Early-arrival waveform inversion for near-surface velocity and anisotropic parameters: modeling and sensitivity kernel analysis

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ABSTRACT

Understanding the relationships between vertical velocity, anisotropic parameters ϵ, δ and traveltime is important both for parametrization of waveform inversion and proper interpretation of waveform inversion results. Forward modeling indicates that traveltime is more sensitive to vertical velocity and ϵ than to δ. This suggests that δ may be fixed during an inversion for vertical velocity and ϵ. When vertical velocity and ϵ are directly parametrized, ϵ changes little during the inversion. A more balanced sensitivity kernel can be obtained by using different parametrizations, such as the vertical and horizontal velocities, or the logarithms of slowness and ϵ.

INTRODUCTION

Full Waveform Inversion (FWI) (Tarantola, 1984; Pratt et al., 1998; Mora, 1987) iteratively updates model by trying to match input data with modeled data. It estimates subsurface velocity more accurately than conventional techniques, such as ray-based methods (Hampson and Russell, 1984; Olson, 1984; White, 1989), especially in geologically complex areas. This is because FWI predicts kinematics of recorded data more accurately by using finite-difference wave-equations, compared with ray-based methods using high frequency approximations of wave propagation. However, the dynamics of recorded data are not very accurately predicted by current FWI methods. In other words, successful field-data application of FWI usually relies more on matching kinematics of recorded data.

With longer-offset data ($>$ 10km) commonly acquired these days, matching kinematics means matching data traveltime over the entire offset range. For such large offset ranges, anisotropic effects, if they exist, are no longer negligible. In the presence of anisotropy, if isotropic FWI is used, the inversion results will not correctly reflect the true subsurface attributes (Ghilami et al., 2011). More specifically, isotropic FWI of diving waves mostly recovers the horizontal velocity in anisotropic media. Migration using such a velocity will place reflectors at incorrect depths. To avoid this, anisotropic parameter estimation should be part of the inversion process. Such inversion can be carried out in several ways. One way is to perform single-parameter inversion for each parameter sequentially. Alternatively, joint inversion performs simultaneous inversion of multiple parameters, called joint inversion. Joint inversion is usually better, since the results of the first approach are sensitive to the order in which the inversion is performed. On the other hand, given the original definition of anisotropic parameters (Thomsen, 1986), direct changes in those parameters themselves usually result in very small changes in data kinematics, where as changes in velocity affect the data kinematics much more significantly (Plessix and Cao, 2011). For the purpose of simultaneous inversion, it is important to understand quantitatively how much the data kinematics change as a function of anisotropic parameters and velocity, and to come up with an effective parametrization of the model.

In this paper, I first describe the acoustic vertical transversely isotropic (VTI) wave-equation and the sensitivity kernel calculation. Then I use synthetic data examples to illustrate the sensitivity of the data kinematics to the anisotropic parameters and the velocity, in terms of both forward modeling and sensitivity-kernel calculation.

FORWARD MODELING

Exact anisotropic wave equations are in the form of elastic wave equations. Acoustic anisotropic wave equations can be obtained by various approximations of the exact elastic equations. One way to do this is to set shear-wave velocity to zero in the exact elastic wave equations. Detailed derivations can be found in several papers (Zhang and Zhang, 2009; Crawley et al., 2010; Duveneck et al., 2008). The resulting acoustic anisotropic wave equations are a system of second-order equations:

$$\begin{aligned} \frac{\partial^2 p}{\partial t^2} &= {v_p}^2 (1+2\epsilon) \frac{\partial^2 p}{\partial x^2} + {v_p}^2 \sqrt{1+2\delta} \frac{\partial^2 r}{\partial z^2} \\ \frac{\partial^2 r}{\partial t^2} &= {v_p}^2 \sqrt{1+2\delta} \frac{\partial^2 p}{\partial x^2} + {v_p}^2 \frac{\partial^2 r}{\partial z^2}, \end{aligned} \tag{1}$$

where p and r are horizontal and vertical stress, respectively, v_p is vertical p-wave velocity and ϵ and δ are anisotropic parameters (Thomsen, 1986).

To illustrate the traveltime sensitivity of data to parameter changes, I use part of the BP 2002 benchmark model. The original synthetic model has only p-wave velocity; the anisotropic parameters were created from the velocity model according to typical Gulf of Mexico anisotropic parameters. Velocity, ϵ and δ models are shown in Figure 1.

Four different modeling experiments with modeled shot records are shown in Figure 2. The first experiment is the VTI anisotropic modeling using all three fields shown in Figure 1. The second experiment is the same VTI modeling but with $\delta = 0$. The third experiment is isotropic modeling using the velocity field only, and the fourth is isotropic modeling using the horizontal p-wave velocity, which is defined as $v_h = v_p\sqrt{1+2\epsilon}$. Figure 3 shows the refraction traveltime difference of the latter three shots compared to the first shot. It can be seen that traveltime is insensitive

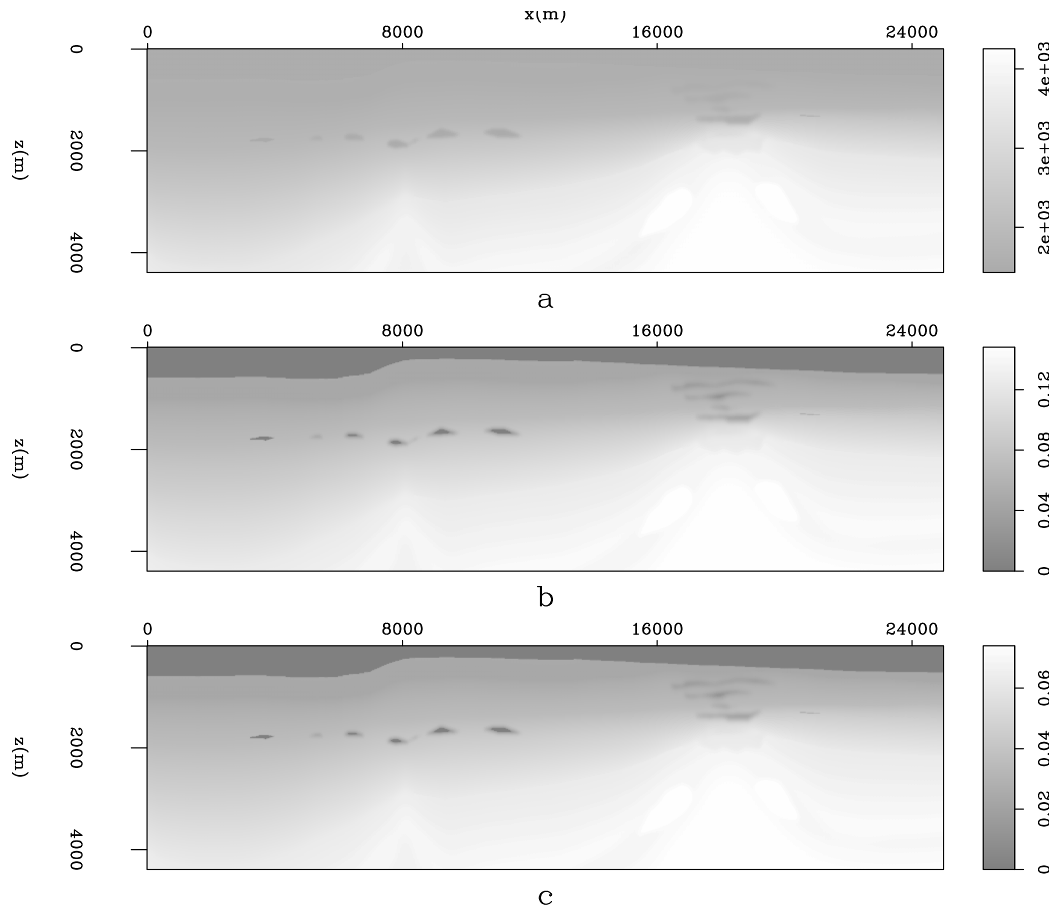

Figure 1: Reference model for various modeling experiments. Top: velocity model; middle: ϵ model; bottom: δ model. [**ER**]

to δ changes, but is sensitive to ϵ changes. Also, isotropic modeling using the horizontal p-wave velocity results in non-trivial traveltime differences, which means that even using isotropic FWI, the retrieved model is not necessarily the horizontal p-wave velocity, as was previously thought (Ghilami et al., 2011).

MODEL PARAMETRIZATION AND SENSITIVITY-KERNEL CALCULATION

For inversion, there are several ways to parametrize the model space, which contains vertical p-wave velocity and the anisotropic parameters ϵ and δ. However, due to insensitivity of the data to the δ parameter, currently I consider a model space consisting of only vertical p-wave velocity and ϵ; i.e. δ is fixed during inversion. Using gradient-based inversion methods, different parametrization leads to different model updates. In joint inversion, it is unfavorable to have updates that result in little or almost no change in one of the model components. Quantitatively, for a

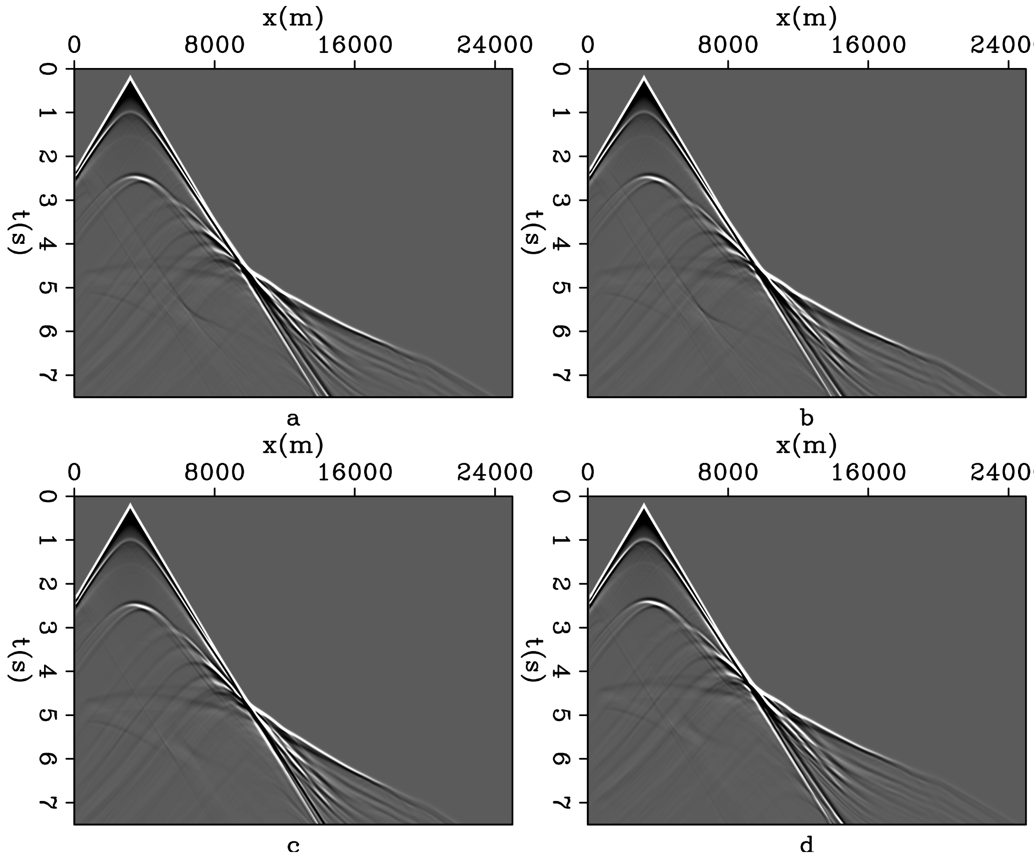

Figure 2: Shots from different modeling experiments: a) VTI modeling using all three fields shown in Figure 1; b) same as a) except that $\delta = 0$; c) isotropic modeling using only the velocity field; d) isotropic modeling using horizontal p-wave velocity $v_h = v_p\sqrt{1+2\epsilon}$. [**ER**]

model space that contains two components m_1 and m_2, update directions g_{m1} and g_{m2} should be chosen such that $g_{m1}/m_1 \approx g_{m2}/m_2$. This can be achieved by using proper parametrization.

I compare three different parametrizations and their corresponding model updates (sensitivity kernels). For sensitivity-kernel calculation, I first calculate the full data residual by subtracting d_{cal}, which is modeled from the smoothed version of the true model (Figure 4) from d_{obs}, which is modeled from the true model. I consider only a single trace refraction of the total data residual for this study.

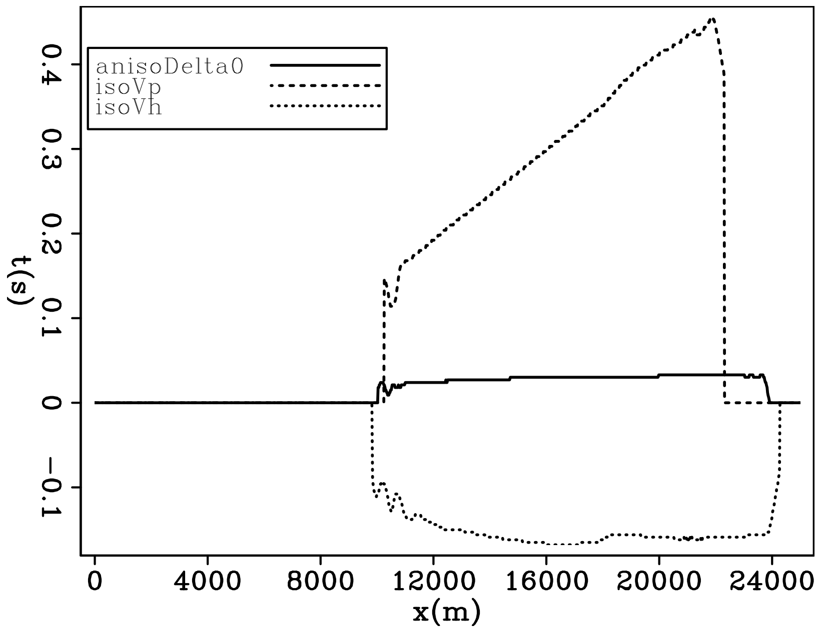

Figure 3: Refraction traveltime difference of shots compared to the VTI case. [**ER**]

Naive parametrization

The most naive parametrization directly inverts velocity and ϵ. However, to avoid higher-order term involving both variables, equation 1 can be rewritten as follows:

$$\begin{aligned} m_1 \frac{\partial^2 p}{\partial t^2} &= m_2 \frac{\partial^2 p}{\partial x^2} + \sqrt{1+2\delta} \frac{\partial^2 r}{\partial z^2} \\ m_1 \frac{\partial^2 r}{\partial t^2} &= \sqrt{1+2\delta} \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 r}{\partial z^2}, \end{aligned} \tag{2}$$

where $m_1 = {v_p}^{-2}$ and $m_2 = 1 + 2\epsilon$ are the two model variables, assuming

$$\begin{aligned} m_1 &= m_{1,0} + \Delta m_1 \\ m_2 &= m_{2,0} + \Delta m_2 \\ p &= p_0 + \Delta p \\ r &= r_0 + \Delta r \\ m_{1,0} \frac{\partial^2 p_0}{\partial t^2} &= m_{2,0} \frac{\partial^2 p_0}{\partial x^2} + \sqrt{1+2\delta} \frac{\partial^2 r_0}{\partial z^2} \\ m_{1_0} \frac{\partial^2 r_0}{\partial t^2} &= \sqrt{1+2\delta} \frac{\partial^2 p_0}{\partial x^2} + \frac{\partial^2 r_0}{\partial z^2}, \end{aligned} \tag{3}$$

By combining this with equation 2, we can obtain

$$\begin{aligned} m_{1,0} \frac{\partial^2 \Delta p}{\partial t^2} - m_{2,0} \frac{\partial^2 \Delta p}{\partial x^2} - \sqrt{1+2\delta} \frac{\partial^2 \Delta r}{\partial z^2} &= -\Delta m_1 \frac{\partial^2 p_0}{\partial t^2} + \Delta m_2 \frac{\partial^2 p_0}{\partial x^2} \\ m_{1,0} \frac{\partial^2 \Delta r}{\partial t^2} - \sqrt{1+2\delta} \frac{\partial^2 \Delta p}{\partial x^2} - \frac{\partial^2 \Delta r}{\partial z^2} &= -\Delta m_1 \frac{\partial^2 r_0}{\partial t^2}, \end{aligned} \tag{4}$$

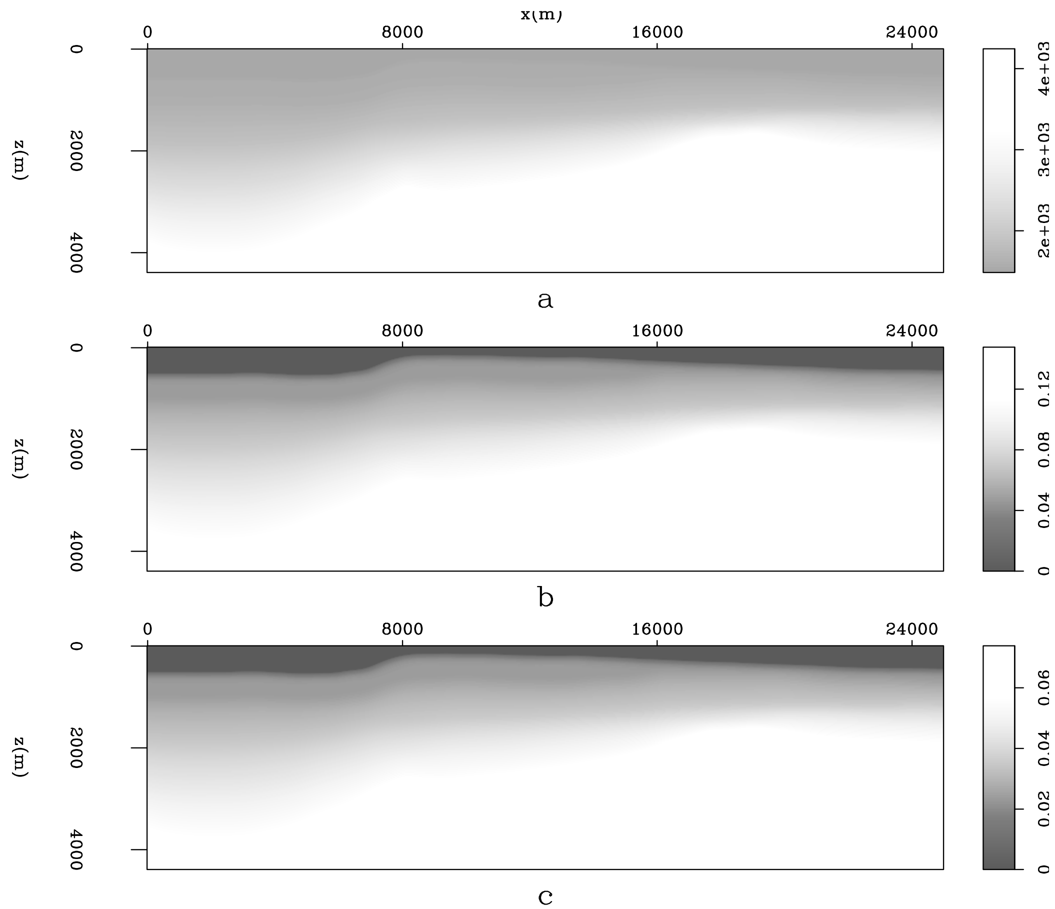

Figure 4: Smooth model for generating d_{cal}: a) velocity model; b) ϵ model; c) δ model. [**ER**]

which can be written in the following matrix form:

$$\begin{vmatrix} m_{1,0}\frac{\partial^2}{\partial t^2} - m_{2,0}\frac{\partial^2}{\partial x^2} & -\sqrt{1+2\delta}\frac{\partial^2}{\partial z^2} \\ -\sqrt{1+2\delta}\frac{\partial^2}{\partial x^2} & m_{1,0}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \end{vmatrix} \begin{vmatrix} \Delta p \\ \Delta r \end{vmatrix} = \begin{vmatrix} -\frac{\partial^2 p_0}{\partial t^2} & \frac{\partial^2 p_0}{\partial x^2} \\ -\frac{\partial^2 r_0}{\partial t^2} & 0 \end{vmatrix} \begin{vmatrix} \Delta m_1 \\ \Delta m_2 \end{vmatrix}, \tag{5}$$

This establishes a linear relationship between model perturbation and data perturbation, and can be used to calculate the sensitivity kernel. Figure 5 shows the relative sensitivity kernels of the two model parameters, which are defined as

$$k_{m_i} = g_{m_i}/m_{i,0}, \tag{6}$$

where g_{m_i} is the sensitivity kernel, and k_{m_i} is the relative sensitivity kernel. the clipping value of the top figure is over sixteen orders of magnitude larger than the bottom figure, which means if we are to use this parametrization for our inversion, there will be almost no updates of the anisotropic parameter.

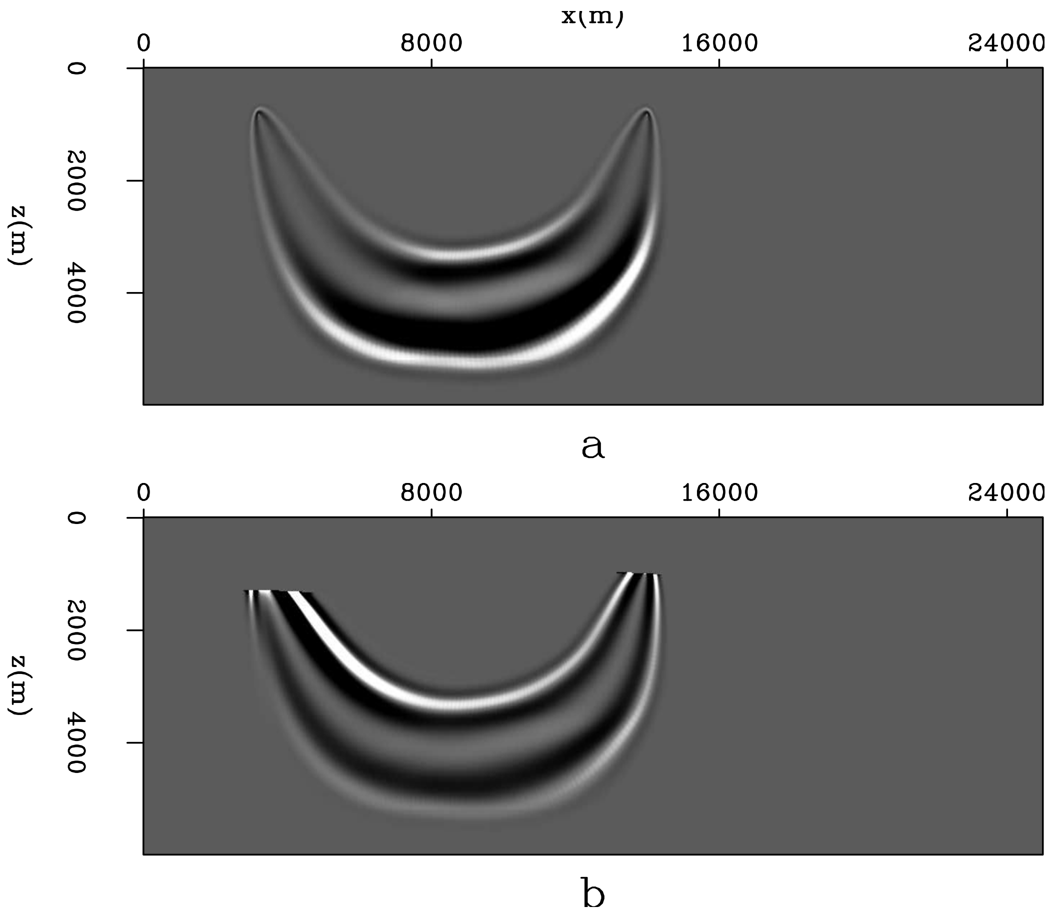

Figure 5: Relative sensitivity kernel of parameter: a) ${v_p}^{-2}$; b) $1+2\epsilon$. Clipping value of the top figure is $1e10$, clipping value of the bottom figure is $2e-6$. [**ER**]

Velocity parametrization

Another parametrization is to use velocities for both variables. Defining $m_1 = {v_p}^2$ and $m_2 = {v_p}^2(1+2\epsilon)$, equation 1 can be rewritten as follows:

$$\begin{aligned} \frac{\partial^2 p}{\partial t^2} &= m_2 \frac{\partial^2 p}{\partial x^2} + m_1\sqrt{1+2\delta}\frac{\partial^2 r}{\partial z^2} \\ \frac{\partial^2 r}{\partial t^2} &= m_1\sqrt{1+2\delta}\frac{\partial^2 p}{\partial x^2} + m_1\frac{\partial^2 r}{\partial z^2}, \end{aligned} \quad (7)$$

Using similar procedure to the one described in the previous section, we can obtain a matrix form expression:

$$\begin{vmatrix} \frac{\partial^2}{\partial t^2} - m_{2,0}\frac{\partial^2}{\partial x^2} & -m_{1,0}\sqrt{1+2\delta}\frac{\partial^2}{\partial z^2} \\ -m_{1,0}\sqrt{1+2\delta}\frac{\partial^2}{\partial x^2} & \frac{\partial^2}{\partial t^2} - m_{1,0}\frac{\partial^2}{\partial z^2} \end{vmatrix} \begin{vmatrix} \Delta p \\ \Delta r \end{vmatrix} = \begin{vmatrix} \sqrt{1+2\delta}\frac{\partial^2 r_0}{\partial z^2} & \frac{\partial^2 p_0}{\partial x^2} \\ \sqrt{1+2\delta}\frac{\partial^2 p_0}{\partial x^2} + \frac{\partial^2 r_0}{\partial z^2} & 0 \end{vmatrix} \begin{vmatrix} \Delta m_1 \\ \Delta m_2 \end{vmatrix}, \quad (8)$$

This is the linear relationship between model perturbation and data perturbation, and can be used to calculate the sensitivity kernel. Figure 6 shows the relative sensitivity

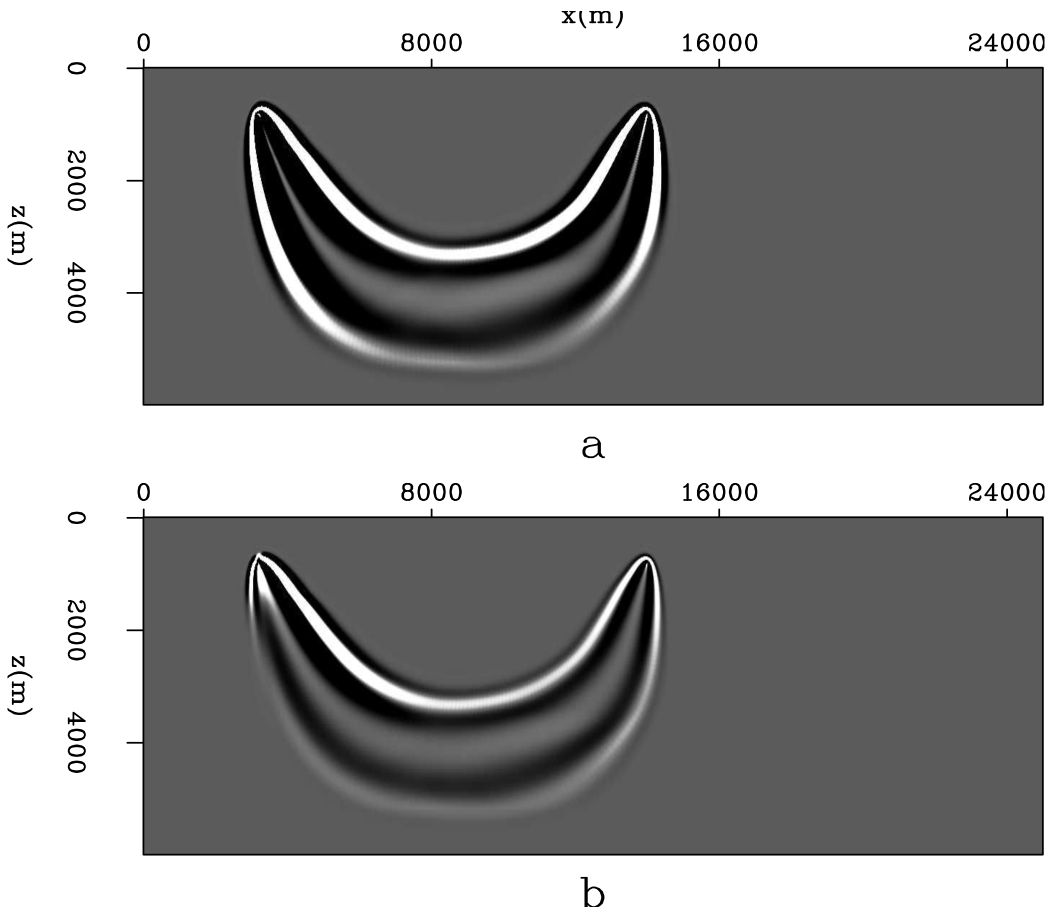

Figure 6: Relative sensitivity kernel of parameter: a) ${v_p}^2$; b) ${v_p}^2\,(1+2\epsilon)$. Both figures are clipped at the same value. [**ER**]

kernel of the two model parameters. Both figures are clipped to the same value. Since both variables are parametrized as velocities, their updates are of similar strength, and this is an effective parametrization.

Logarithmic velocity parametrization

A slightly different parametrization is to define $m_1 = ln\,({v_p}^{-2})$ and $m_2 = 1 + 2\epsilon$. Using this parametrization, equation 1 can be rewritten as follows:

$$\begin{aligned} e^{m_1}\frac{\partial^2 p}{\partial t^2} &= m_2\frac{\partial^2 p}{\partial x^2} + \sqrt{1+2\delta}\frac{\partial^2 r}{\partial z^2} \\ e^{m_1}\frac{\partial^2 r}{\partial t^2} &= \sqrt{1+2\delta}\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 r}{\partial z^2}, \end{aligned} \tag{9}$$

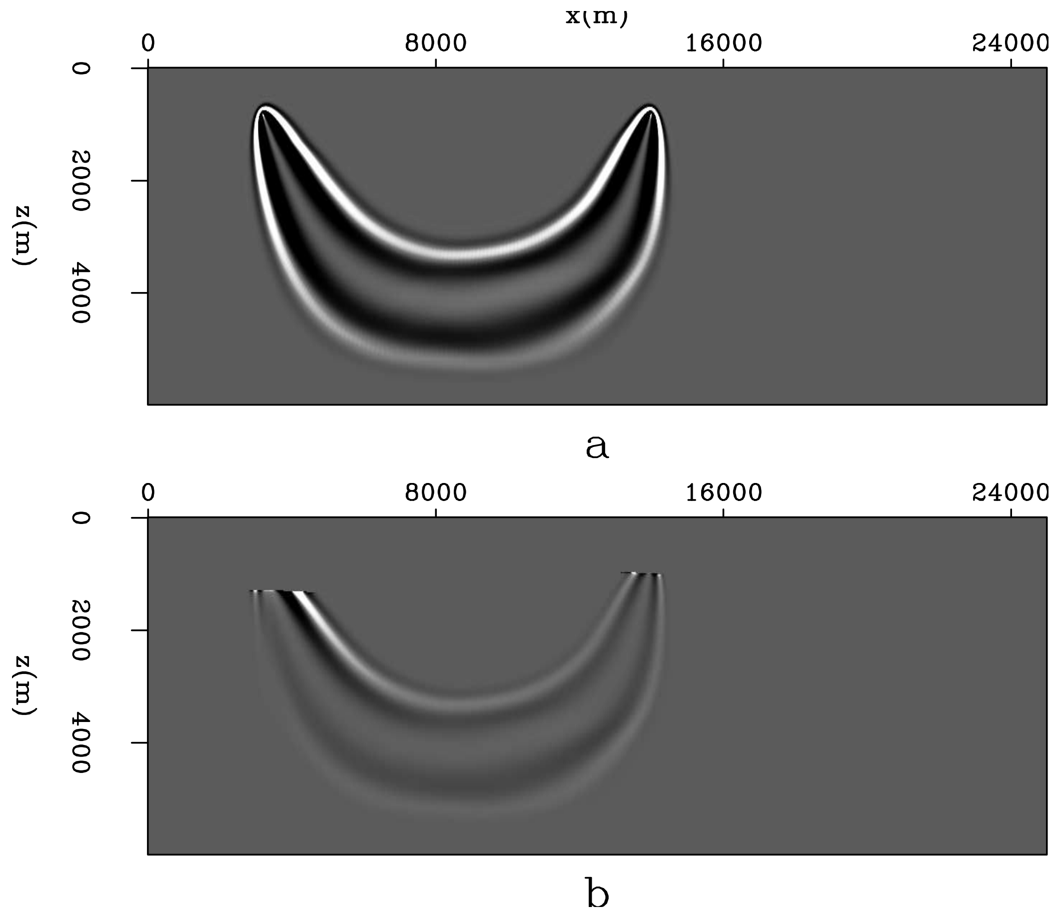

Figure 7: Relative sensitivity kernel of parameter: a) $a^{-1}\ln\left(v_p{}^{-2}\right)$; b) $(1+2\epsilon)$. Both figures are clipped at the same value. [**ER**]

Using a procedure similar to the one described in the previous section, we can obtain a matrix form expression:

$$\begin{vmatrix} e^{m_{1,0}}\frac{\partial^2}{\partial t^2} - m_{2,0}\frac{\partial^2}{\partial x^2} & -\sqrt{1+2\delta}\frac{\partial^2}{\partial z^2} \\ -\sqrt{1+2\delta}\frac{\partial^2}{\partial x^2} & e^{m_{1,0}}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} \end{vmatrix} \begin{vmatrix} \Delta p \\ \Delta r \end{vmatrix} = \begin{vmatrix} -e^{m_{1,0}}\frac{\partial^2 p_0}{\partial t^2} & \frac{\partial^2 p_0}{\partial x^2} \\ -e^{m_{1,0}}\frac{\partial^2 r_0}{\partial t^2} & 0 \end{vmatrix} \begin{vmatrix} \Delta m_1 \\ \Delta m_2 \end{vmatrix}, \qquad (10)$$

This is the linear relationship between model perturbation and data perturbation, and can be used to calculate the sensitivity kernel. Figure 7 shows relative sensitivity kernel of the two model parameters. Both figures are clipped to the same value. With this parametrization, updates of both variables are of the same order of strength. I show in another paper [citation] that such parametrization results in very good inversion results.

CONCLUSIONS

The forward modeling experiment suggests that it is important to include anisotropy as part of the inversion. For joint inversion, proper parametrization can give balanced updates to each model variable. Velocity parametrization and logarithmic slowness parametrization are both good candidates in that sense. Actual inversion using these parametrization needs to be further studied.

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