Random boundary condition for low-frequency wave propagation

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SUMMARY

Random boundaries are an attractive computational alternative to damping boundary conditions. In this paper we present a random boundary that is effective at both high-frequencies (Reverse Time Migration) and low-frequencies (waveform inversion). This boundary condition uses larger, irregular shaped randomized velocities that is effective in introducing incoherency in wavefronts at a large range of wave lengths. We compare this boundary condition to alternative implementations on source functions with a range of peak frequencies.

INTRODUCTION

A random boundary condition (Clapp, 2009) is attractive in Reverse Time Migration (RTM) imaging, since it generates time-reversible wavefields that are incoherent. The advantage of the random boundary condition is that the receiver wavefield can be first propagated from max-time to 0, then simultaneously propagated with the source wavefield from 0, to max-time. This eliminates the need to either store the entire wavefield on disk, or use checkpointing schemes at the cost of an extra propagation. Such reduction in memory requirement is particularly suitable for implementation on novel hardwares such as GPUs (Micikevicius, 2008). The incoherent nature of the reflections off the random boundary result in random correlations when the imaging condition is applied resulting little to no degradation of the final image.

In theory, this random boundary condition can be easily extended to wavefields at all temporal frequency ranges. However, due to memory and computational power concerns, overly large boundary regions are unfavorable. Thus, problems arise when we move to low frequency. In that case, increased spatial wavelengths mandate larger boundary regions if we are to use the same random boundary scheme. For low-frequency applications such as waveform inversion, if we can keep the size of the random boundary approximately the same as that used in RTM, the computational efficiency will be greatly improved by avoiding the I/O cost associated with transferring wavefields. In this paper, we introduce a scheme of perturbing the grain shape of the random boundary condition to keep boundary region small while still effective with low frequency wave propagation. The same random boundary is also effective for high-frequency wave propagation. We briefly discuss the theory and illustrate the method using synthetic examples.

BOUNDARY DESIGN

Randomness in a velocity boundary has different effects at different temporal frequencies. Because lower-frequency signals have longer wavelengths, the averaging effect within the dominant wavelength makes the same random boundary appear less random than at higher frequencies. This can be easily understood by taking the extreme case; zero frequency has infinite wavelength, so it sees a random boundary as a constant velocity, no matter what velocity distribution is used. For a random velocity field to appear random to a low-frequency signal, a coarser 'grain' of random velocity anomalies is necessary. If grains are bigger than a single cell, two adjustable parameters become important in determining the effectiveness of such random boundaries.

The first parameter is the size of grains. A simple way to determine the grain size is:

$$l_{grain} \approx \frac{f_{RTM}}{f_{lowfreq}} \tag{1}$$

where l_{grain} is the effective length of the random velocity grain, f_{RTM} is the dominant frequency in the RTM that uses the random boundary condition, and $f_{lowfreq}$ is the dominant low frequency used in modeling. The above equation holds because of the inverse relationship between frequency and wavelength at a given velocity. The second parameter is the shape of grains, the easiest of which is cubic, with side lengths equal to the effective length l_{grain}. Although this works much better than a single-cell random velocity anomaly, its effectiveness is diminished by its regular shape. To further increase the randomness of reflected and scattered wavefields, we propose randomly shaped grains in place of cubic grains. We generate randomly shaped grains by perturbing cubic grains of certain lengths. In this case, the effective length of such a randomly shaped grain is equal the the side length of the cubic grain being perturbed. The perturbed grains will have similar volume or grain size to the cubic grains, but have random shapes that more effectively scatter coherent wavefields. It will be shown next that randomly shaped grains also work well with higher-frequency signals, because the irregular, small-scale features at grain boundaries scatter shorter wavelengths effectively.

EXAMPLES

To illustrate the idea, we show examples of acoustic modeling using constant velocity fields with random boundaries of different 'grain' size and shape. Each example is compared with a constant boundary condition, where wavefields propagate through boundary regions that have the same velocity values as the inner region. In addition, we use zero-value boundary conditions when wavefields reach the end of each boundary region. The entire velocity field, including the boundary regions, is 6000 meters in x,y and z, with the same 20-meter spacing used in all directions. Boundary regions are 1000 meters (50 gridpoints) in each direction, and either the velocity is constant throughout the region, or velocity values are randomly assigned different grain sizes. In the inner region of the velocity fields, the velocity is constant at 3 km/s; in the boundary regions the velocity is at least 1.5 km/s and no greater than

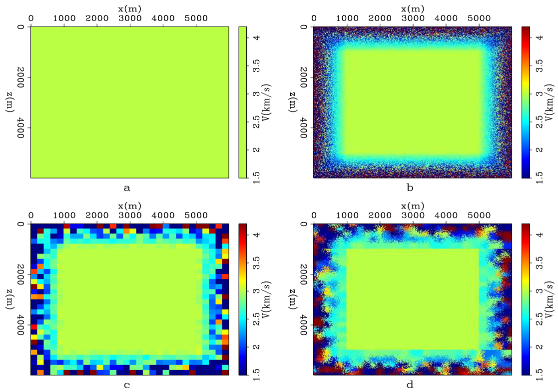


Figure 1: Velocity with different boundary regions: a) constant velocity ; b) cubic grains with 20 m side length ; c) cubic grains with 200 m side length; d) randomly shaped grains with 200 m effective length.

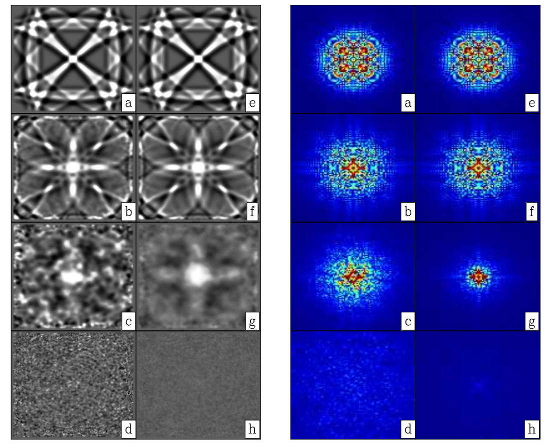
4.2 km/s. Two types of grains are used in random boundaries, shown in Figure 1: cubic grains with two different side lengths: 20 m (1 grid point)and 200 m (10 grid point), and randomly shaped grains with effective length of 200 m (10 grid points). 3-D modeling is used for all examples, but for display purposes, we only show 2D slices of 3D velocity and wavefields.

Low-frequency modeling

Figure 2 shows wavefields at 3.5 seconds using a point source with a peak frequency of 6 Hz in velocity fields with the different boundary regions mentioned in Figure 1. Figure 2(a) left shows 2D slices of the wavefield using one realization of each velocity. For a 6 Hz peak frequency (which is typical in waveform inversion), a random boundary with small grain size is only slightly better than constant velocity. In this case, the dominant wavelength is 500 m, and only when grain size become comparable to the dominant wavelength does the wavefield start to appear random. However, there are some low-frequency residuals remaining in the images using big cubic grains. Figure 2(a) right shows a stacked wavefield of the same source and record time using 16 different realizations of random boundaries. It is now obvious that the low-frequency residuals are evidence of poor performance from using big cubic grains. The regular shape of these cubic grains cannot scatter certain incident wavefronts, thus leaving coherent energy at these angles. Randomly shaped grains, on the other hand, scatter those low-frequency components quite well. This can be seen in the wavenumber domain amplitude spectrum of these wavefields (Figure 2(b)), where the randomly shaped grains reduce lower-wavenumber components much more effectively than cubic grains.

Broadband modeling application

In this section, we test a broadband point source with a peak frequency of 25 Hz, using the same boundary as in the previous example. Figure 3 shows the amplitude spectrum of the point source used, which contains non-trivial high- and low-frequency components. Figure 4(left column) shows 2D slices



(a) (b)

Figure 2: Time-domain and wave-number-domain wavefield snapshots for a point source with 6 Hz peak frequency in velocity fields with different random boundary conditions. The left panel shows time-domain wavefield snapshots; the right panel shows the corresponding wave-number-domain amplitude spectrum. The left column of each panel shows one realization of velocity with a) constant velocity, b) cubic grains with 20 m side length, c) cubic grains with 200 m side length, and d) randomly shaped grains with effective length 200 m. The right column shows average wavefields using 16 realizations of velocity with e) constant velocity, f) cubic grains with 20 m side length, g) cubic grains with 200 m side length, and h) randomly shaped grains with 200 m effective length.

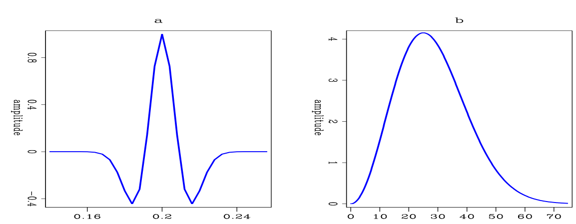


Figure 3: Time-domain and frequency-domain broadband source with a peak frequency of 25 Hz: a) Time-domain wavelet; b) Frequency-domain amplitude spectrum.

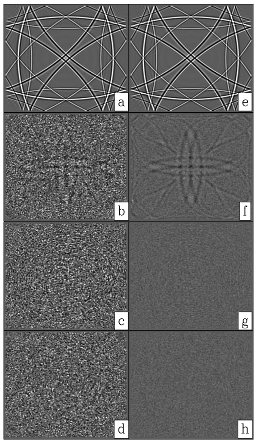


Figure 4: Time domain wavefield snapshots for a broadband point source with a peak frequency of 25 Hz, in velocity fields with different random boundary conditions. The left column shows one realization of velocity with a) constant velocity, b) cubic grains with 20 m side length, c) cubic grains with 200 m side length, and d) randomly shaped grains with effective length 200 m. The right column shows average wavefields using 16 realizations of velocity with e) constant velocity, f) cubic grains with 20 m side length, g) cubic grains with 200 m length, and h) randomly shaped grains with effective length 200 m.

of the wavefield using one realization of each velocity. In this case, the proposed random boundaries still work quite well. A random boundary with a small grain size is effective at high frequency, but has difficulty eliminating low-frequency components of the source. This is more obvious in the right column of Figure 4, which is the stacked wavefield of the same source and record time, using 16 different realizations of random boundaries.

Spectra of random boundaries

Another way to analyze the previous two examples is to consider the spectra of different random boundaries. Figure 5 shows a velocity field filled with different random boundaries. We can look at their k spectra by Fourier transform along both the x and z axes and take the absolute amplitudes (Figure 5). It can be seen that randomly shaped grains at high k values have an amplitude similar to that of single-cell random boundary, and at low k values have an amplitude similar to that of a large cubic-grain random boundary. The large cubic-grain random boundary, on the other hand, has a far lower high-k component, and there are even notches at certain k values. These features become obvious in stacked k_z and k_x spectra of each random field (Figure 6). Large amplitude at high k values means more detailed randomness, which is useful for scattering high-frequency waves. Large amplitude at low k values means many coarse 'grains', which is useful for scattering low-frequency waves. Random fields that have both can effectively scatter broadband signals, and this is the case for randomly shaped grains. The large volume of the grains effectively scatters low-

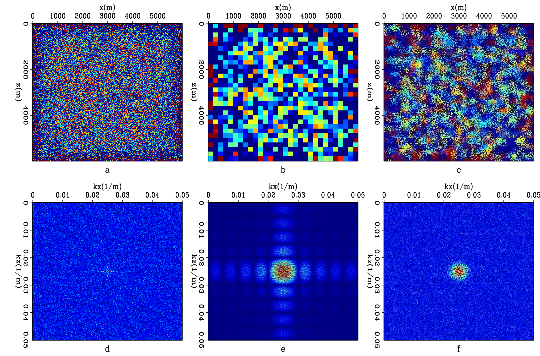


Figure 5: Velocity field with different random boundaries: a) cubic grains with 20 m length; b)cubic grains with 200 m length; and c) randomly shaped grains with 200 m effective length. The bottom row shows their corresponding k spectra: d) cubic grains with 20 m side length, e) cubic grains with 200 m side length, and f) randomly shaped grains with 200 m effective length.

frequency signals, while the edges of grains effectively scatter high-frequency signals. On the other hand, regular notches in the spectrum means that the random field (cubic grains in this case) displays certain patterns that will render it unable to deal with certain wavefronts; this is analogous to the null space in inversion.

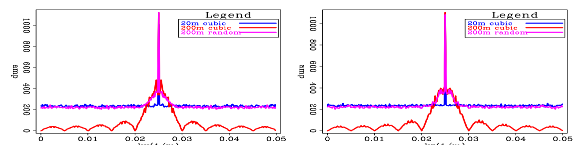


Figure 6: Stacked k_x and k_z spectra of each random field obtained by stacking the amplitude k spectrum along the x and z directions. On the left is the k_x spectrum; on the right is the k_z spectrum. The blue is from cubic grains of side length 20 m,red is from cubic grains of side length 200 m, and pink is from randomly shaped grains of length 200 m.

CONCLUSIONS

Large grain size in a random boundary condition is effective for low-frequency data. Randomly shaped grains work better than regularly shaped grains, and are also very effective in dealing with broadband sources. Of key importance is matching the grain size in the random boundary to the spectrum of the wavefield used in modeling. Applying such frequency-matched boundary conditions extends the utility of the random boundary condition beyond RTM, with possible applications to waveform inversion and others.

ACKNOWLEDGMENTS

We would like to thank Stanford Exploration Project for the financial support of this research.

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