Least-squares migration/inversion of blended data

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SUMMARY

We present a method based on wave-equation least-squares migration/inversion to directly image data collected from recently developed wide-azimuth acquisition geometry, such as simultaneous shooting and continuous shooting, where two or more shot records are often blended together. We show that by using least-squares migration/inversion, we not only enhance the resolution of the image, but more importantly, we also suppress the crosstalk or acquisition footprint, without any pre-separation of the blended data. We demonstrate the concept and methodology in 2-D and apply the data-space inversion scheme to the Marmousi model, where an optimally reconstructed image, free from crosstalk artifacts, is obtained.

INTRODUCTION

High quality seismic images are extremely important for subsalt exploration, but data collected from conventional narrow-azimuth towed streamer (NATS) often produce poor subsalt images due to insufficient azimuth coverage. Recently developed wide-azimuth towed streamer (WATS) (Michell et al., 2006) and multi-azimuth towed streamer (MATS) (Keggin et al., 2006; Howard and Moldoveanu, 2006) acquisition technologies have greatly improved the subsalt illumination and hence better subsalt images are obtained. However, acquiring WATS or MATS data is expensive. One main reason is the inefficiency of the conventional way of acquiring data, which allows sufficient time sharing between shots to prevent interference (Beasley et al., 1998; Beasley, 2008; Berkhout, 2008). As a consequence, the source domain is often poorly sampled to save the survey time.

To gain efficiency, simultaneous shooting (Beasley et al., 1998; Beasley, 2008; Hampson et al., 2008) and continuous shooting, or more generally, blended acquisition geometry (Berkhout, 2008), have been proposed to replace the conventional shooting strategy. In the blended acquisition geometry, we try to shoot and record continuously, and consequently, the time sharing between shots would be minimized and a denser source sampling can be obtained. However, this shooting and recording strategy results in two or more shot records blending together and brings processing challenges. A common practice of processing these blended data sets is to first separate the blended shot gathers into individual ones (Spitz et al., 2008; Akerberg et al., 2008), as called ”deblending” by Berkhout (2008). Then conventional processing flows are applied to these deblended shot gathers. The main issue with this strategy is that it might be extremely difficult to separate the blended gathers when the shot spacing is close and many shots are blended together.

In this paper, we present an alternative method of processing these blended data sets. Instead of deblending the data prior to the imaging step, we propose to directly image them without any pre-separation. The simplest way for direct imaging would be migration, however, migration of blended data generates images contaminated by crosstalk. The crosstalk is due to the introduction of the blending operator (Berkhout, 2008), which makes the corresponding combined Born modeling operator far from unitary, thus its adjoint, also known as migration, gives poor reconstruction of the reflectivity. A possible solution is to go beyond migration by formulating the imaging problem as a least-squares migration/inversion (LSI) problem, which uses the pseudo inverse of the combined Born modeling operator to reconstruct the reflectivity of the subsurface.

We extend the LSI theory from the conventional acquisition geometry (Nemeth et al., 1999; Clapp, 2005; Valenciano, 2008; Tang, 2008b) to the blended acquisition geometry and develop inversion schemes in both data space and model space. By comparing the pros and cons of both inversion schemes, we show that the data-space approach is preferred over the model-space approach if the combined Born modeling operator is far from unitary, whose normal operator, i.e., the Hessian, has many non-negligible off-diagonal elements. Hence an approximate Hessian with a limited number of off-diagonal elements can not capture the characteristics of the crosstalk, making it less effective in removing the crosstalk in the model space. Big Hessian filters, which sufficiently capture the information of the crosstalk, are too expensive for practical applications. Therefore, the data-space inversion approach, where the Hessian is implicitly computed, becomes more attractive. We demonstrate our ideas with simple synthetic examples and we also test the data-space inversion scheme on the Marmousi model to illustrate how the crosstalk is suppressed through inverting the combined Born modeling operator. Application to 4-D time-lapse inversion using blended data sets is also discussed in a companion paper by Ayeni et al. (2009).

This paper is organized as follows: we first describe the problem of directly imaging the blended data through migration; then we develop the theory of LSI in both data space and model space for blended data, and compare their pros and cons for imaging blended data. Finally, we apply the data-space inversion approach to the Marmousi model to test its performance on complex model.

PROBLEMS WITH DIRECT MIGRATION

In general, primaries can be modeled by the Born modeling equation (ignoring multiple scattering and assuming an accurate background velocity model) as follows:

$$\mathbf{d} = \mathbf{L}\mathbf{m}, \tag{1}$$

where $\mathbf{d}$ is the modeled data, $\mathbf{L}$ is the forward Born modeling operator, and $\mathbf{m}$ denotes the reflectivity, a perturbed quantity from the background velocity. Equation 1 models the data for

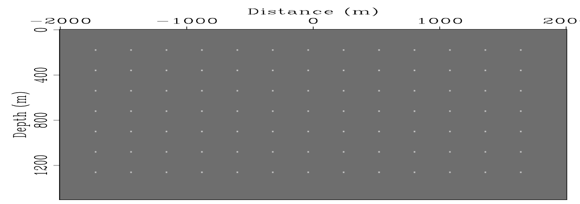


Figure 1: A reflectivity model containing many point scatters.

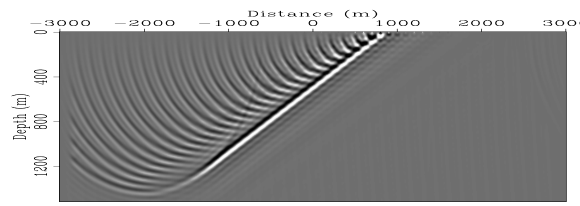


Figure 2: Source wavefield after linear-time-delay blending.

conventional acquisition geometry, i.e., there is no interference from different shots. For blended acquisition geometry, however, two or more shot records are often blended together, creating one or more super areal shot record(s). This blending process can be described by a linear transform as follows:

$$\widetilde{\mathbf{d}} = \mathbf{B}\mathbf{d}, \tag{2}$$

where $\mathbf{B}$ is the so called blending (Berkhout, 2008) or encoding (Romero et al., 2000; Tang, 2008a) operator, and $\widetilde{\mathbf{d}}$ is the super areal shot records after blending. Substituting equation 1 into equation 2 leads to the modeling equation for the blended acquisition geometry:

$$\widetilde{\mathbf{d}} = \mathbf{B}\mathbf{L}\mathbf{m} = \widetilde{\mathbf{L}}\mathbf{m}, \tag{3}$$

where $\widetilde{\mathbf{L}} = \mathbf{B}\mathbf{L}$ is defined as the combined Born modeling operator.

There are many choices of the blending operator, which one produces the optimal imaging result might be case dependent and is beyond the scope of this paper. In this paper, we mainly consider two different blending operators, i.e., linear-time-delay blending operator and random-time-delay blending operator. The first one seems to be common and easy to implement in practice, while the second one is interesting and has been partially adopted in acquiring data with simultaneous shooing (Hampson et al., 2008). For example, Figure 1 shows a scattering reflectivity model with a constant velocity 2000 m/s. Figure 2 and Figure 3 show the snapshots of the corresponding blended source wavefields. 41 point sources with a equal spacing 100 m are blended into one composite source for both cases. Figure 4 shows the modeled blended data. Given the complexity of the super areal shot gathers shown in Figure 4, it might be very difficult or even impossible to deblend them.

We can directly use the adjoint of the combined modeling operator, which is also widely known as the migration operator, to reconstruct the reflectivity as follows:

$$\widetilde{\mathbf{m}}_{\mathrm{mig}} = \widetilde{\mathbf{L}}'\widetilde{\mathbf{d}}_{\mathrm{obs}} = \mathbf{L}'\mathbf{B}'\mathbf{B}\mathbf{d}_{\mathrm{obs}}, \tag{4}$$

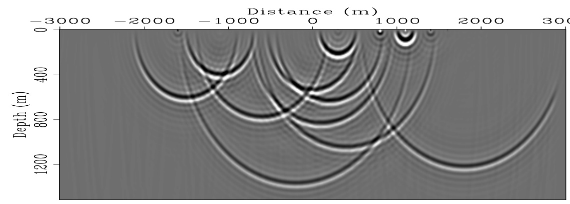
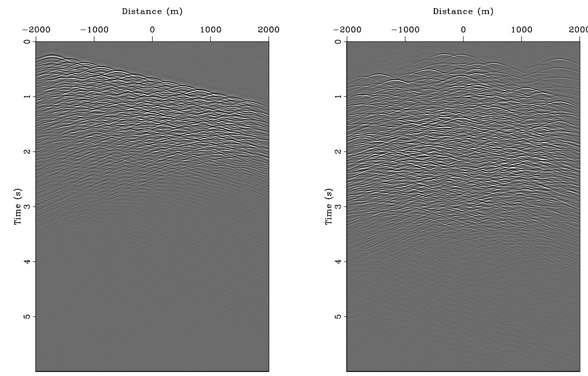


Figure 3: Source wavefield after random-time-delay blending.



(a) (b)

Figure 4: Modeled blended shot gather. (a) linear-time-delay blending and (b) random-time-delay blending.

where the superscript $'$ stands for conjugate transpose and the subscript $_{\mathrm{obs}}$ stands for the observed data. Different from the imaging formula in conventional acquisition geometry, now we have an extra $\mathbf{B}'\mathbf{B}$ in the imaging formula, which has a direct impact on the imaging quality of the blended data. If $\mathbf{B}'\mathbf{B}$ is close to unitary, i.e., $\mathbf{B}'\mathbf{B} \approx \mathbf{I}$ with $\mathbf{I}$ being the identity matrix, then direct migration of blended data would produce exactly the same results as migration of conventional data, the blending process would produce little impact on the final image we obtain. However, in reality, $\mathbf{B}'\mathbf{B}$ is often far from unitary due to the fact that $\mathbf{B}$ is usually a short matrix (its number of rows is much smaller than its number of columns), and its normal operator, i.e., $\mathbf{B}'\mathbf{B}$, is rank deficient. In other words, there are many non-negligible off-diagonal elements in $\mathbf{B}'\mathbf{B}$. As a consequence, direct migration using equation 4 produces crosstalk artifacts. Examples are demonstrated in Figure 5 and Figure 6, which illustrate the migrated images for the blended data shown in Figure 4, the images are severely degraded by crosstalk artifacts. For comparison, Figure 7 shows the image when migrating the conventional data (when $\mathbf{B} = \mathbf{I}$), no crosstalk presents in the result.

DIRECT IMAGING THROUGH INVERSION

One attempt to reduce the crosstalk is to go beyond migration by formulating the imaging problem as a LSI problem. The motivation behind LSI is that the pseudo inverse of the combined Born modeling operator $\widetilde{\mathbf{L}}$ should be able to optimally reconstruct the reflectivity, hence the image would be minimally affected by the crosstalk artifacts. The LSI can be per-

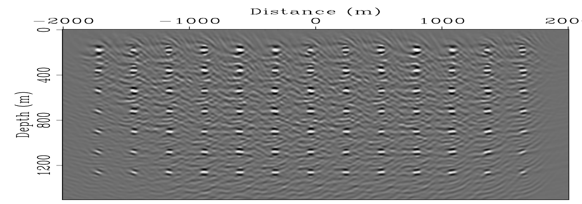


Figure 5: Migration of the linear-time-delay blended data.

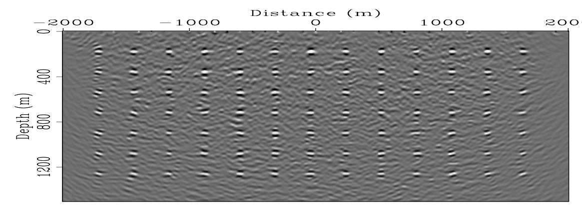


Figure 6: Migration of the random-time-delay blended data.

formed either in the model space or in the data space, each of which has its own pros and cons, we will analysis both of them for imaging the blended data.

LSI in the model space

The least-squares solution of equation 3 can be formally written as follows:

$$\mathbf{m} = \widetilde{\mathbf{H}}^{-1}\widetilde{\mathbf{L}}'\widetilde{\mathbf{d}}_{\text{obs}} = \widetilde{\mathbf{H}}^{-1}\widetilde{\mathbf{m}}_{\text{mig}}, \tag{5}$$

where $\widetilde{\mathbf{H}} = \widetilde{\mathbf{L}}'\widetilde{\mathbf{L}} = \mathbf{L}'\mathbf{B}'\mathbf{B}\mathbf{L}$ is the Hessian for the blended acquisition geometry. However, equation 5 has only symbolic meaning, because the Hessian is often singular and its inverse is not easy to obtain directly. A more practical way would be reconstructing the reflectivity $\mathbf{m}$ through iterative inverse filtering by minimizing a model-space objective function defined as follows:

$$J(\mathbf{m}) = ||\widetilde{\mathbf{H}}\mathbf{m} - \widetilde{\mathbf{m}}_{\text{mig}}||_2^2 + \varepsilon||\mathbf{A}\mathbf{m}||_2^2, \tag{6}$$

where $||\cdot||_2$ stands for ℓ_2 norm, and $\mathbf{A}$ is a regularization operator that imposes prior information that we know about the model $\mathbf{m}$, and ε is a trade-off parameter that controls the strength of regularization.

The advantage of the model-space formulation is that it can be implemented in a target-oriented fashion, which can substantially reduce the size of the problem and hence the computational cost (Valenciano, 2008). However, it requires explicitly computing the Hessian operator, which is considered to be expensive without certain approximations. As demonstrated by Valenciano (2008), for a typical conventional acquisition geometry, i.e., when $\mathbf{B} = \mathbf{I}$, the Hessian operator $\mathbf{L}'\mathbf{L}$ is diagonal dominated for areas of good illumination, for areas of poor illumination, the diagonal energy spreads along its off-diagonals. The spreading is limited and can almost be captured by a limited number of off-diagonal elements. That is why Valenciano (2008) suggests computing a truncated Hessian filter to approximate the exact Hessian for inverse filtering. By doing so, the cost of the model-space inversion scheme becomes affordable for practical applications. Figure 8(a) shows the local Hessian operator located at $x = 0$ m and $z = 750$ m in the

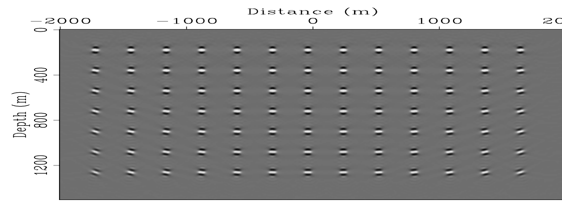


Figure 7: Migration of the data acquired with the conventional acquisition geometry.

subsurface (a row of the entire Hessian matrix) for the previous scattering model with the conventional acquisition geometry. The origin of this plot denotes the diagonal element of the Hessian, while locations not at the origin denote the off-diagonal elements of the Hessian. As expected, the Hessian is well focused around its diagonal part, and hence can be approximated by a filter with a small size. However, for the blended acquisition geometry, the combined modeling operator $\widetilde{\mathbf{L}}$ becomes far from unitary and hence the Hessian $\widetilde{\mathbf{H}}$ has non-negligible off-diagonal energy, which can spread over many of the off-diagonal elements. This phenomenon is confirmed by Figure 8(b) and Figure 8(c), which show the local Hessian operators at the same image point for the blended acquisition geometries. It is clear that a filter with a small size could not capture all the important characteristics of the crosstalk that is presented in the migrated image, therefore inverse filtering would fail to remove the crosstalk. Big filters may be too expensive to compute. Therefore, we seek an inversion approach that does not require explicitly computing the Hessian, and hence we do not have too worry about the size of the Hessian filter. This important consideration leads us to the following data-space inversion approach.

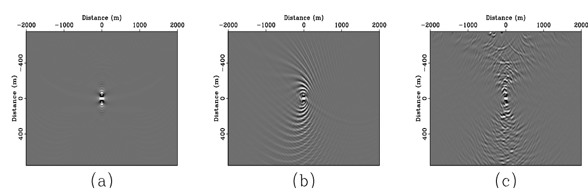


Figure 8: The local Hessian operator located at $x = 0$ m and $z = 750$ m in the subsurface. (a) for conventional acquisition geometry, (b) for linear-time-delay blended acquisition geometry and (c) for random-time-delay blended acquisition geometry.

LSI in the data space

The data-space LSI minimizes the following objective function:

$$F(\mathbf{m}) = ||\widetilde{\mathbf{L}}\mathbf{m} - \widetilde{\mathbf{d}}_{\text{obs}}||_2^2 + \varepsilon||\mathbf{A}\mathbf{m}||_2^2. \tag{7}$$

The data-space objective function $F(\mathbf{m})$ can be minimized through gradient-based optimization schemes, which iteratively reconstruct the model parameters. The advantage of this data-space formulation is that it does not require explicitly building the Hessian operator, hence all crosstalk information is captured implicitly. However, the data-space formulation lacks flexibility and can not be implemented in a target-oriented fashion. Its cost is another concern, because each iteration costs about two migration, making it challenging for large scale applications. The cost can be significantly reduced by using proper preconditioners, which may speed up the convergence considerably.

NUMERICAL EXAMPLES

We test our data-space inversion scheme on the Marmousi model. Figure 9 shows the reflectivity and velocity model used for the Born modeling under the blended acquisition geometry. 51 shots are modeled with a uniform spacing 100 m ranging from 4000 m to 9000 m. The receiver spread ranges from 4000 m to 9000 m with a 10 m sampling and is fixed for all shots. Similar to the previous example for the scattering model, we also simulate linear-time-delay and random-time-delay blended acquisition geometries. In both cases, all 51 gathers are blended into one super areal shot gather.

The migration and data-space LSI results are shown in Figure 10 and Figure 11. In the inversion results, a horizontal laplacian operator that imposes horizontal continuities of the reflectivity is used as the regularization operator **A**.

For both blended acquisition geometries, migration produces poor images (Figure 10(a) and Figure 11(a)), which are seriously contaminated by crosstalk artifacts. The data-space LSI, on the contrary, successfully removes the crosstalk and we get good reconstruction of the reflectivity in the subsurface (Figure 10(b) and Figure 11(b)).

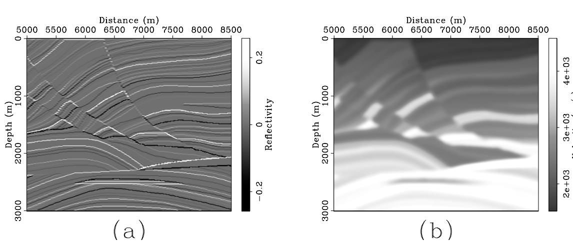


Figure 9: Marmousi model. (a) Reflectivity model; (b) background velocity model.

CONCLUSIONS

We present a method based on LSI to directly image the subsurface using blended data sets. This method does not require any pre-separation of the blended shot gathers, and the crosstalk is effectively removed by formulating the imaging problem as a least-squares inverse problem. The inversion examples on the Marmousi model show that LSI can successfully remove the crosstalk that migration generates and optimally reconstruct the reflectivity in the subsurface.

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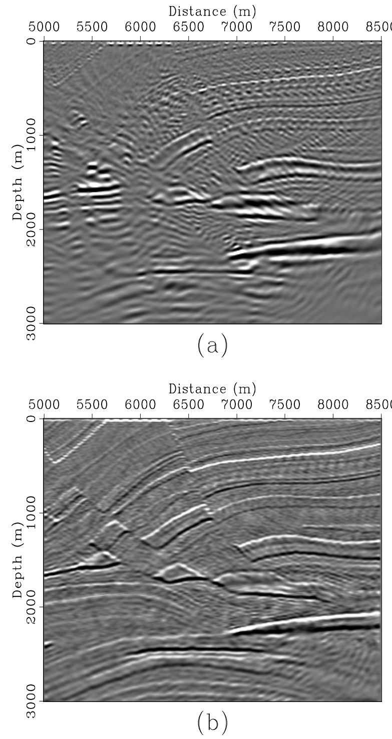

Figure 10: Comparison between (a) migration and (b) data-space LSI for the linear-time-delay blended acquisition geometry.

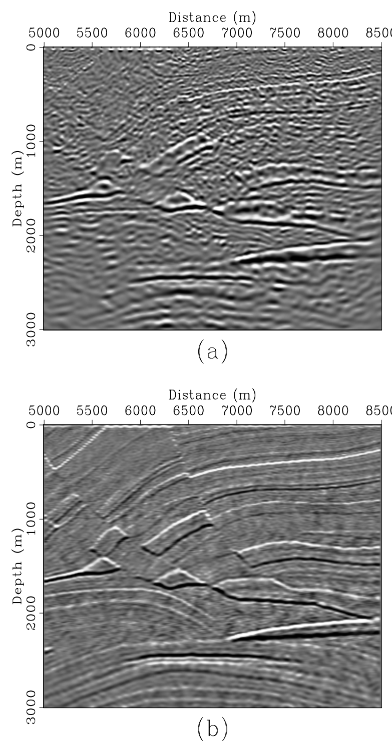

Figure 11: Comparison between (a) migration and (b) data-space LSI for the random-time-delay blended acquisition geometry.

EDITED REFERENCES

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REFERENCES

Akerberg, P., G. Hampson, J. Rickett, H. Martin, and J. Cole, 2008, Simultaneous source separation by sparse radon transform: 78th Annual International Meeting, SEG, 2801–2805.

Ayeni, G., Y. Tang, and B. Biondi, 2009, Preconditioned joint inversion of simultaneous source time-lapse seismic data sets: 79th Annual International Meeting, SEG, submitted.

Beasley, C. J., 2008, A new look at marine simultaneous sources: The Leading Edge, **27**, 914–917.

Beasley, C. J., R. E. Chambers, and Z. Jiang, 1998, A new look at simultaneous sources: 68th Annual International Meeting, SEG, 133–135.

Berkhout, A. J. G., 2008, Changing the mindset in seismic data acquisition: The Leading Edge, **27**, 924–938.

Clapp, M. L., 2005, Imaging under salt: Illumination compensation by regularized inversion: Ph.D. thesis, Stanford University.

Hampson, G., J. Stefani, and F. Herkenhoff, 2008, Acquisition using simultaneous sources: The Leading Edge, **27**, 918–923.

Howard, M. S., and N. Moldoveanu, 2006, Marine survey design for rich-azimuth seismic using surface streamers: 76th Annual International Meeting, SEG, 2915–2919.

Keggin, J., T. Manning, W. Rietveld, C. Page, E. Fromyr, and R. van Borselen, 2006, Key aspects of multi-azimuth acquisition and processing: 76th Annual International Meeting, SEG, 2886–2890.

Michell, S., E. Shoshitaishvili, D. Chergotis, J. Sharp, and J. Etgen, 2006, Wide azimuth streamer imaging of mad dog; Have we solved the subsalt imaging problem?: 76th Annual International Meeting, SEG, Expanded Abstracts, 2905–2909.

Nemeth, T., C. Wu, and G. Schuster, 1999, Least-squares migration of incomplete reflection data: Geophysics, **64**, 208–221.

Romero, L. A., D. C. Ghiglia, C. C. Ober, and S. A. Morton, 2000, Phase encoding of shot records in prestack migration: Geophysics, **65**, 426–436.

Spitz, S., G. Hampson, and A. Pica, 2008, Simultaneous source separation: A prediction-subtraction approach: 78th Annual International Meeting, SEG, Expanded Abstracts, 2811–2815.

Tang, Y., 2008a, Modeling, migration and inversion in the generalized source and receiver domain: SEP-136, 97–112.

———, 2008b, Wave-equation Hessian by phase encoding: 78th Annual International Meeting, SEG, Expanded Abstracts, 2201–2205.

Valenciano, A., 2008, Imaging by wave-equation Inversion: Ph.D. thesis, Stanford University.