Automatic denoising by 2-D continuous wavelet transform

Fantine Huot, Anthony Lichnewsky, and Carlos Boneti

ABSTRACT

We present a seismic processing workflow for identification, separation, and removal of specific wave modes, combining feature extraction by two-dimensional continuous wavelet transforms (2-D CWT) with machine learning algorithms. It addresses the challenges arising from temporally and spatially transient phases, which cannot be effectively addressed using conventional stationary filtering. We first transform the seismic data into a domain in which the signal of interest and the noise are well-separated. We establish a representation of the 2-D CWT output that is intuitive to understand, visualize and label. We characterize the noise in this domain and use a machine learning classifier to automate the noise identification process. We then design a filter to remove the unwanted noise modes and transform the data back to its original time domain. We demonstrate the effectiveness of the method by applying it to noisy marine acquisition shot gathers. The described method is computationally robust and its theory can be extended to higher dimensions. As a consequence, the methodology is applicable to any temporally and spatially continuous seismic dataset, both pre-stack and after imaging.

INTRODUCTION

Noise estimation, separation, and removal are core aspects of the seismic processing workflow. However, traditional signal processing techniques, commonly based on stationary filters (Claerbout, 1985; Yilmaz, 2001), do not perform well when confronted to noise modes that have temporal and spatial non-stationarity. In fact, they may even deteriorate some of the input signals frequency content in such cases. They often also involve human decision and dataset-dependent manual fine-tuning of the filter parameters. As a result, they are time-consuming, prone to errors, and the designed filters can be difficult to define analytically.

In this study, we present a signal processing workflow that separates and removes noise modes that are temporally and spatially non-stationary. Using two-dimensional continuous wavelet transforms, we first transform the data into a domain in which the signal of interest and the noise are well-separated. We establish a data representation that allows us to quickly identify and characterize the noise modes, and train a machine learning classifier to automate the noise identification process. We then filter the unwanted noise modes and use the inverse 2-D CWT to reconstruct the data in its original time domain. We demonstrate the effectiveness of the method by applying it to noisy marine acquisition shot gathers.

THE TWO-DIMENSIONAL CONTINUOUS WAVELET TRANSFORM

The suggested approach is applied on shot records, acquired in a marine survey. In the top row of Figure 1 we show two severely corrupted single shot gathers with different levels of noise. The signals of interest are heavily polluted by cable vibration noise. The signal and noise overlap, and the noise displays spatial and temporal coherence. It is clearly non-stationary and thus poses challenges to conventional noise removal techniques.

We use continuous wavelet transforms to account for the temporal and spatial non-stationarity of the different wave modes. The CWT is a multi-scale analysis method that compares the input signal to a stretched and compressed analysing wavelet usually called the mother wavelet (Daubechies, 1992; Farge, 1992; Mallat, 2008). While CWT is commonly used with a one-dimensional (1-D) wavelet (Torrence and Compo, 1998), herein we project the input data upon a basis of rotating 2-D wavelets.

A 2-D wavelet is a function ψ with a zero average:

$$\int_{\mathbb{R}^2} \psi(x)dx = 0.$$

To ensure the existence of the inverse 2-D CWT, the mother wavelet is chosen such that it satisfies the following admissibility condition:

$$C_\psi = (2\pi)^2 \int_{\mathbb{R}^2} \frac{|\hat{\psi}(\omega)|^2}{|\omega|} d\omega < +\infty,$$

where $\hat{\psi}$ denotes the Fourier transform of ψ.

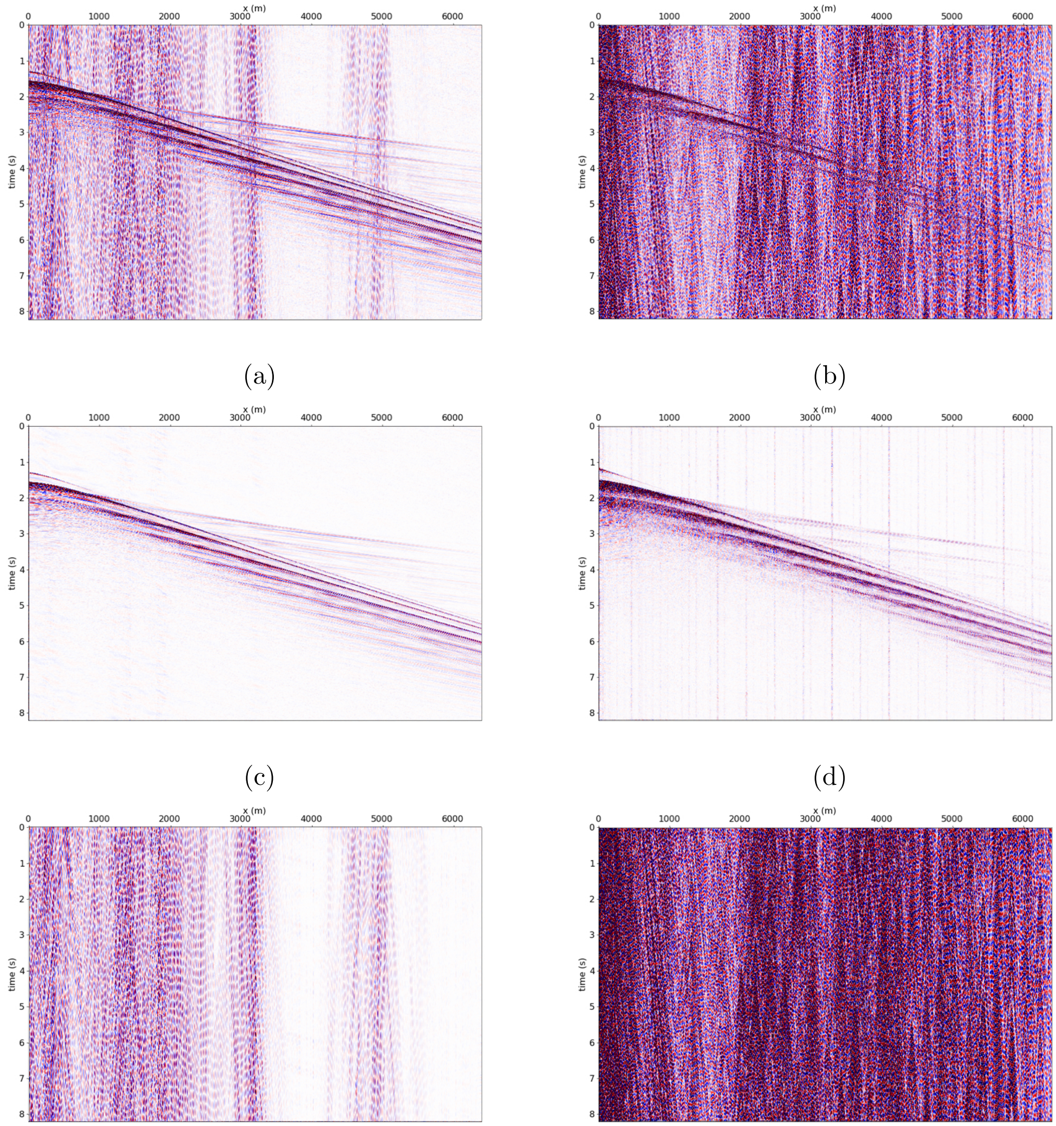
Given a 2-D mother wavelet ψ, we then compute the analysing wavelets $\psi_{s,u,\theta}$ by scaling by s, rotating by θ, and translating by u:

$$\psi_{s,u,\theta}(x) = \frac{1}{s}\psi\left(R_\theta^{-1}\left(\frac{x-u}{s}\right)\right),$$

where R_θ is a rotation matrix.

Given an input signal f, we then define the CWT coefficients W_f as:

$$W_f(s,u,\theta) = \langle f, \psi_{s,u,\theta}\rangle = \int_{\mathbb{R}^2} f(x)\psi^*_{s,u,\theta}(x)dx,$$



(a) (b)

(c) (d)

(e) (f)

Figure 1: (Top) Two marine acquisition seismic shot gathers, with different levels of noise. The signals of interest are severely corrupted and heavily polluted by cable vibration noise, which is non-stationary. (Middle) Noise removal results using the described methodology. We observe good signal protection. Note the direct arrivals, head waves and additional reflection phases that are invisible in the original data, especially in the low signal-to-noise ratio example on the right. The remaining regular vertical patterns are artifacts of the acquisition hardware and can be filtered with traditional methods. (Bottom) Corresponding noise models. Note the absence of signal leakage. [**NR**]

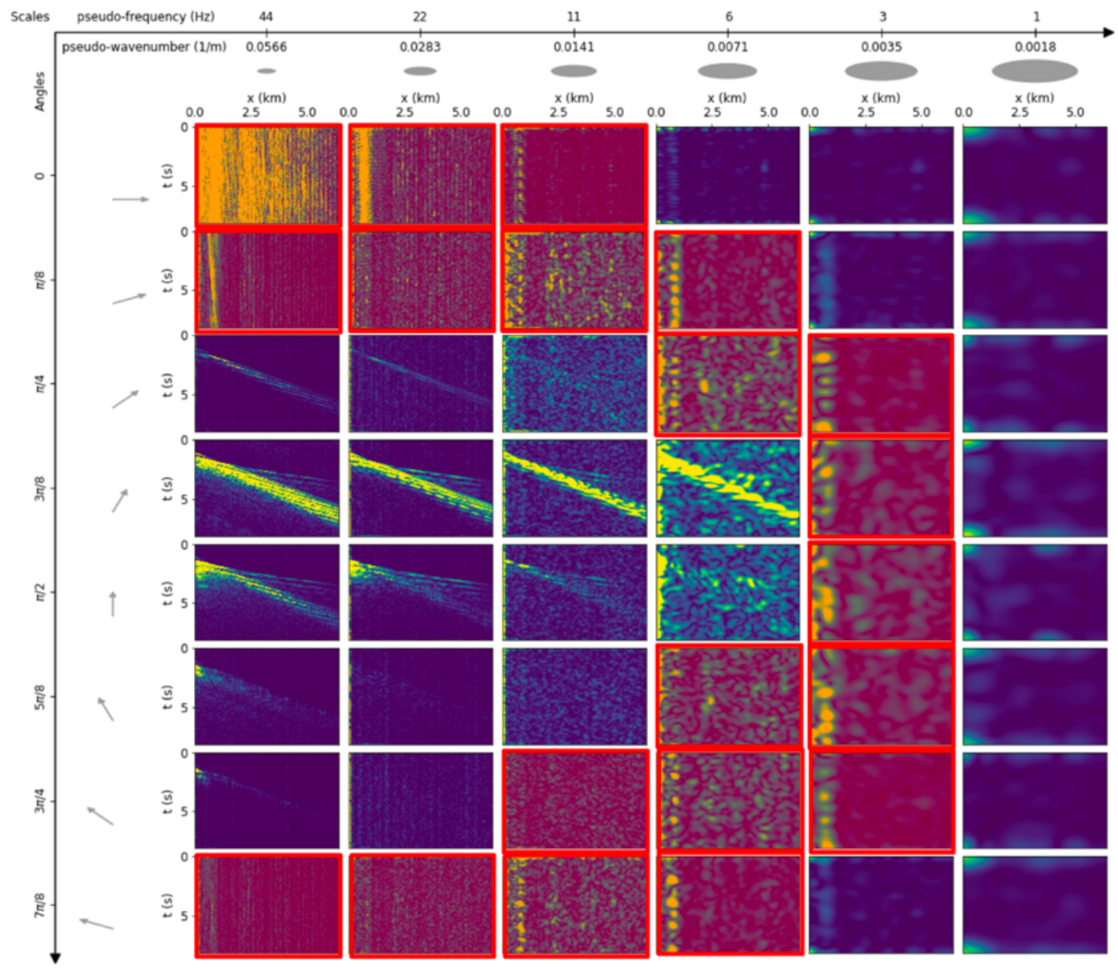


Figure 2: CWT coefficients computed on the shot gather from the top-right panel in Figure 1. Each small panel represents the amplitude of the complex CWT coefficients over travel time and offset. The panels are displayed by increasing scale (from left to right) and different rotation angles (from top to bottom) of the analysing wavelet. The wavelet scales are converted into equivalent pseudo-frequencies and pseudo-wavenumbers. For this dataset, the CWT seems to separate signal and noise efficiently. We can visually identify and separate different wave modes. The red squares denote the 2-D panels that the machine learning classifier flagged as noise. [**NR**]

where $*$ denotes the complex conjugate.

For this example, we use the two-dimensional Morlet kernel as 2-D mother wavelet (Daubechies, 1992; Mallat, 2008).

Since s and u are both 2-D variables, the output of this operation is a highly redundant 5-D tensor of time-frequency scales. 5-D data can be challenging to analyse, visualize, and interpret. Therefore, we first reduce the data dimensionality to 4-D by adopting the same wavelet scales over the time and space axes. This is achieved by converting the scales to their respective pseudo-frequency and pseudo-wavenumber equivalents, effectively transforming s from a 2-D variable into a 1-D variable. The values on this 1-D axis have to be computed with care as to match the different discretizations of the original axes to avoid introducing computing artifacts.

CWT coefficients are usually plotted with the wavelet scales as one of the axes, but this representation can be difficult to relate to physical properties. Instead, we project the CWT coefficients onto 2-D panels over axes that are equivalent to those of the input data. We then arrange these 2-D panels over a grid by increasing scales (left to right) and varying rotation angles (top to bottom) of the analysing wavelet, as illustrated in Figure 2. Under this representation, the various panels are interpretable, comparable, and most importantly, observed signal and noise modes are separated. This result is important since under this specific representation, the various wave modes can easily be identified and labelled. Since all the panels have matching dimensions, this feature representation is suitable for machine learning purposes.

WAVE MODE IDENTIFICATION

A clustering algorithm can be used to automatically identify similar CWT panels, letting the user then choose which wave mode cluster to filter or to keep (Hastie et al., 2005; Martin et al., 2018; Huot et al., 2017). Herein we manually label a subset of CWT panels for one of the shot gathers, and use a support vector machine (Hastie et al., 2005; Pedregosa et al., 2011), to separate the CWT panels for the remaining shot gathers (Huot et al., 2018). While classifying at the panel level shows decent separation, more sophisticated classification schemes can also be considered.

FILTERING AND INVERSE CWT

We use a simplistic filtering scheme, classifying the CWT coefficients at the panel level. We then mute the panels corresponding to the noise modes and proceed to reconstructing the signal. The CWT has an exact analytical inverse which allows us to transform the data back to its original domain:

$$f(x) = \frac{1}{C_\psi} \int_0^{+\infty} \frac{ds}{s^3} \int_{\mathbb{R}^2} du \int_o^{2\pi} W_f(s,u,\theta)\psi_{s,u,\theta}(x) d\theta.$$

The CWT can be computed efficiently using FFTs and its inverse is a computationally robust operation since it is based on weighted summations. The redundant coefficients make the transformation robust even when many coefficients are muted. As a typical seismic survey can represent terabytes of data, such computational properties are highly desirable. The reconstructed data after the filtering process is presented in the middle row of Figure 1. We observe good signal protection, despite using a simplistic automated filtering scheme without explicit signal preservation. Several recorded phases can be seen and analysed, e.g. direct waves, head waves, and later reflection events. The remaining regular vertical patterns are artifacts of the acquisition hardware and can be filtered with traditional methods.

DISCUSSION AND CONCLUSIONS

The described seismic processing workflow allows for identification, separation, and removal of specific noise modes. It addresses the challenges arising from temporally and spatially transient phases, which cannot be effectively addressed using conventional stationary filtering. It is computationally robust and its theory can be extended to higher dimensions. As a consequence, the methodology is applicable to any temporally and spatially continuous seismic dataset, both pre-stack and after imaging. We establish a representation of the 2-D CWT output that is intuitive to understand, visualize and label. It is thus suited for machine learning algorithms. We apply the inverse transform using automatically chosen CWT panels and retrieve time-domain data. Noise is suppressed and many new seismic phases are visible.

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