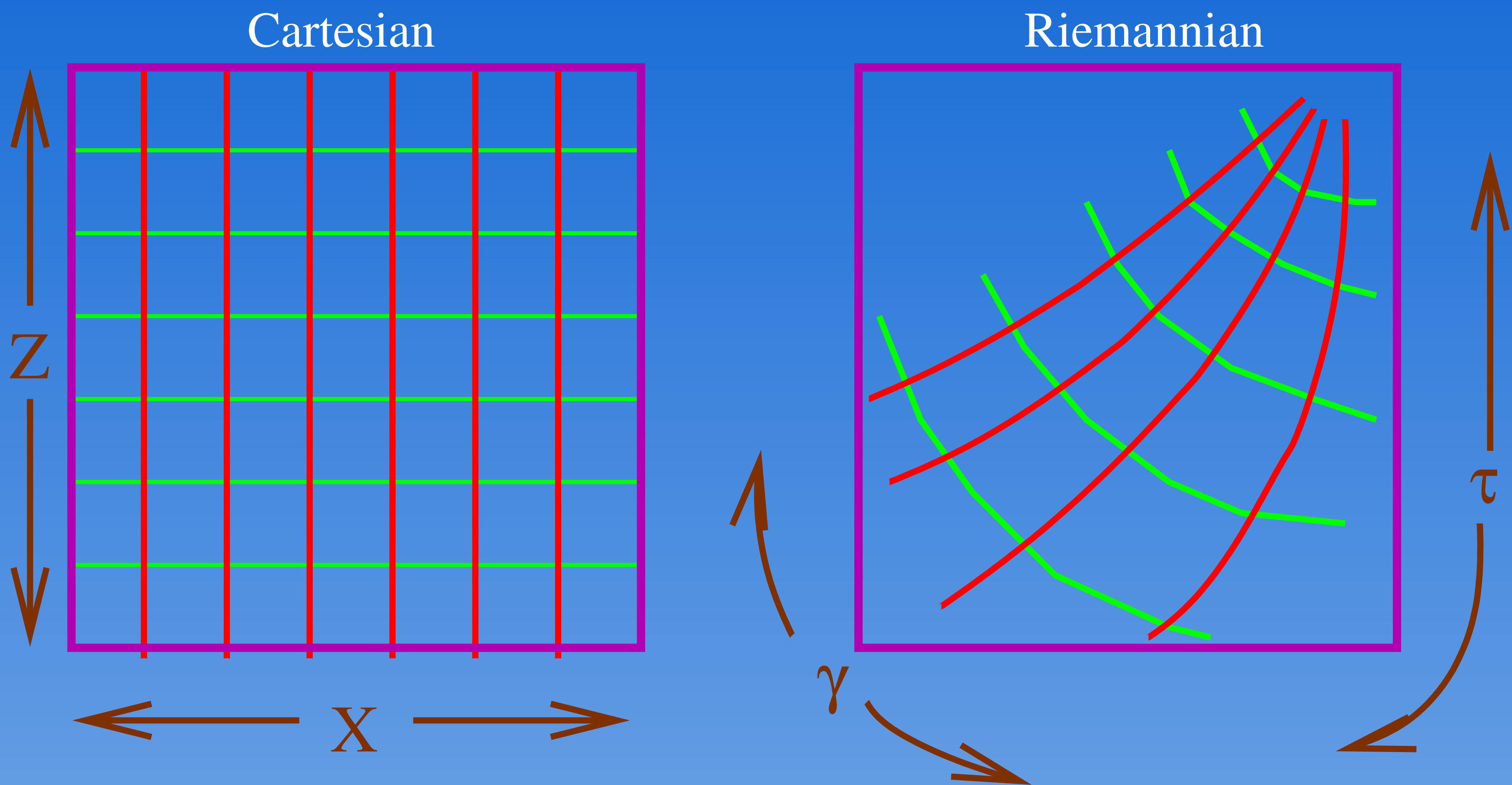
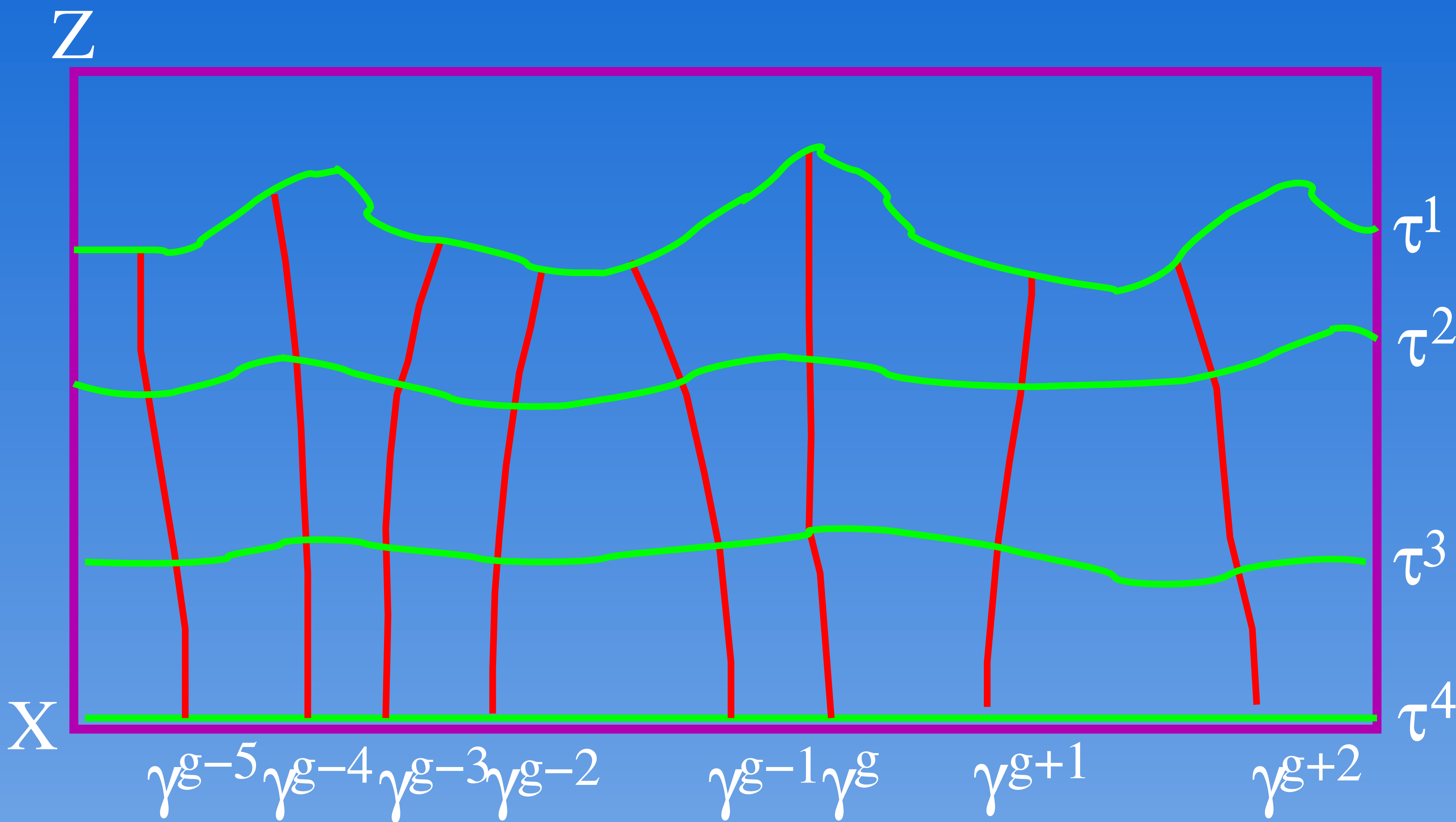
RWE in 2-D



Z Extrapolation Direction τ

X Orthogonal Direction γ

RWE in 2-D


Z
X
τ^1
τ^2
τ^3
τ^4
γ^{g-5} γ^{g-4} γ^{g-3} γ^{g-2} γ^{g-1} γ^{g} γ^{g+1} γ^{g+2}

RWE: Helmholtz Equation

- $\frac{1}{\alpha^2}\frac{\partial^2 U}{\partial \tau^2} + \frac{1}{\alpha J}\frac{\partial}{\partial \tau}\left(\frac{J}{\alpha}\right)\frac{\partial U}{\partial \tau} + \frac{1}{\alpha J}\frac{\partial}{\partial \gamma}\left(\frac{\alpha}{J}\right)\frac{\partial U}{\partial \gamma} + \frac{1}{J^2}\frac{\partial^2 U}{\partial \gamma^2} = -\omega^2 s^2 U$
- Ray-coordinate interpretation:
  - ⋆ $\alpha \longrightarrow$ extrapolation direction stretch factor
  - ⋆ $J \longrightarrow$ geometric spreading (i.e., Jacobian)
- Leads to dispersion relation
  - ⋆ $k_\tau = k_\tau\left(\omega, k_\gamma; \alpha, J, s\right)$
  - ⋆ Function of 3 mixed-domain variables (Cartesian has 1)
  - ⋆ $\hat{k}_\tau$: Phase-screen mixed domain approximation of k_τ

Shot-profile RWE

- Extrapolate Source and Receiver wavefields using
  - ⋆ $S(\tau+\Delta\tau,\gamma,\omega;s_i) = S(\tau,\gamma,\omega;s_i)\mathrm{e}^{-i\hat{k}_\tau\Delta\tau}$
  - ⋆ $R(\tau+\Delta\tau,\gamma,\omega;s_i) = R(\tau,\gamma,\omega;s_i)\mathrm{e}^{i\hat{k}_\tau\Delta\tau}$
- Apply imaging condition at each τ level
  - ⋆ $I(\tau,\gamma) = \sum_i \sum_\omega S(\tau,\gamma,\omega;s_i) R^\star(\tau,\gamma,\omega;s_i)$
- $I(\tau,\gamma)$ interpolated to Cartesian image $I(x,z)$
  - ⋆ Use sinc-based interpolation operators weighted by inverse Jacobian J

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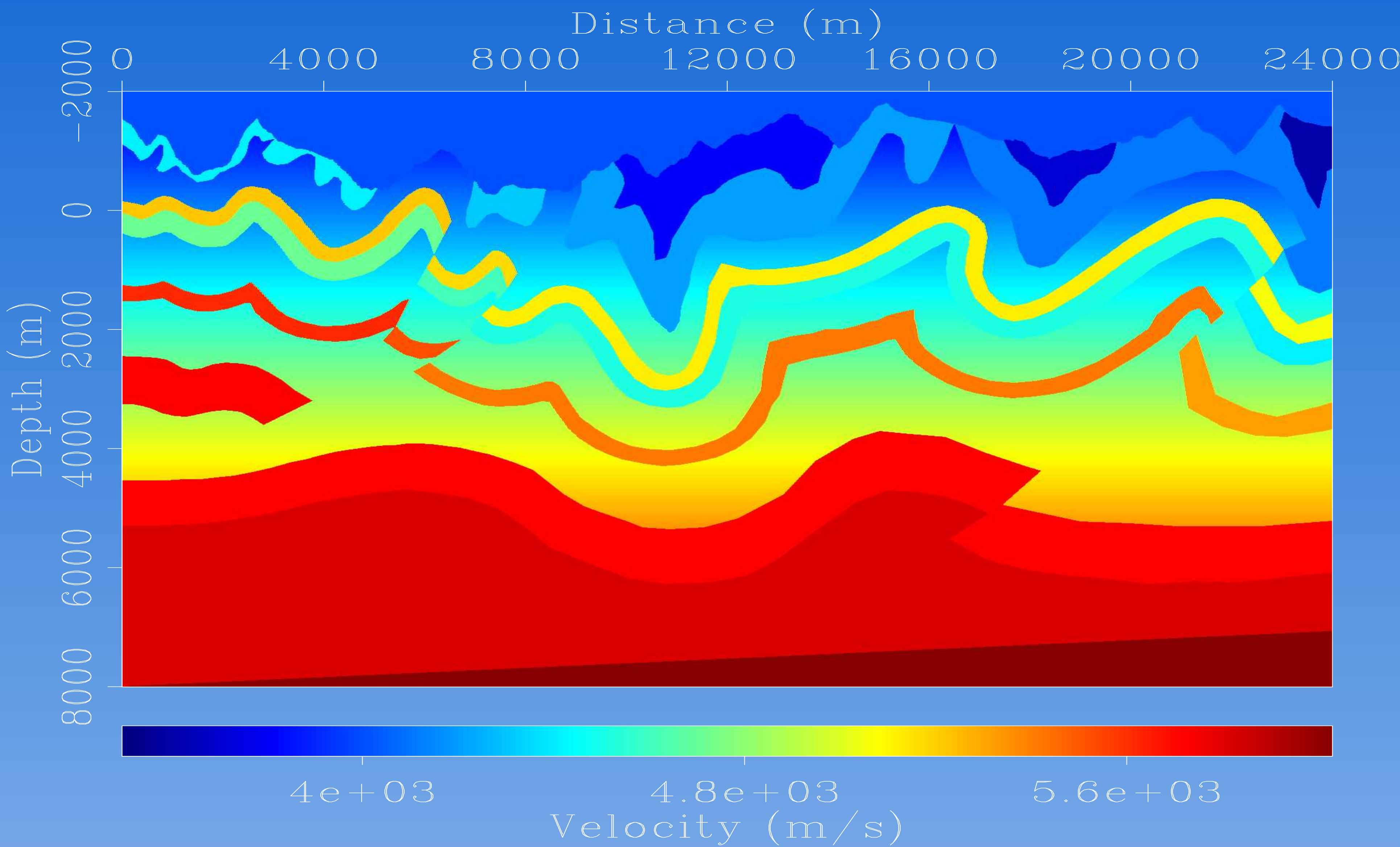
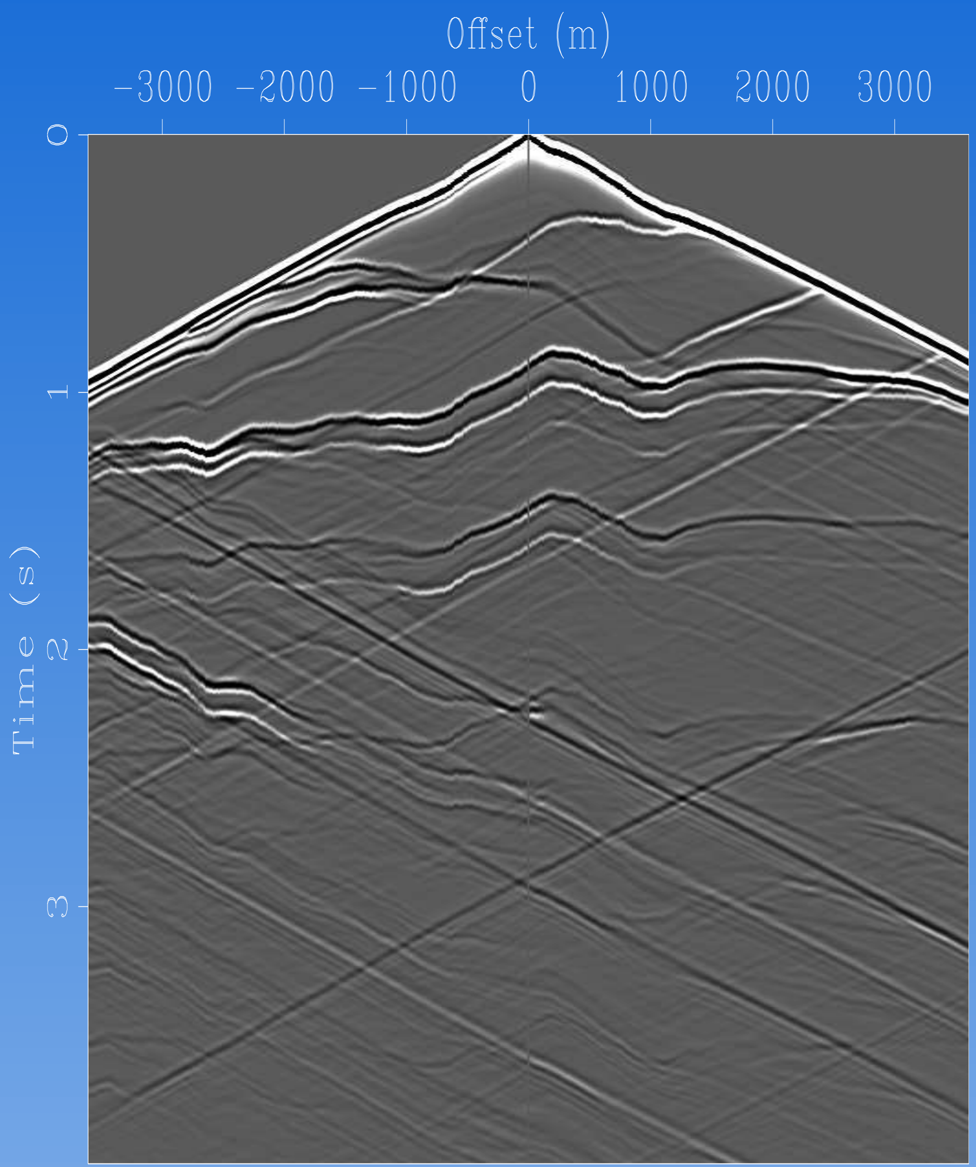
Foothills Model

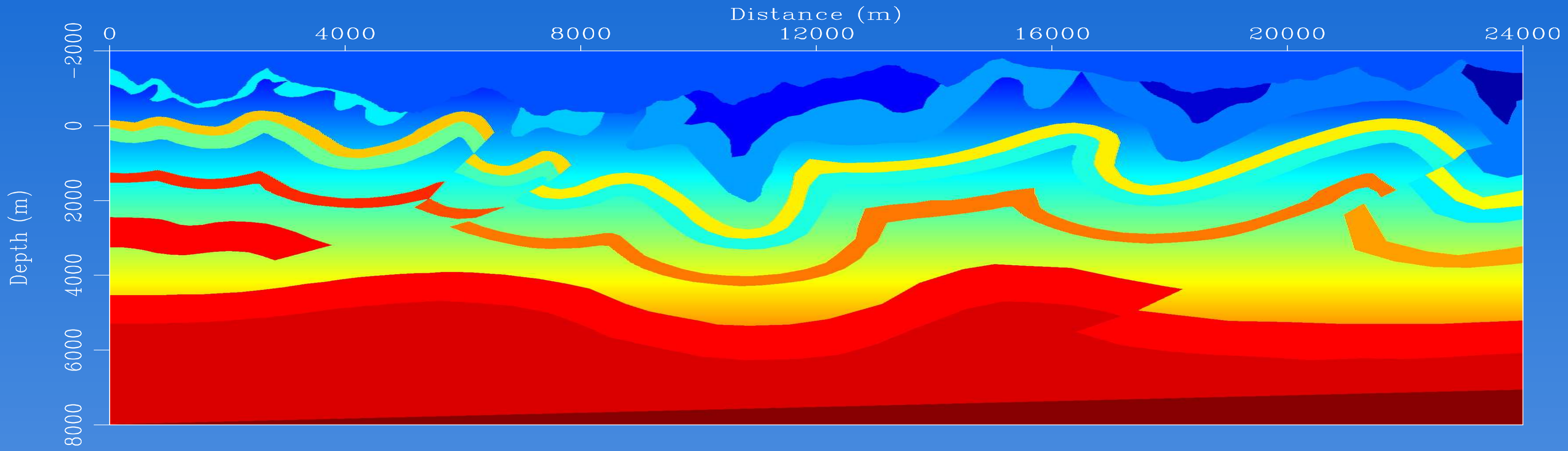


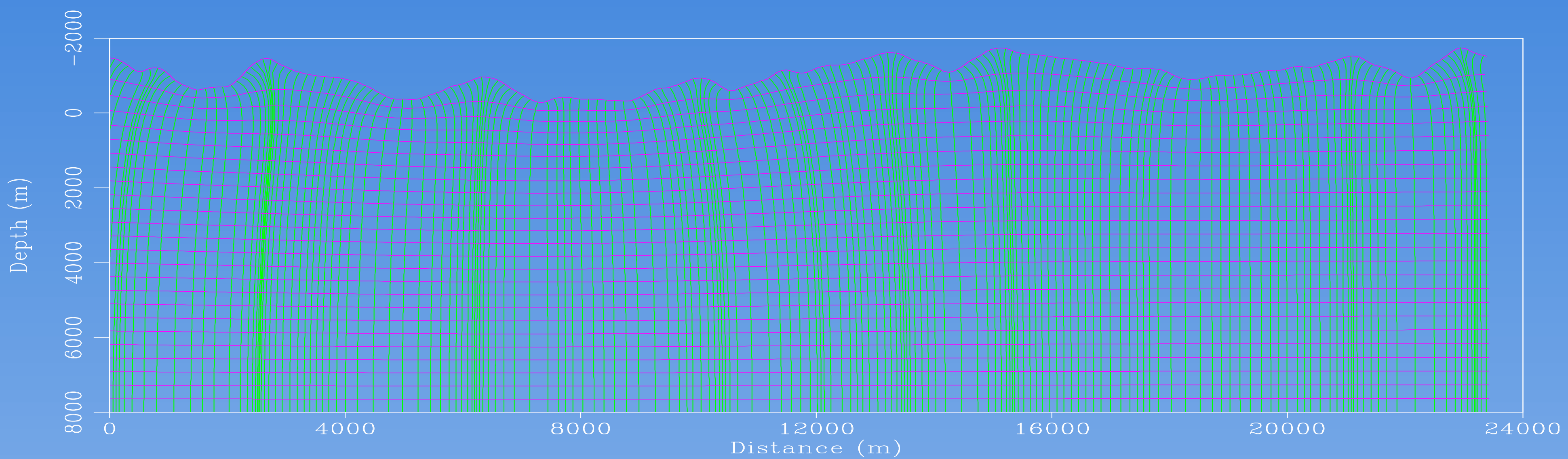
Foothills Data Set



- 278 shot gathers
- Δs=90m
- Δr=15m
- Max.offset = ±3600m
- Data generated on Cartesian mesh
  - ⋆ Interpolated to uniform spacing on topography

Foothills Coordinate System





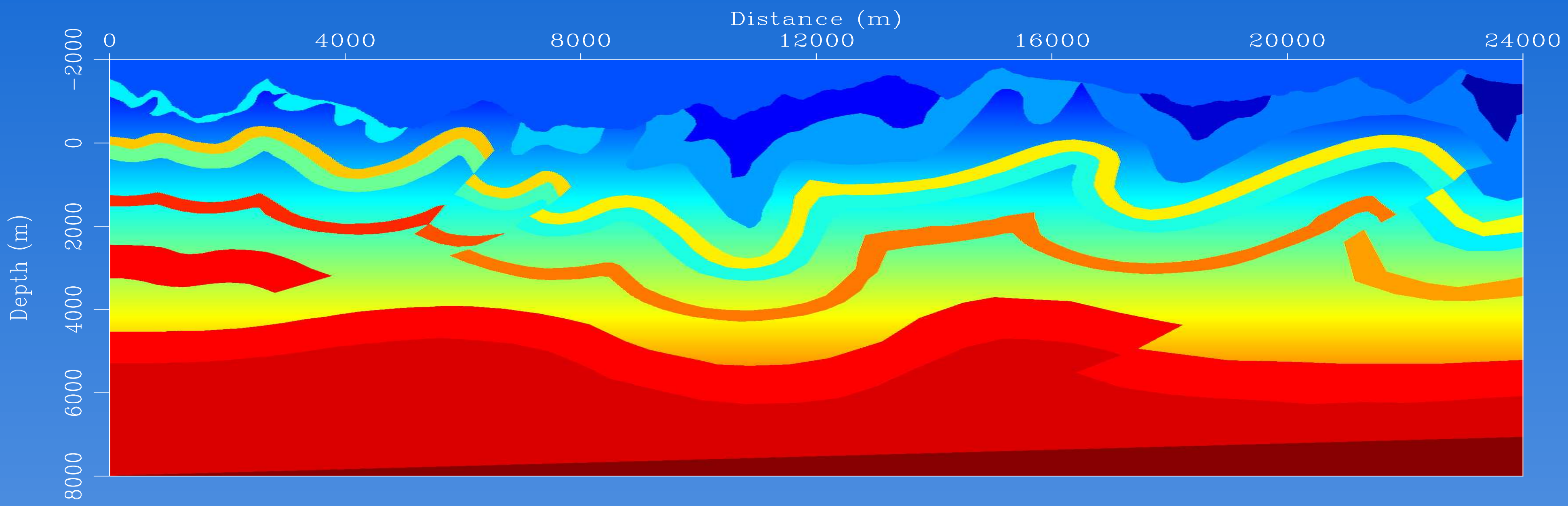
Wavenumber Approximation

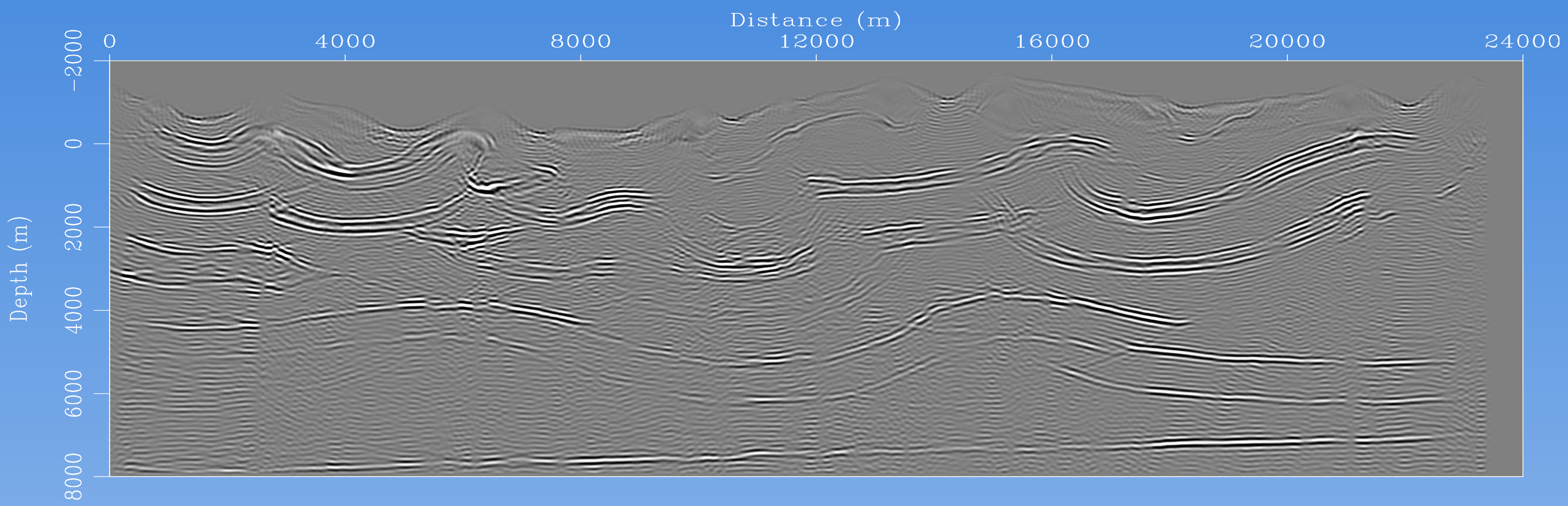
- Expand $k_\tau = k_\tau\left(\omega, k_\gamma; \alpha, J, s\right)$ about 3 reference slownesses

- Used 1 α reference value of - good approximation

- Used 1 J reference value - not so good approximation

Foothills Prestack Image





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- k_τ approximation is not accurate enough for large topographic system variation

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- Incorporate multiple reference J values
  - ⋆ Multidimensional interpolation between multiple expansions about J, s, and α
  - ⋆ Costs: 2 references for each parameter $\longrightarrow \approx N_{ref}^3$

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- k_τ approximation is not accurate enough for large topographic system variation
- Incorporate multiple reference J values
  - ⋆ Multidimensional interpolation between multiple expansions about J, s, and α
  - ⋆ Costs: 2 references for each parameter $\rightarrow \approx N_{ref}^3$
- Upward wavefield datuming (i.e., flood the topography)
  - ⋆ Remove the expansion about s
  - ⋆ Cartesian-based wave-equation migration from new datum

Conclusions

- Wave-equation migration directly from topography is achievable with minimal data preprocessing
- Conformal mapping generates orthogonal topographic coordinate systems
- Mappings help define appropriate wavefield extrapolation equations.
- Need multiple reference media to accurately represent extrapolation wavenumber

SEP 120: 3-D Coordinate Generation