Interpolation has become of more importance recently, largely due to increased reliance on algorithms that require dense and regular data sampling, such as wave-equation migration and 3D surface-related multiple elimination (SRME). (van Dedem and Verschuur, 2005) Examples of current methods include Fourier (Liu and Sacchi, 2005), Radon transform (Trad, 2004), and filter-based (Spitz, 1991; Crawley, 2000) methods. Other methods that rely on the underlying physics (and typically also a velocity model) include migration/demigration (Pica et al., 2005), DMO-based methods (Biondi and Vlad, 2002), and the focal transform (Verschuur and Berkhout, 2005), which requires an input focal operator instead of velocity.

In this paper, we describe a hybrid approach that combines both non-stationary prediction-error filters (Crawley, 1999) and pseudo-primaries generated from surface-related multiples (Shan and Guitton, 2004) in order to interpolate missing near offsets. The pseudo-primaries are generated by a surface-consistent cross-correlation of a multiple model with the input data. Once the pseudo-primaries have been generated, a non-stationary prediction-error filter (PEF) is estimated on the pseudo-primaries by solving a least-squares problem. A second least-squares problem is then solved where the newly found PEF is used to interpolate the missing data (Claerbout, 1999).

The data used in this example is from the Sigsbee2B synthetic dataset with the first 2000' of offset were removed. Near-offset data is typically missing from marine data, and large gaps can exist when undershooting obstacles like drilling platforms. Estimating a PEF on the original (missing) data produces an ideal reconstruction. Estimating a PEF on the pseudo-primaries, which are generated without the near offset data, gives promising results, which can be quality controlled with the output of the convolution of the pseudo-primary-derived PEF with the recorded data.

Pseudo-primaries from Multiples

Pseudo-primaries (W) can be generated by computing

$$W\left(x_p, x_m, \omega\right) = \sum_{x_s} M\left(x_s, x_m, \omega\right) \overline{P}\left(x_s, x_p, \omega\right), \qquad (1)$$

where W is the pseudo-primary data, ω is frequency, x_s is the shot location, x_p is the surface location, $\overline{P}$ is the complex conjugate of the original trace at (x_s, x_p) and M is the multiple reflection recorded at x_m. In this equation, the cross-correlation of the first-order multiples in M with the primaries and first-order multiples in $\overline{P}$ produces primaries and zero-lag components, respectively. Cross-correlation of the second-order multiples in M with the primaries, first-order, and second-order multiple reflections in $\overline{P}$ produces first-order multiples, primaries, and zero-lag components, respectively. With higher orders of multiples this trend continues.

Pseudo-primaries generated in this fashion contain subsurface information that would not be recorded with a non-zero minimum offset. One example of this is a first-order multiple that reflects at the free surface within the recording array, resulting in near offsets being recorded when that wave returns to the surface. An example of this is shown in Figure 1.

Figure 1 (a) is a single Sigsbee 2B shot, including the negative offsets that are predicted by reciprocity, but with offsets less than 2000' removed. Figure 1(b) is the corresponding pseudo-primaries for the same area, generated in part with Figure 1(a). We can see where the first and second-order multiples in P map to in the zero-lag at the top of the image. We can also see a lot of near-offset information present in the pseudo-primaries that is not present in the recorded primaries. However, simply replacing the missing near offsets of the primaries with the corresponding pseudo-primaries would not yield a satisfactory result due to the crosstalk and noise in the pseudo-primary shot.

Interpolation with non-stationary prediction-error filters

Interpolation can be cast as a series of two inverse problems where a prediction-error filter is estimated on known data and is then used to interpolate missing data. A prediction-error filter (PEF) can be estimated by minimizing the output of convolution of known data with an unknown filter (except for the leading 1), which can be written in matrix form as

$$\min_f \left\| \begin{bmatrix} d_7 & 0 & 0 \\ d_6 & d_7 & 0 \\ d_5 & d_6 & d_7 \\ d_4 & d_5 & d_6 \\ d_3 & d_4 & d_5 \\ d_2 & d_3 & d_4 \\ d_1 & d_2 & d_3 \end{bmatrix} \begin{bmatrix} 1 \\ f_1 \\ f_2 \end{bmatrix} \right\|^2 \qquad (2)$$

where f_i are unknown filter values and d_i are known data values.

The filters used in this paper are all multidimensional, which is implemented by the helical coordinate (Claerbout, 1998). In the case of a stationary multidimensional PEF, this is an over-determined least-squares problem with a unique solution.

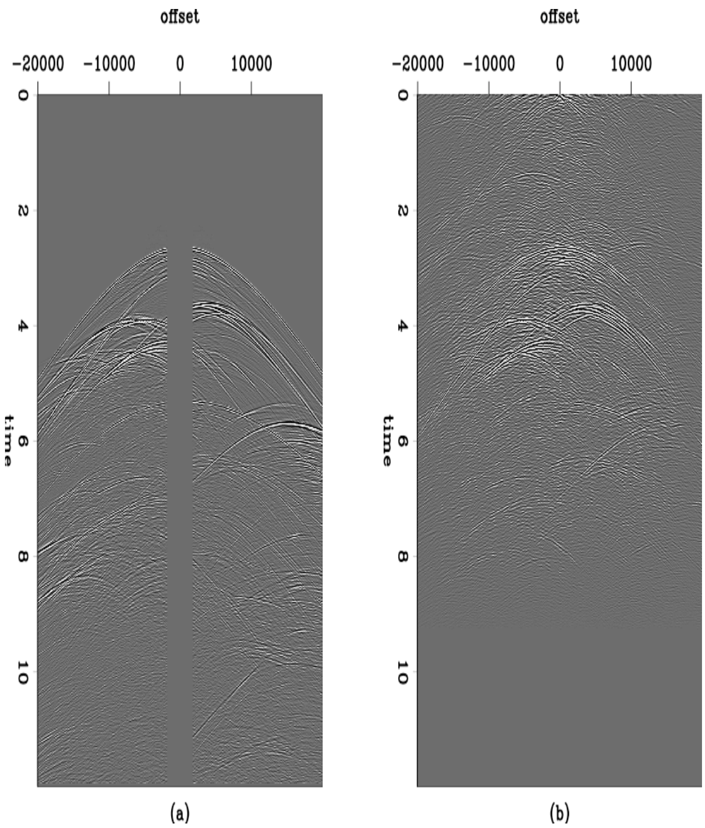


Figure 1. (a) Original shot record and (b) pseudo-primaries for the same area from the Sigsbee2B dataset.

Seismic data is non-stationary in nature, so a single stationary PEF is not adequate for the many changing dips present. We estimate a single spatially-variable nonstationary PEF and solve a global optimization problem (Crawley, 1999, Guitton, 2003). In that case the problem is now underdetermined, and a regularization operator is introduced to the least-squares problem (in matrix notation) so that

$$\min_{\mathbf{f}} \left\| \mathbf{DKf} + \mathbf{d} \right\|^2 + \varepsilon_f{}^2 \left\| \mathbf{Af} \right\|^2, \qquad (3)$$

where **D** represents non-stationary convolution with the data, **f** is now a non-stationary PEF, **K** (a selector matrix) and **d** (a copy of the data) both constrain the value of the first filter coefficient to 1, **A** is a regularization operator (a Laplacian operating over space) and ε_f is a tradeoff parameter for the regularization. Solving this system will create a smoothly non-stationary PEF.

Once the PEF has been estimated, it can be used in a second least squares problem, that matches the output model to the known data while simultaneously regularizing the model with the newly found PEF,

$$\min_{\mathbf{m}} \left\| \mathbf{Sm} - \mathbf{d} \right\|^2 + \varepsilon_m{}^2 \left\| \mathbf{Fm} \right\|^2, \qquad (4)$$

where **S** is a selector matrix which is 1 where data is present and 0 where it is not, **F** represents convolution with the non-stationary PEF, ε_m is now a trade-off parameter and **m** is the desired model.

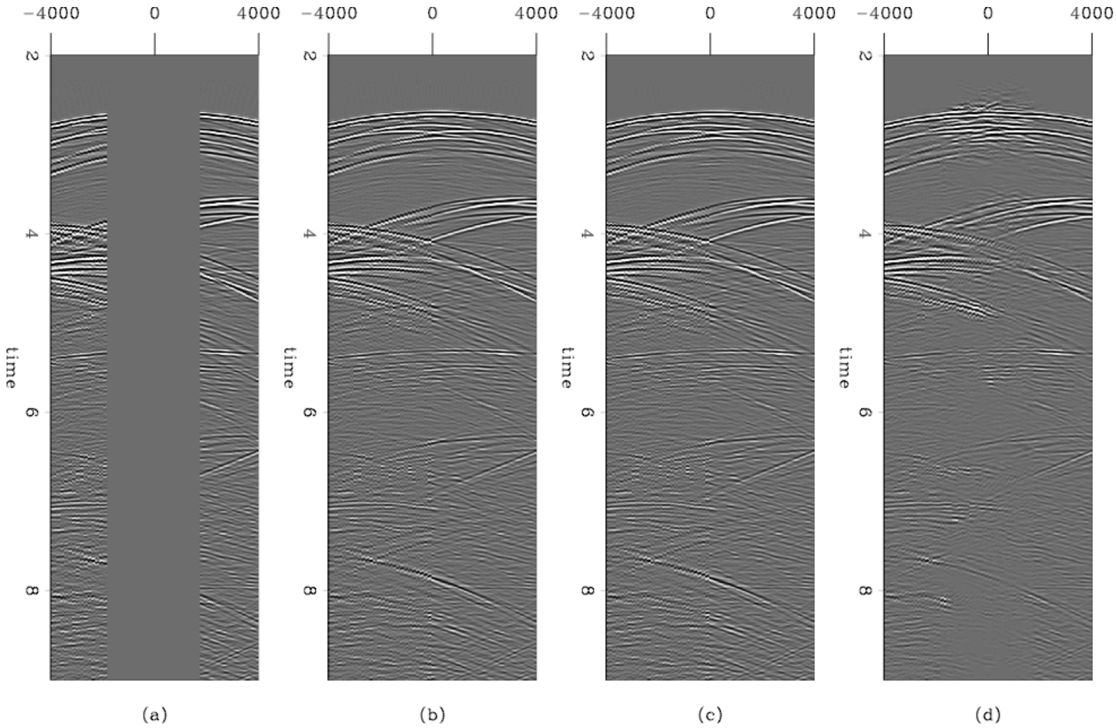
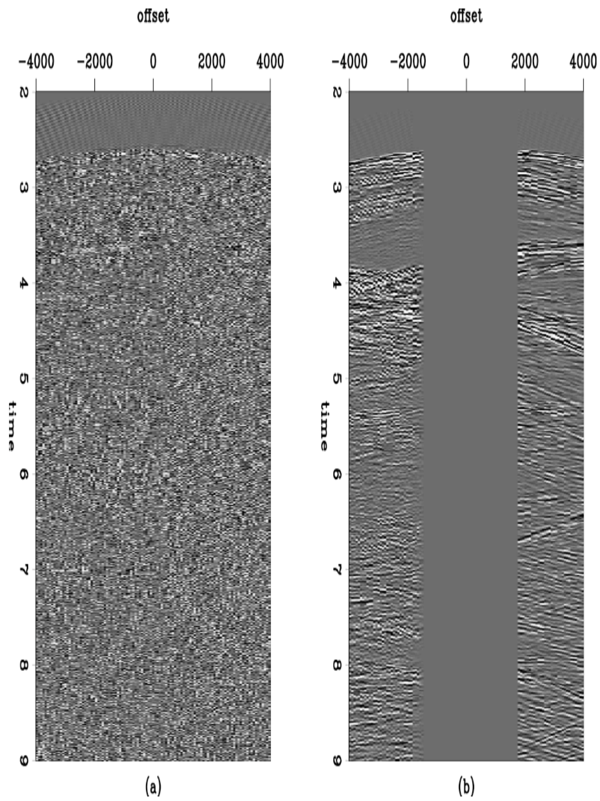


Figure 2. (a) Original data with near offsets (<2000') missing. (b) Original complete data. (c) Interpolation with PEF based upon complete data. (d) Interpolation with PEF based upon pseudo-primaries.



Results

To interpolate by an integer factor, PEFs are estimated on the input data, which will not work for larger gaps. Instead, we estimate the PEF on the pseudo-primaries with equation 2 and then use that PEF to interpolate the recorded data with equation 3. The results of this experiment are shown in Figure 2.

The near offset gap is 4000' or 53 traces, as shown in Figure 2(a). The complete data in Figure 2(b) is used as input to equation 3 with Figure 2(a) as input to equation 4, which produces Figure 2(c), which illustrates that if we estimate the PEF on the answer we can perfectly recreate the data. Figure 2(d) shows the main result of this paper, which is when the pseudo-primaries are used as input to equation (2).

Figure 3. (a) PEF estimated on pseudo-primaries convolved with pseudo-primaries. (b) PEF after convolution with input data.

The results in Figure 2(d) are promising, but not ideal. Most of the events are successfully continued through the data, but some interference is present. One method to quality control the PEF estimation is to examine the residual of equation 3. If this result is uncorrelated and low in amplitude, the PEF has captured all of the useful information in the pseudo-primaries. Similarly, the PEF can then be convolved with the recorded data, with the output shown in Figure 3.

Figure 3(b) shows that while the PEF estimated on the pseudo-primaries does a good job of whitening the recorded data, the result is not perfect. Differences in spectral content of the data are the most obvious cause, and any adjustments to this algorithm can be quality controlled by looking at this intermediate result.

Conclusions and Future Work

Incorporating pseudo-primary data into a non-stationary prediction-error filter based interpolation method gives promising results for large gaps in the near offset. While most interpolation algorithms suffer from an objective measure of the quality of interpolation in practical applications, the usefulness of the pseudo-primaries can be judged in a relatively objective manner by looking at the convolution of the pseudo-primary based PEF with the recorded data. Future work includes reducing this difference and examining the final results after SRME compared to a high-resolution parabolic radon transform.

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