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Geologically Constrained Full Waveform Inversion - Theory

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SUMMARY

The waveform inversion problem is inherently ill-posed. Traditionally, regularization terms are used to address this issue.
For waveform inversion, where the model is expected to have many details reflecting the physical properties of the Earth, regularization and data fitting can work in opposite directions, slowing down convergence. In this paper, we constrain the velocity model with a model-space preconditioning scheme based on directional Laplacian filters. This preconditioning strategy preserves the details of the velocity model while smoothing the solution along known geological dips. The Laplacian filters have the property to smooth or kill local events according to a local dip field. By construction, these filters can be inverted and used in a preconditioned waveform-inversion scheme to yield geologically meaningful models. We illustrate on a 2-D synthetic example how preconditioning with non-stationary directional Laplacian filters outperforms traditional waveform inversion when sparse data are inverted for. We think that preconditioning could benefit waveform inversion of real data where (for instance) irregular geometry, coherent noise and lack of low frequencies are present.

Introduction

The goal of waveform inversion is to derive physical properties of the Earth, such as P-wave velocity, S-wave velocity, or density. These properties can be related to the presence of hydrocarbons in the subsurface and their estimation is one of the most important goal in seismic processing. In practice, we try to minimize the function $f(\mathbf{m}) = \|\mathbf{u}_{obs} - \mathbf{u}_{mod}\|^{norm}$ where $\mathbf{m}$ is a vector of physical properties (what we are looking for), $\mathbf{u}_{obs}$ the observed and $\mathbf{u}_{mod}$ the modeled data. Note that f is a 1-D function defined by the choice of a norm. In practice, the ℓ^2 norm is often chosen, but the ℓ^1 norm seems to have more practical needs for its robustness to non-Gaussian noise present in the data [Crase et al. (1990)]. The minimization of $f(\mathbf{m})$ can be achieved using iterative methods. Tarantola (1984) establishes that the model can be updated as follows:

$$\mathbf{m}_{n+1} = \mathbf{m}_n - \alpha_n.\nabla f(\mathbf{m}_n) \tag{1}$$

where $\nabla f(\mathbf{m})$ is the gradient of $f(\mathbf{m})$ and α_n a step-length that needs to be estimated. It turns out that the computation of the gradient is equivalent to a reverse-time migration [Lailly (1984)].

Unfortunately, although a promising approach, waveform inversion is hampered by many issues. The main one is the presence of local minima in f. To circumvent this problem, the data can be inverted in a multi-scale fashion in the time [Bunks et al. (1995)] or frequency domain [Sirgue and Pratt (2004)]. Second, time damping of the input data offer opportunities to focus the inversion on different parts of the data, thus reducing the effects of local minima [Brenders et al. (2009)].

Traditionally, ill-posed problems can be solved by adding a regularization term to the objective function. Very often, a regularization term that can penalize differences between neighboring points is selected. However, whereas waveform inversion tends to add details to a velocity model, regularization tends to smooth them out, thus working against our primary goal: fitting the data. One way to address these somewhat conflicting goals is to use preconditioning. Here, we show how we can geologically constrain the velocity model by using a non-stationary preconditioning approach. This method requires two ingredients: a dip estimation method and a local dip filtering technique. We use the method of Fomel (2002) for the former and of Hale (2007) for the later.

In this paper we first introduce the waveform inversion approach, with and without preconditioning. We show that preconditioning amounts to a simple change of variable which, in effect, changes the gradient direction. Then, we present our method of local dip filtering, which follows Hale's. Finally, we present synthetic results on a modified version of the Marmousi model. These results demonstrate that non-stationary, preconditioned inversion yields geologically plausible models.

Method

In this paper, we use a time domain approach for solving the scalar acoustic wave equation (parametrized in terms of P-wave velocity v_p):

$$\frac{\partial^2 u(\mathbf{x},t)}{\partial t^2} - v_p(\mathbf{x})^2\nabla^2 u(\mathbf{x},t) = v_p(\mathbf{x})^2 s(\mathbf{x},t). \tag{2}$$

with the source term $s(\mathbf{x},t) = \delta(\mathbf{x}-\mathbf{x}_s)S(t)$ where $S(t)$ is the source function at $\mathbf{x}_s$ and u the pressure field. Tarantola (1984) gives the definition of the gradient for the acoustic equation (2) for each component of m (equal to v_p only in this case).

$$\nabla f(\mathbf{m}_n) = \frac{2}{\mathbf{v_p}_n^3}\sum_{shots}\sum_t \frac{\partial^2 \overrightarrow{\mathbf{u}_n}}{\partial t^2}\cdot \overleftarrow{\delta\mathbf{u}_n} \tag{3}$$

where $\overleftarrow{\delta\mathbf{u}_n}$ is the *backward* propagated residual at iteration n such that $\delta\mathbf{u_n} = \mathbf{u}_{obs} - \mathbf{u}_n$ and $\overrightarrow{\mathbf{u}_n}$ is the *forward* propagated synthetic source. For our iterative method, we opted for the L-BFGS approach of

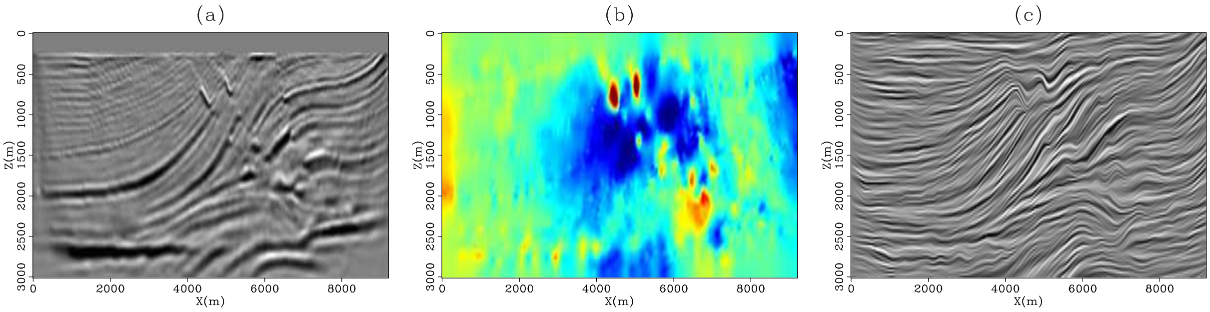

Figure 1 *Illustration of the preconditioning operator* $\mathbf{S}$*. We first migrate the synthetic Marmousi dataset in (a). (b) shows the dip field estimated from the migration result in (a). Applying* $\mathbf{S}$ *estimated from (b) to a random field shows in (c) the texture of the migrated section in (a).*

Nocedal (1980) where the model is updated as follows:

$$\mathbf{m}_{n+1} = \mathbf{m}_n - \alpha_n \mathbf{H}_n^{-1} \nabla f(\mathbf{m}_n), \tag{4}$$

where $\mathbf{m}_{n+1}$ is the updated solution, α_n the step length computed by a line search that ensures a sufficient decrease of $f(\mathbf{m})$ and $\mathbf{H}_n \approx \nabla^2 f(\mathbf{m}_n)$ the approximate Hessian. To improve chances of not falling into a local minimum, we follow a multi-scale approach [Bunks et al. (1995)] where the source and data are bandpassed prior to inversion.

Preconditioning

Preconditioning amounts to a change of variable $\mathbf{m} = \mathbf{S}\mathbf{p}$ where $\mathbf{p}$ is the new variable and $\mathbf{S}$ an operator (non-stationary deconvolution with local dip-filters). With the new variable $\mathbf{p}$, the iterative scheme in Equation (4) is modified as follows:

$$\mathbf{p}_{n+1} = \mathbf{p}_n - \alpha_n \tilde{\mathbf{H}}_n^{-1} \nabla \tilde{f}(\mathbf{p}_n), \tag{5}$$

where

$$\nabla \tilde{f}(\mathbf{p}_n) = \mathbf{S}' \nabla f(\mathbf{m}_n) = \mathbf{S}' \nabla f(\mathbf{S}\mathbf{p}_n) \tag{6}$$

and $\mathbf{S}'$ is the transpose of $\mathbf{S}$. Therefore, with preconditioning, we obtain a new gradient direction which will steer our result toward a geologically constrained solution. Note that in Equation (5) the approximate Hessian is blind to this change of variable as it is built from gradient and solution step vectors. Assuming that we can find $\mathbf{S}$ and compute its adjoint and inverse (to accommodate any starting guess $\mathbf{p}_0 = \mathbf{S}^{-1}\mathbf{m}_0$), preconditioning can be easily introduced in any waveform inversion scheme. Now we present our choice of preconditioning operator $\mathbf{S}$.

Defining the preconditioning operator S

As stated above, we follow the approach of Hale (2007) for the construction of the operator $\mathbf{S}$. This operator is a non-stationary deconvolution with directional Laplacian filters. Directional Laplacian filters are built from small wavekill filters $\mathbf{A}$, similar to those of Claerbout (1995). From these filters, new operators $\mathbf{A}'\mathbf{A}$ are formed by autocorrelation. These new operators are then factorized into minimum-phase filters $\tilde{\mathbf{A}}$ such that $\tilde{\mathbf{A}}'\tilde{\mathbf{A}} \approx \mathbf{A}'\mathbf{A}$. Having minimum-phase filters, we can build a stable non-stationary deconvolution operator $\mathbf{S} = \tilde{\mathbf{A}}^{-1}\tilde{\mathbf{A}}'^{-1}$ and its inverse $\mathbf{S}^{-1} = \tilde{\mathbf{A}}'\tilde{\mathbf{A}}$. The wavekill filter $\mathbf{A}$ is dip dependent. In practice, we estimate and use these filters as follows: first, we estimate a dip field following the approach of Fomel (2002); then we estimate a bank of directional Laplacian filters for a range of angles; finally we apply the appropriate inverse Laplacian filter on each sample according to the local dip.

To illustrate the preconditioning operator $\mathbf{S}$, we show in Figure 1a the migration result of a synthetic dataset based on the 2-D Marmousi model. This result is obtained with Reverse Time Migration (RTM).

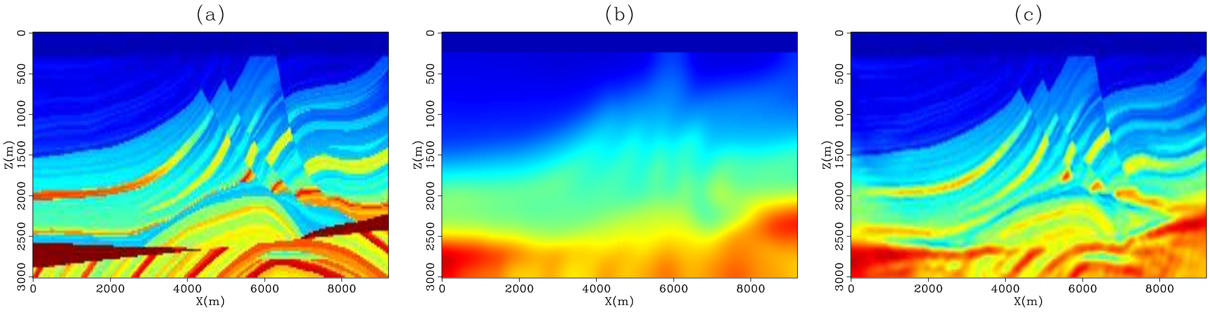

Figure 2 *(a) True velocity model used to generate the synthetic dataset. (b) Initial guess obtained by smoothing the true model in (a). (c) Estimated model after waveform inversion. No preconditioning is applied in this case.*

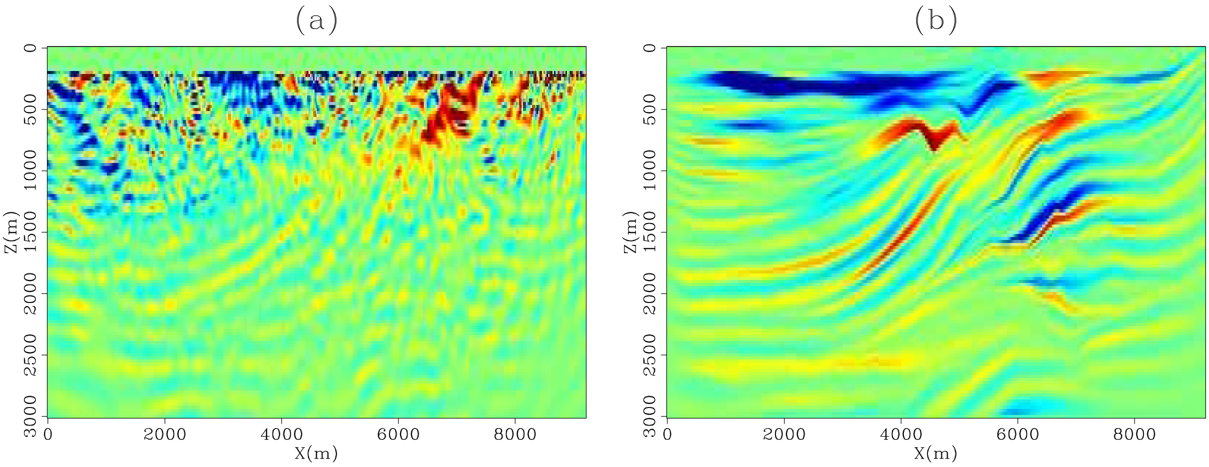

Figure 3 *(a) Gradient $\nabla f(\mathbf{m}_n)$ of the unpreconditioned inversion after 4 iterations of the 0-8Hz scale (4 shots, 2.5 Km. apart). (b) Reprojected gradient $\mathbf{S}\nabla \tilde{f}(\mathbf{p}_n)$ of the preconditioned inversion after 4 iterations of the 0-8Hz scale (4 shots, 2.5 Km. apart). With preconditioning, the gradient follows the information captured in the dip field and looks more geologically appealing than in (a).*

From this image, we can estimate the local dip field (Figure 1b). This dip field is obtained iteratively with some smoothing. Now, we apply the operator $\mathbf{S}$ to a random field the size of the migrated image in Figure 1a to obtain Figure 1c: it shows the texture of the image clearly. In the next section, we demonstrate that this operator can be used to constrain the solution of waveform inversion.

Examples

We illustrate the geologically-constrained waveform inversion method on a synthetic dataset. We modified the Marmousi 2-D velocity model by adding a 250 m. thick water layer (Figure 2a). We created 184 shots 50 m. apart with a fixed receiver array (369 in total) at the surface using the scalar wave equation (no density). The source is a Ricker-2 wavelet with a fundamental frequency of 8Hz. To illustrate that our inversion works (without preconditioning), we show in Figure 2c the result of waveform inversion with four frequency scales (0-4Hz,0-8Hz,0-12Hz,0-16Hz) using the starting guess in Figure 2b (obtained by smoothing the true model in Figure 2a) and using all the shots. There is a very good match between the inverted and true model. Because all the data was used, little would be gained by using preconditioning.

To make a compelling case, we kept only four shots, 2.5 Km. apart. First, we show in Figure 3 a comparison between the gradient without preconditioning $\nabla f(\mathbf{m}_n)$ and the gradient with preconditioning back in the velocity space $\mathbf{S}\nabla \tilde{f}(\mathbf{p}_n)$. Because only 4 shots are present, the unpreconditioned gradient looks

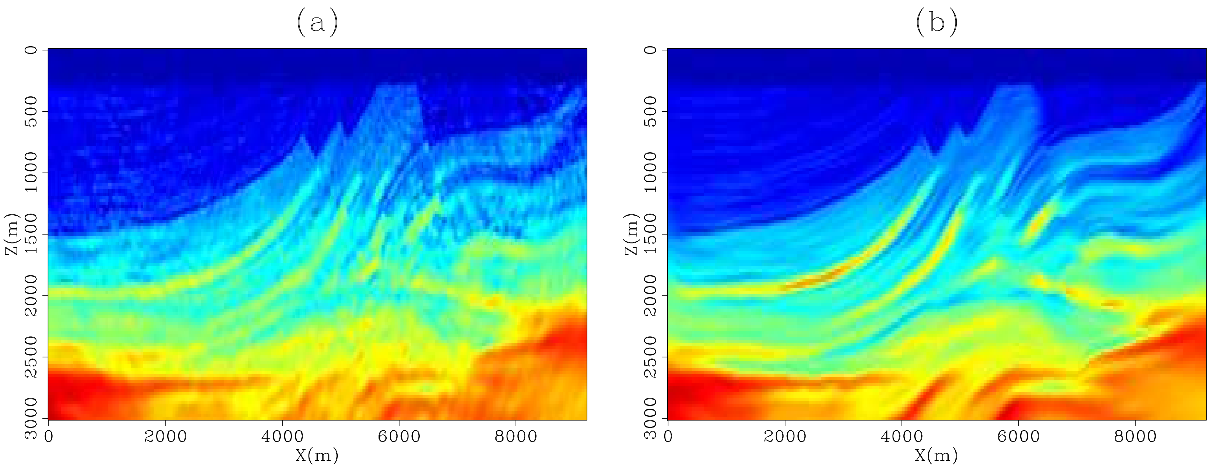

Figure 4 *(a) Inversion result for the unpreconditioned inversion. (b) Inversion result for the preconditioned inversion with directional Laplacian filters. The geology at the reservoir level is recovered very well in (b). The inversion result in (a) looks fairly good considering the number of shots: the multi-scale approach, together with the low-frequency content of the data, act as natural regularization operators.*

noisy and resemble geology in very few locations only. On the contrary, thanks to the preconditioning with directional Laplacian filters, the reprojected gradient in Figure 3b shows the geology captured in the dip field of Figure 1b very well.

Now, we show in Figure 4 the inversion results for the unpreconditioned (Figure 4a) and preconditioned inversion (Figure 4b). Although quite noisy, the unpreconditioned result shows the geology very well: the presence of low frequencies in the data, along with the multi-scale approach, act as regularization operators. This effect will be less pronounced with real data where low frequencies are often missing. The preconditioned inversion result in Figure 4b is much cleaner: the geology at the reservoir level is recovered very well.

Conclusions

Preconditioning waveform inversion with non-stationary directional Laplacian filters can yield geologically meaningful velocity models. It can help decrease artifacts due to acquisition geometry or inconsistencies in the data (not shown here). We anticipate that preconditioning can play a bigger role with real data where low frequencies are often lacking, where data are noisy and where the acquisition geometry is irregular.

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