Regularized least-squares inversion for 3-D subsalt imaging

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SUMMARY

Obtaining seismic images of the subsurface near and beneath salt is very difficult due to seismic energy that is lost by propagating outside of the survey area or becoming evanescent at salt boundaries (poor illumination). We demonstrate an iterative regularized least-squares inversion for imaging that helps to compensate for illumination problems. We show the use of a regularization operator that acts to regularize amplitudes along reflection angles (or equivalent offset ray parameters) to compensate for the sudden, large amplitude changes caused by poor illumination. This regularization operator has the effect of filling in the gaps created in the reflection angle range due to the lost seismic energy. We demonstrate the use of this regularization operator in an iterative least-squares inversion scheme to improve imaging for a poorly illuminated 3-D seismic dataset.

INTRODUCTION

In an ideal world, a 3-D seismic survey would have infinite extents and dense shot and receiver grids over the entire x-y plane. This would provide the best illumination possible everywhere in the subsurface. In our world, our limited source-receiver geometries allow energy to leave the survey and the density of our shot and receiver arrays depends on the equipment available. For 3-D surveys, the geometry leads to limited azimuth ranges dependent on the direction in which the survey is shot. The limited extents of the 3-D survey will allow seismic energy to escape as it is redirected by subsurface structures, such as salt bodies. These illumination problems make it very difficult to properly image the subsurface near and beneath salt bodies.

Migration techniques try to put the energy recorded in our seismic data back where it belongs in the subsurface. When such energy has not been recorded, due to its escape from the survey area or becoming evanescent (as it may at salt boundaries), the migrated image will have shadow zones (Muerdter et al., 1996) where the signal is weak or non-existent. Even improved migration methods such as downward-continuation migration that generates angle-domain common image gathers (Prucha et al., 1999) cannot properly image such areas.

To improve imaging in areas with poor illumination, we can use migration and its adjoint process to perform an iterative least-squares inversion. Unfortunately, the "missing" data that escaped from the survey can render this process unstable, as it is one reason that the iterative inversion problem may have a null space. The inversion can be stabilized through the addition of regularization (Tikhonov and Arsenin, 1977) that allows us to impose some type of regularization on the resulting image. This regularization can take many forms. In this abstract, we will discuss *geophysical regularization* in which amplitudes are regularized along reflection angles for every point in the subsurface (Prucha and Biondi, 2002; Kuehl and Sacchi, 2001).

We will begin by explaining our scheme for regularized inversion for 3-D seismic data. We will demonstrate this algorithm on a real 3-D dataset from the Gulf of Mexico.

REGULARIZED INVERSION

Seismic migration can be thought of as an operator and therefore used in a conjugate-gradient inversion. We choose a regularized least-squares inversion in which we minimize this objective function:

$$min\{Q(\mathbf{m}) \;=\; ||\mathbf{Lm}-\mathbf{d}||^2+\epsilon^2\;||\mathbf{Am}||^2\}. \qquad (1)$$

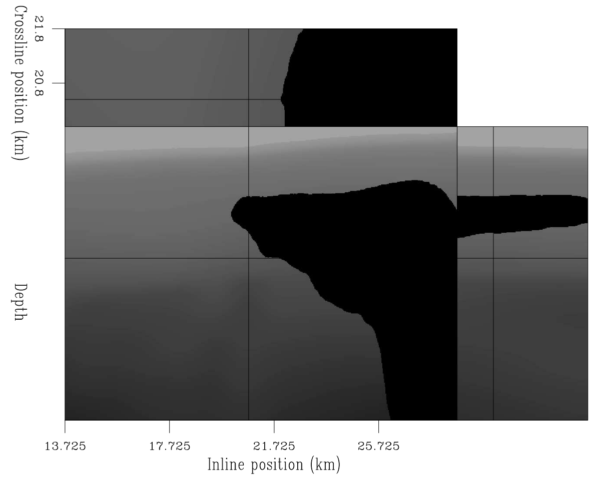

Figure 1: Velocity model for the Gulf of Mexico dataset. It is shown as a flattened cube, with the slices taken from the lines seen on each face of the cube.

Here, $\mathbf{d}$ is the input data and $\mathbf{m}$ is the image obtained through inversion. $\mathbf{L}$ is a linear operator, which may be a migration operator such as downward-continuation migration (Prucha et al., 1999), or in the case of 3-D imaging it can be common azimuth migration (CAM) (Biondi and Palacharla, 1996). $\mathbf{A}$ is a regularization operator. ϵ controls the strength of the regularization.

The first part of equation (1) can be thought of as the "data fitting goal", meaning that it is responsible for making a model that is consistent with the data. The second part is the "model styling goal", meaning that it allows us to impose some idea of what the model should look like using the regularization operator $\mathbf{A}$. The model styling goal helps to keep the solution from diverging due to the null space during the inversion.

We can reduce the necessary number of iterations by preconditioning the model. This incorporates the preconditioning transformation $\mathbf{m}=\mathbf{A}^{-1}\mathbf{p}$ (Fomel and Claerbout, 2003) into equation (1). $\mathbf{A}^{-1}$ is obtained by mapping the multi-dimensional regularization operator $\mathbf{A}$ to helical space and applying polynomial division (Claerbout, 1998). We refer to this inversion scheme as regularized inversion with model preconditioning (RIP).

The choice of regularization operator $\mathbf{A}$ depends on the information we have in the data and our expectations for the resulting model. We know that poor illumination will result in holes in the angle-domain common image gathers (ADCIGs) that are produced by the migration operator. To reduce the effects of this poor illumination, our regularization operator can be designed to penalize large changes in amplitude along the reflection angle (or equivalent offset ray parameter) axis (Prucha and Biondi, 2002). We refer to this operator as *geophysical regularization*.

GULF OF MEXICO DATASET

To experiment with our 3-D regularized least-squares inversion, we required a 3-D dataset with poor illumination and an accurate velocity model. Fortunately, BP and ExxonMobil were able to provide us with such a dataset. The Gulf of Mexico dataset is a real deepwater dataset over a typical GOM salt structure. A flattened cube showing the velocity model can be seen in Figure 1. The velocities are believed to be accurate and range from a water velocity of 1.5 km/s to a salt velocity of greater than 4 km/s. The data itself has an inline common midpoint (CMP) spacing of .025 km, a crossline CMP spacing of .025 km, and an inline offset range from .3 km to 9.4 km sampled every .05 km.

In order to run regularized inversion on this 3-D dataset, we must select an appropriate linear operator **L** and regularization operator **A** for the objective function in equation (1). In this case, we choose to make **L** a common azimuth migration (CAM) operator. This means that we will only be handling offsets in the inline direction. Therefore, the regularization operator **A** can simply be the geophysical regularization previously discussed, acting along the inline offset ray parameter axis of the image. As seen in the 2-D examples of Prucha and Biondi (2002), this regularization will help to reduce artifacts and fill in the shadow zones along the ray parameter axis.

3-D RESULTS

We performed common azimuth migration (CAM) and 7 iterations of regularized inversion with model preconditioning (RIP) on the 3-D dataset. The improvements made by RIP can be best seen by zooming in under the salt. Figure 2 shows the stacked 3-D migration on top and the stacked RIP result after 7 iterations on the bottom. These have been stacked over the inline offset ray parameter axis (equivalent to reflection angle). They are displayed as flattened cubes where a depth slice is shown above an inline section and the crossline section is displayed to the right of the inline section. Oval "A" indicates reflectors that can be followed under the salt nose after imaging with RIP. Ovals "B", "C", and "D" show areas on the inline section where the reflectors can be traced almost entirely through the shadow zones after RIP. In the crossline section, many more reflectors are seen after RIP, particularly in oval "E".

The migration and RIP results for an unstacked inline section can be seen in Figure 3. They are taken at a crossline location halfway across the 3-D volume. It shows a common inline p_h section from $p_h = .1875$ and a common image gather from inline position 20.375 km to the right of the inline section. A depth slice is shown above the inline sections. Comparing these inline sections, the RIP result is considerably cleaner. The effects of the regularization are clear in the common image gather (right part of the figures), where the unlabeled oval indicates gaps in the events that are filled by RIP. In the common inline p_h section, the ovals indicate particular areas of the shadow zones that are being filled in. Oval "A" highlights a reflector that, in the migration result, is discontinuous and has inconsistent amplitudes where it does exist. In the RIP result, this reflector is continuous with strong amplitudes along its full extent. Oval "B" extends across one of the shadow zones. The shadow zone is considerably cleaner in the RIP result, with almost none of the up-sweeping artifacts seen in the migration result. Also, the reflectors themselves extend farther into the shadow zone, particularly on the right side of the oval. The events also extend farther into the shadow zone indicated by oval "C".

CONCLUSIONS

We have explained the use of iterative regularized least-squares inversion for imaging. We demonstrated the use of a geophysical regularization operator for filling in poorly illuminated areas and show its application to a 3-D seismic imaging problem. Although it took several iterations, the seismic image after RIP showed improvements throughout the shadow zones, even after stacking over the inline offset ray parameter axis. The use of the regularization operator is an integral part of using iterative least-squares inversion to improve imaging in poorly illuminated areas.

ACKNOWLEDGMENTS

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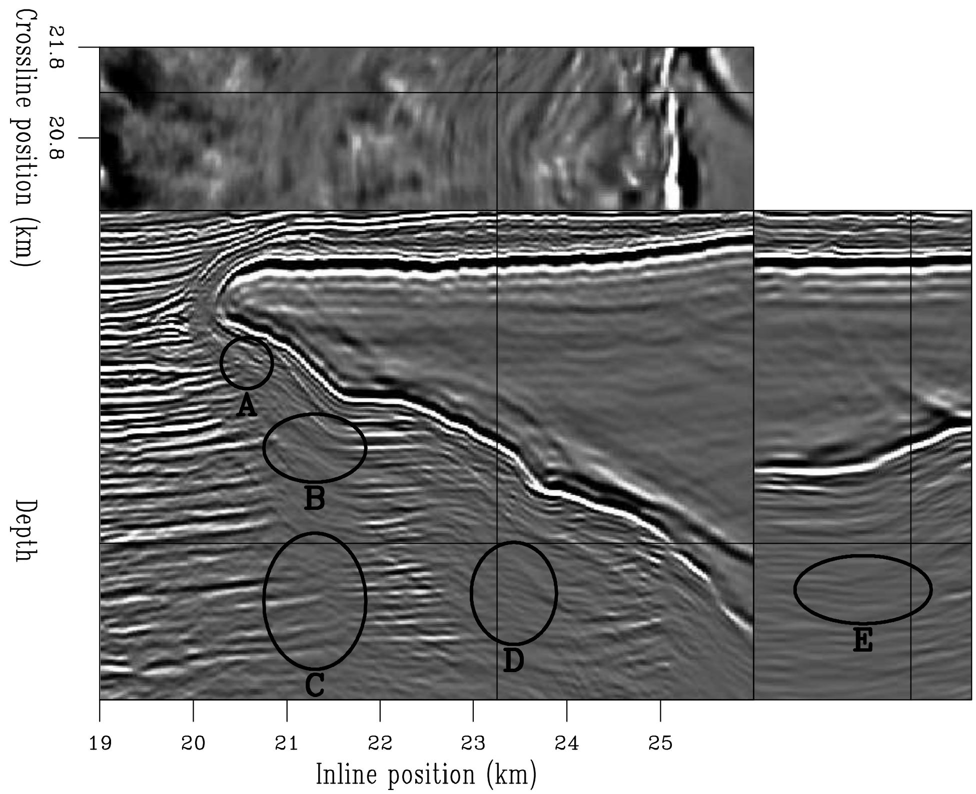

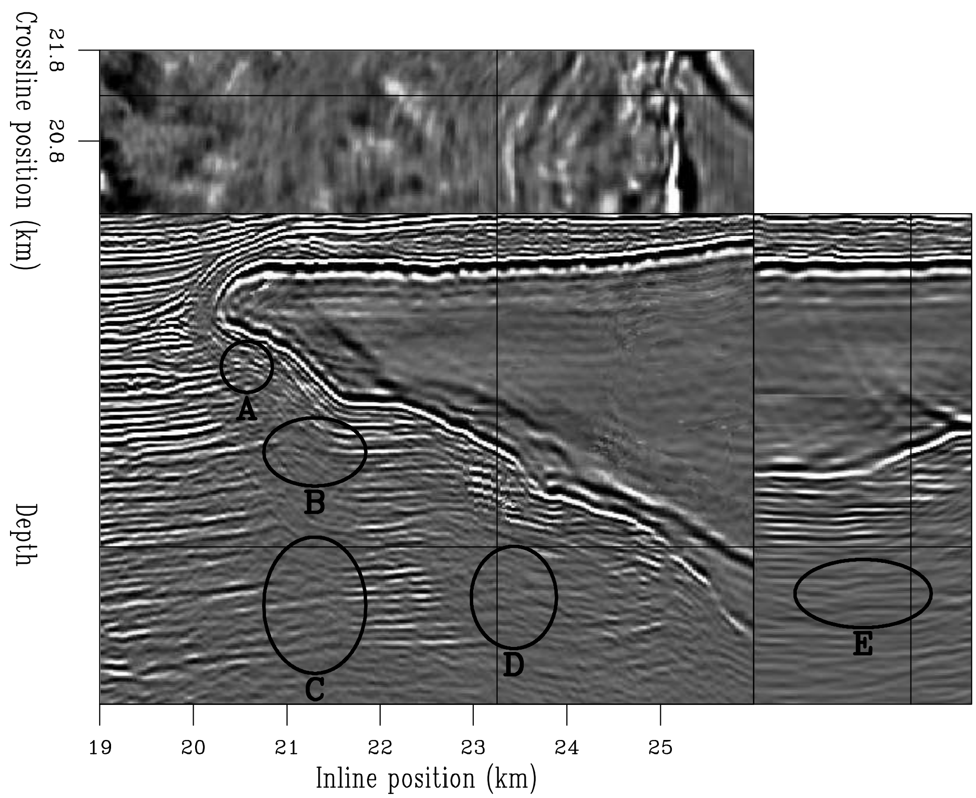

Figure 2: 3-D stacked results. Top: results of CAM. Bottom: result of 7 iterations of RIP.

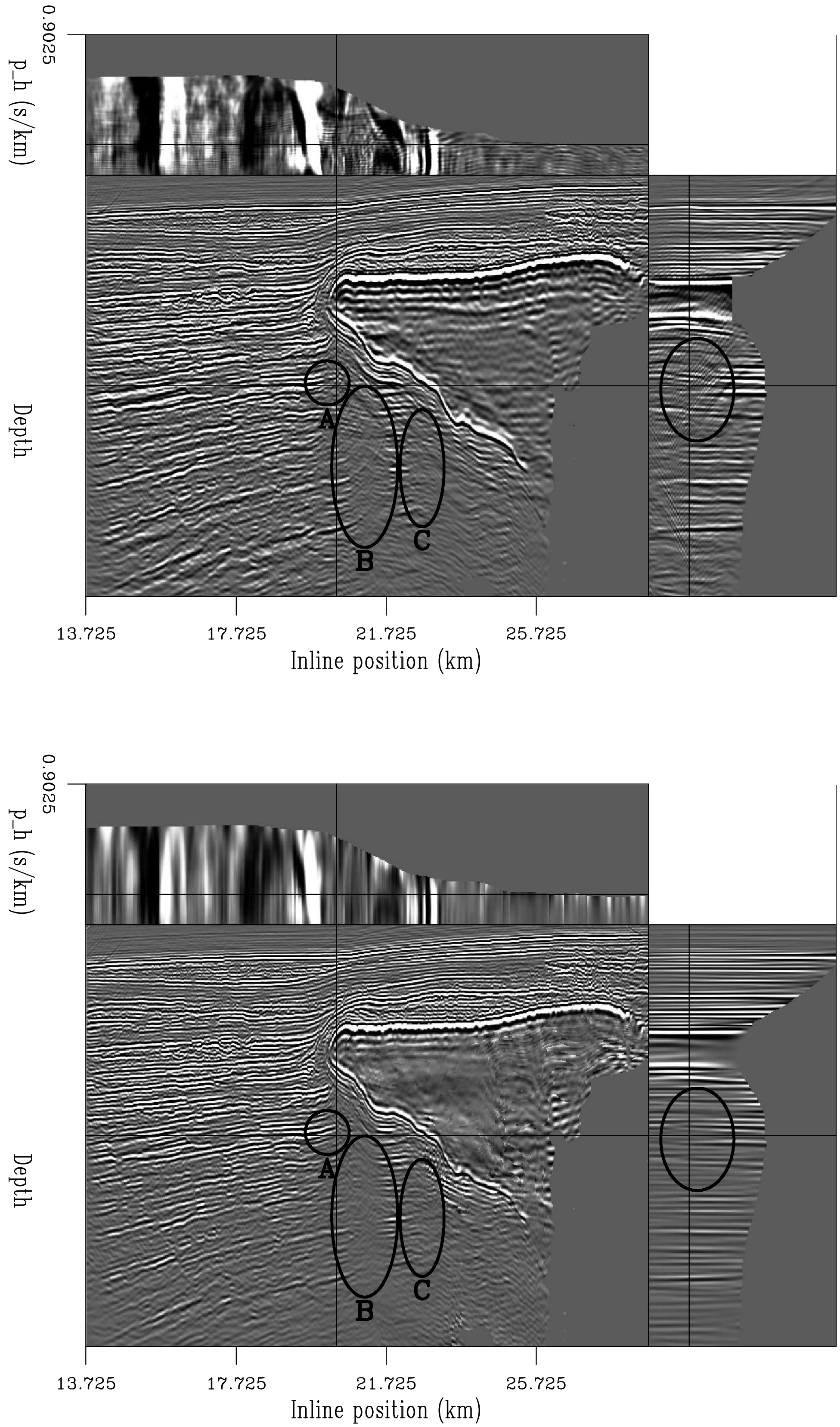

Figure 3: Unstacked inline volumes taken from the 3-D results. Top: CAM result. Bottom: results of 7 iterations of RIP.