Distance Regularized Level Set Salt Body Segmentation

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ABSTRACT

Segmentation of seismic images using a Distance Regularized Level Set Evolution (DRLSE) scheme maintains numerical stability of our implicit surface without the expense and accuracy issues associated with reinitialization approaches. In this work I apply the DRLSE algorithm to the Sigsbee salt model as well as an offshore salt data set. I then apply a modified energy functional which includes a Frobenius norm term that further improves the segmentation results. These applications of DRLSE demonstrate promising results using a very simplified energy functional.

INTRODUCTION

Delineating the boundaries of salt bodies is important to sub-salt depth imaging as well as inversion schemes such as Full Waveform Inversion (FWI). Explicit parametrization approaches such as "Snakes" (Kass et al., 1988) and "Intelligent Scissors" (Mortensen and Barrett, 1998) methods encounter difficulty when changes of topology are necessary, or if there is a need to adapt to sharp corners and cusps on an object boundary. The implicit approach of the level set method avoids these issues (Lee, 2005), but requires a mathematically regular implicit surface to be successful (no sharp edges or "creases"). Standard level set evolution schemes lack an intrinsic regularization of the implicit surface itself. When the level set equation is derived from a variational approach, an energy functional is used which can accommodate a single or double-potential well term. Including this term maintains regularity of the implicit surface, without having to resort to reinitialization schemes as in standard level set algorithms. Furthermore, the variational approach to deriving the level set evolution equation allows for a conceptually straight-forward framework for adding additional terms to guide the surface evolution. In this work, I begin by analyzing the shortcomings of explicit parametrization approaches as well as the standard level set formulation. Next, I derive the DRLSE evolution equation using a variational approach. After that, I explain the use of a Frobenius norm term as particularly suited to salt body delineation. Last, I show the results of applying the Distance Regularized Level Set Evolution (DRLSE) algorithm to a salt model and salt data set, followed by a discussion of future work.

LEVEL SET FUNDAMENTALS

Explicit methods of parametrizing curve boundaries expose us to a number of numerical problems. These include node distribution, evolving curves at sharp corners and cusps, as well the challenge of preventing evolving curve fronts from overlapping. By adding an extra dimension to the optimization problem, we can eliminate these issues, which is the main attraction behind using level set methods. In the case of segmenting a 2D object, level set methodology expands the optimization space from a 2D (explicitly defined) curve to a 3D surface (which implicitly defines a curve). We evolve a surface ϕ such that the curve represented by the contour at $\phi = 0$ (the zero level set) is the boundary of the object that we wish to segment (Osher and Fedkiw, 2003). We choose force functions to evolve this surface in order to meet this end. These force functions are designed to conform the surface such that it delineates the segmentation object at $\phi = 0$. Equation 1 shows us the standard form of the level set equation, with F representing a directional force applied to the implicit surface ϕ.

$$\frac{\partial \phi}{\partial t} = |F \nabla \phi| \tag{1}$$

Since this equation lacks an intrinsic means to maintain regularity, irregular or sharp features can form on the surface as it evolves, causing further evolution to be unstable. One approach to remediating this is to periodically reinitialize to a regular surface (Adalsteinsson and Sethian, 1995). When the implicit surface begins to become unstable (develops very sharp or flat shapes), the function is reinitialized by solving

$$\frac{\partial \phi}{\partial t} = \mathrm{sign}(\phi_o)(1 - |\nabla \phi|) \ . \tag{2}$$

However, this approach is not always very accurate, especially when the implicit surface is not smooth or if there is a strong difference between the signed distance function and the surface being reinitialized. Furthermore, the question of how often reinitialization should be applied makes such approaches ad-hoc at best, and often expensive to implement well. For these reasons, the simplicity and robustness of deriving the evolution equation from a variational approach is a very attractive option.

DISTANCE REGULARIZED LEVEL SET EVOLUTION

The DRLSE method is derived using a variational method. In our case, we intend to find the stationary value (minimum) of an energy functional. A standard approach to minimizing an energy functional is to find the steady state solution to the gradient flow equation

$$\frac{\partial \phi}{\partial t} = -\frac{\partial E}{\partial \phi} \ . \tag{3}$$

From this, we substitute E with an energy functional that we minimize, as described in the next section. Because E is an energy functional derived using calculus of variation, the partial derivative shown in equation 3 becomes a Gâteaux derivative. The Gâteaux derivative is a generalization of directional derivative, and is used to formalize the functional derivative used in calculus of variations.

External Energy

The energy functional that I use in this work can be represented very simply as the sum of a regularization term coupled with an external energy term

$$E(\phi) = R_p(\phi) + E_{ext}(\phi) \ . \tag{4}$$

The external energy term can be defined to include any term that utilizes information pertinent to the data of interest. We begin with a simple external energy term that is based on an active contour model, utilizing edge-based information, and composed of two terms that work in conjunction

$$E_{ext}(\phi) = \lambda L_g(\phi) + \alpha A_g(\phi) \ . \tag{5}$$

L_g is an "edge" term acting to direct the curve evolution towards areas of strong contrast, while A_g is an "area" term that acts to accelerate the curve evolution by providing a ballooning or shrinking force (depending on how α is chosen). The "edge" term computes a line integral of the function g along the zero level set contour,

$$L_g(\phi) = \int_\Omega g\delta_\epsilon(\phi)\,|\nabla\phi|\,dx \ , \tag{6}$$

while the "area" term (equation 7) computes a weighted area of the region inside the zero level set contour

$$A_g(\phi) = \int_\Omega gH(-\phi)dx \ . \tag{7}$$

Within the external energy functional terms (equations 6 and 7), I use a function g as the actual edge indicator

$$g = \frac{1}{1 + |\nabla(G_\sigma * I)|^2} \ . \tag{8}$$

In this case I is the amplitude of a pixel in the image we are segmenting, and G is a Gaussian smoothing function. By taking the gradient of this convolution, g tends to have smaller values at edge boundaries, acting as a very simple edge indicator function.

To detect the zero level set curve in our energy functional, it is necessary to construct a modified Dirac delta function that smoothly indicates the boundary

$$\delta_\epsilon(x) = \begin{cases} \frac{1}{2\epsilon}(1 + \cos(\frac{\pi x}{\epsilon})), & |x| \leq \epsilon \\ 0, & |x| > \epsilon \end{cases} . \tag{9}$$

We use ϵ to set the window over which our smoothed Dirac delta function is applied.

For the area term as well as the Frobenius norm term (described later), we integrate g over the area enclosed by the zero level set. As with the Dirac delta function, I use a modified Heaviside function to smoothly indicate the boundary of the zero level set within a window defined by ϵ

$$H_\epsilon(x) = \begin{cases} \frac{1}{2}(1 + \frac{x}{\epsilon} + \frac{1}{\pi}\sin(\frac{\pi x}{\epsilon})), & |x| \leq \epsilon \\ 1, & x > \epsilon \\ 0, & x < -\epsilon \end{cases} . \tag{10}$$

Regularization Term

To maintain numerical stability, we need to define the regularization term R_p such that it maintains the property of a function that is mathematically regular. For example, the signed distance function is very commonly used to initialize our level set algorithm because of its simplicity, but more importantly because it represents a regular implicit surface. Within the energy functional (equation 4) I include a regularization term in order to maintain the signed distance property of our surface as we evolve it

$$R_p(\phi) = \int_\Omega p(|\nabla\phi|)dx . \tag{11}$$

The signed distance function has the property of having a constant gradient (equal to one). For this reason, I construct a potential-well function

$$p_1(|\nabla\phi|) = \frac{1}{2}(|\nabla\phi| - 1)^2 , \tag{12}$$

such that R_p is minimized when $|\nabla\phi| = 1$ (Li, 2010).

However, while using equation 12 allows us to maintain the signed distance property (and consequently regularity of the implicit surface), this formulation is not necessarily stable when we take the Gâteaux derivative of R_p and incorporate it into the gradient flow equation. Taking the Gâteaux derivative of equation 11, we get

$$\frac{\partial R_p}{\partial\phi} = -\mathrm{div}(d_p(|\nabla\phi|)\nabla\phi) , \tag{13}$$

where div is the divergence operator, d_p is a function defined by

$$d_p(|\nabla\phi|) = \frac{p^{'}(|\nabla\phi|)}{|\nabla\phi|} \ , \tag{14}$$

and $p^{'}$ is the first derivative of function p. With our choice of single-well potential function, equation 14 is simplified to $d_p(|\nabla\phi|) = 1 - \frac{1}{|\nabla\phi|}$. As a result, in the case where $|\nabla\phi| = 0$, our evolution becomes unstable since $d_p = -\infty$. For this reason, in order to avoid instability in areas where $|\nabla\phi|$ gets close to zero, I modify the potential function to have two wells; one at $|\nabla\phi| = 1$ as before, and a well at $|\nabla\phi| = 0$. This motivation leads us to define a double well equation

$$p_2(|\nabla\phi|) = \begin{cases} \frac{1}{(2\pi)^2}(1 - \cos(2\pi|\nabla\phi|)), & \text{if } |\nabla\phi| \leq 1 \\ \frac{1}{2}(|\nabla\phi| - 1)^2, & \text{if } |\nabla\phi| \geq 1 \end{cases} . \tag{15}$$

When the regularization and external energy terms are included together in equation 4, we derive the following complete energy functional

$$E(\phi) = \mu \int_\Omega p_2(|\nabla\phi|)dx + \lambda \int_\Omega g\delta_\epsilon(\phi)\,|\nabla\phi|\,dx + \alpha \int_\Omega gH_\epsilon(-\phi)dx \ . \tag{16}$$

The constants μ, α and λ act as weights for each of the terms in our energy functional.

Combining the energy functional term above with the gradient flow equation (equation 3) that our derivation is based on, we get an equation that represents the evolution of our implicit surface, and can be directly implemented with a finite difference scheme

$$\frac{\partial\phi}{\partial t} = \mu\mathrm{div}(d_p(|\nabla\phi|)\nabla\phi) + \lambda\delta_\epsilon(\phi)\mathrm{div}\left(g\frac{\nabla\phi}{|\nabla\phi|}\right) + \alpha g\delta_\epsilon(\phi) \ . \tag{17}$$

Use of the Frobenius Norm for Salt Delineation

One attribute of salt that can be useful in segmentation is the lack of stratification that is typically observed. A salt body as viewed in seismic data can usually be characterized as having a chaotic image gradient in the interior. One way that we can make use of this attribute in our DRLSE algorithm is by incorporating an appropriate term in our energy functional to quantify the chaos of the image gradient. By calculating the structural tensor in a neighborhood around each pixel

$$S(p) = \begin{Bmatrix} \sum_i I_{x_i}I_{x_i} & \sum_i I_{x_i}I_{y_i} \\ \sum_i I_{y_i}I_{x_i} & \sum_i I_{y_i}I_{y_i} \end{Bmatrix} \ , \tag{18}$$

and then applying a variation of the Frobenius norm on said tensor, we are able to generate a map of values that represent the strength of stratification (Haukas, 2013).

In the representation shown in equation 18, I_{x_i} and I_{y_i} are the x and y gradient values of the image at index i, where i represents a pixel that exists in the neighborhood surrounding pixel p. For some pixel p, a structure tensor is generated by summing all m of the gradient multiplier elements, where m is equal to the number of terms in the neighborhood stencil used. In this case the neighborhood is a square image of size $m = n \times n$ centered around pixel p.

At this point, we can either calculate the eigenvalues $E(S(p))$ and perform the proper Frobenius norm

$$\|E(S(p))\|_{\text{Frob}} = \sqrt{\sum_{i=1}^{2}\sum_{j=1}^{2} |E(S(p))_{ij}|^2}\ , \tag{19}$$

or we can avoid the eigenvalue decomposition (and its added cost) and sum the square of the diagonal elements of $S(p)$ as an approximate measure. I choose the latter approach for this work.

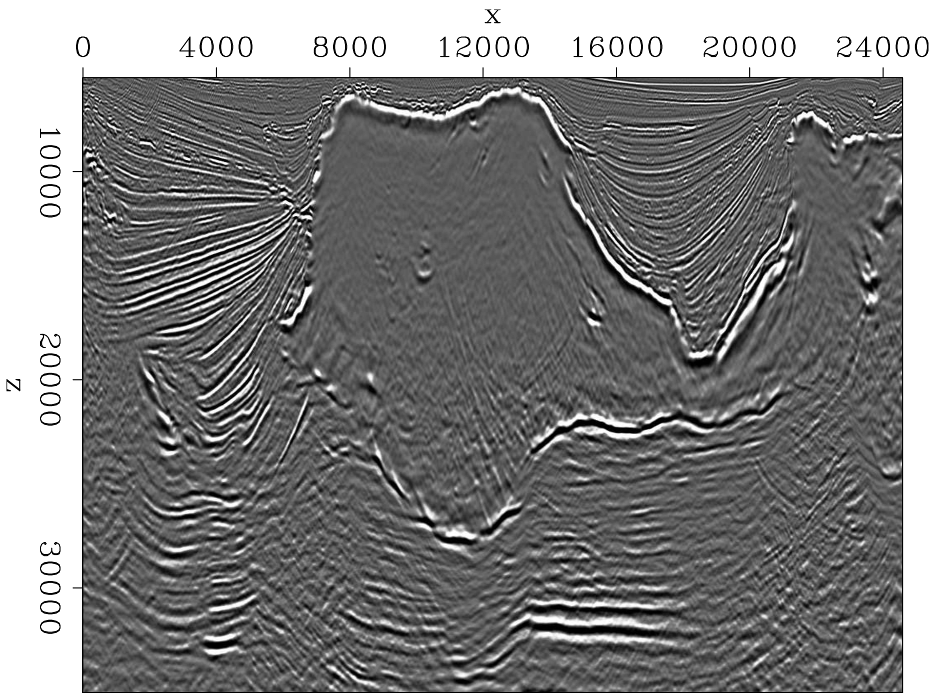
When we compare the stratification in the input data (Figure 1(a)) with the map of the Frobenius norm values (Figure 1(b)), we can see that the term does prove useful in identifying zones of salt.

EXAMPLES AND DISCUSSION

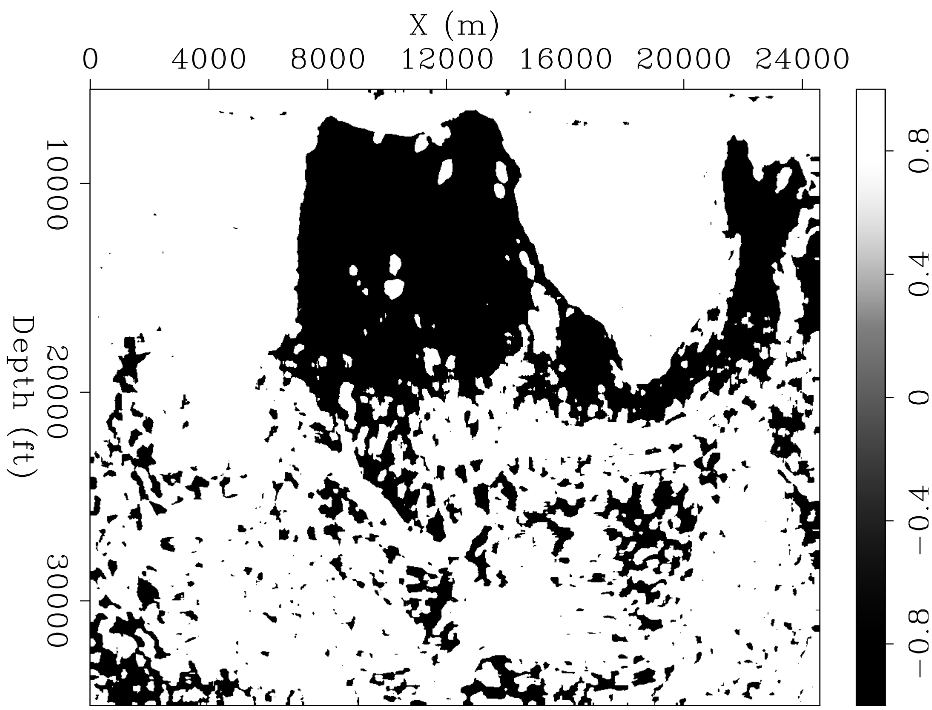
The DRLSE algorithm converges quite well to the Sigsbee salt model (Figure 2(a)). This example demonstrates the effectiveness of the edge finding terms that our simple energy functional includes. What can be noted is that the segmentation is unable to differentiate between the salt body (that was "selected" using the initial surface) and the water body layer above it. This energy functional as currently formulated doesn't utilize the pixels within the initialized area to help determine the evolution of the level set. This is certainly an aspect that could be improved upon through incorporating an appropriate term in equation 4.

One obvious need in the results from this formulation is the lack of smoothness in the zero level set produced. Figure 3 demonstrates this lack of smoothness in the form of a "sprinkled" scattering of small segmented areas. Currently my research includes developing an effective smoothing term to add to the energy functional, namely by minimizing the curvature of the zero level set (Caselles et al., 1997). A term based on this attribute would mitigate the occurrence of "sprinkled" segmentation because these "sprinkled" regions contribute heavily to the total curvature, but are only a small part of the area within the zero level set.

A level set method, in particular the DRLSE method, is well suited to being incorporated into the work-flow of FWI. The gradient of the objective function can be used as a force to evolve the implicit surface (Lewis et al., 2012). When compared to other segmentation methods such as combinatorial graph cuts, the level set approach

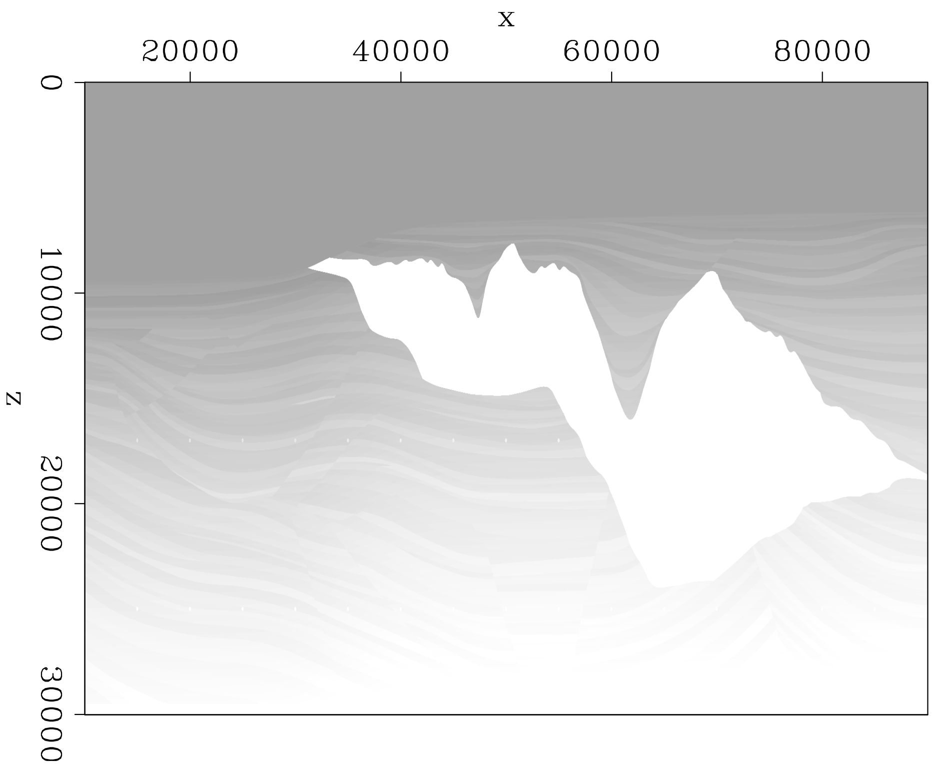


(a)

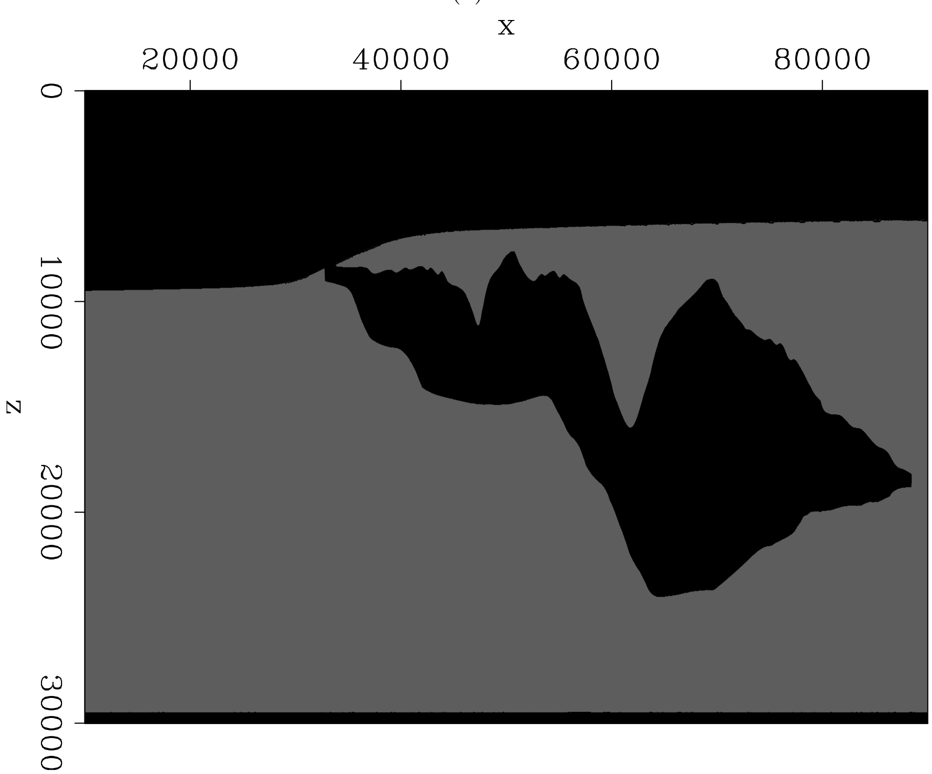


(b)

Figure 1: (a) Initial salt body data input (b) Map of Frobenius norm values, rescaled to binary representation. [**ER**]



(a)



(b)

Figure 2: (a) Initial salt body model input (Sigsbee Model) (b) Level set output (without Frobenius norm). [**ER**]

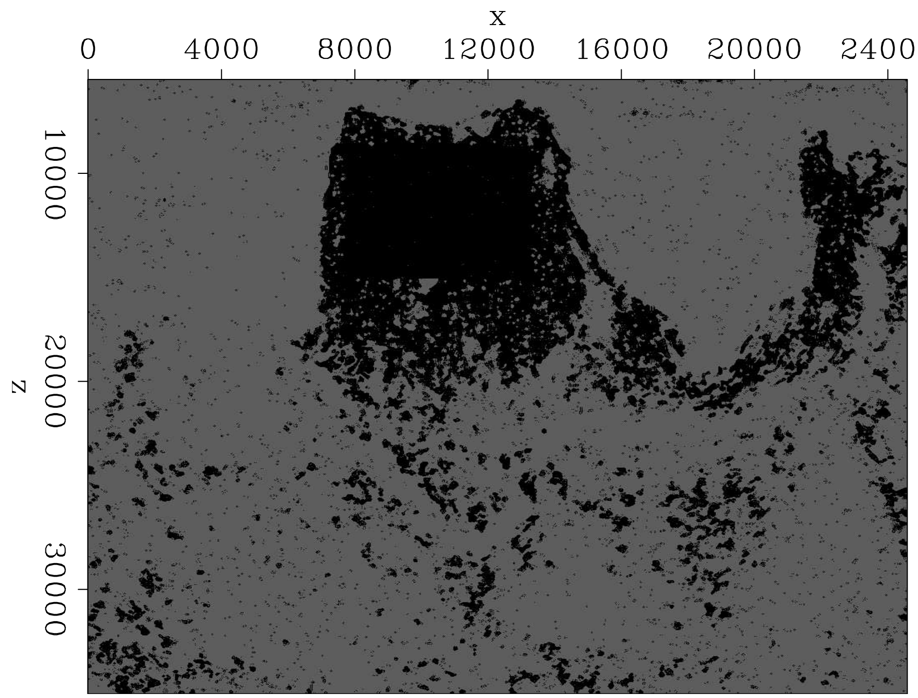


Figure 3: Level set output (with Frobenius norm term included). [**ER**]

is more appropriate for FWI adaptation since it doesn't require an input of hard constraints. The initialized surface that the algorithm uses as input can come from the initial FWI velocity model, or from a human input. However, these initial zero level sets are not hard constraints as in combinatorial graph cuts, and the accuracy of the input relative to the true segmentation is not as critical to a successful segmentation. Future extension of this work aims to incorporate the DRLSE algorithm into a FWI work-flow for semi-automatic salt body segmentation.

CONCLUSION

The level set method provides distinct advantages over explicit parametrization approaches when applied to seismic imaging segmentation problems. When it is derived with variational methods as a regularized algorithm, the problems associated with maintaining numerical stability in the implicit surface are ameliorated. Furthermore, this derivation provides us with a framework that allows us to further define our level set evolution by adding additional terms to our energy functional, as demonstrated by including a Frobenius norm term for salt delineation. Even with the very simple energy functional used in this work, the ability of the DRLSE algorithm to delineate salt bodies is evident in our results. Inclusion of additional external information, as well as smoothing terms, would further refine the salt body segmentation that this method can achieve.

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